Integrating and substituting the limits of integration:

$$u_{avg} = \frac{1}{12} V_\delta E \tag{33}$$

Thus, equations (28) and (33) provide the necessary relationship for the boundary layer thickness δ and the average velocity u_{avg}.

It can be shown (5) that the angular cage velocity ω in terms of the rpm of the inner race is:

$$\omega = \frac{\sin \beta}{\sin \alpha + \sin \beta} \times \frac{2\pi}{60} \times N \tag{34}$$

Hence

$$u_{avg} = \frac{D_w E N \pi}{720} \frac{\sin \beta}{\sin \alpha + \sin \beta} \tag{35}$$

and

$$\delta = \frac{3.09\delta_1}{R_1} \left(\frac{\nu(\sin \alpha + \sin \beta)}{N \sin \alpha \sin \beta} \right)^{1/2} \tag{36}$$

where E and δ_1 are given in Fig. 5 as determined by the numerical solution of the momentum integrals (equations (29) and (30)).

In the previous derivation the moment of momentum (equation (9)) is actually a measure of the torque, which is necessary to propel the oil through the bearing (viscous torque). In reference [3] it is shown that equation (9) can be expressed as:

$$T = \frac{-2\mu V_\delta r_w^2}{\delta} d\theta dx \tag{37}$$

Substituting equation (12) in equation (37) above:

$$T = \frac{-2\mu\omega}{\delta} r_w^3 d\theta dx \tag{38}$$

Substituting equations (13) and (28) into equation (38) and recalling the relationships $R_w = R_0 + x$ and $r_0 = R_0 \sin \alpha$ from Fig. 4, one obtains the integral:

$$T = \frac{-2\rho\nu^{1/2}\omega^{3/2} (\sin \alpha)^{7/2}}{\delta_1 R_0} \int_0^{2\pi} \int_0^l (R_0 + x)^4 d\theta dx \tag{39}$$

Equation (39) is the torque acting on the oil to pump it through the bearing. The reaction torque is the viscous torque acting on the bearing and is the same except that it has a positive value. Hence, integrating, the final viscous torque equation is (note for total viscous torque $x = l$):

$$T = {}^4/_5 \frac{\pi\rho\nu^{1/2}\omega^{3/2} (\sin \alpha)^{7/2}}{\delta_1 R_0} [R_w^5 - R_0^5] \tag{40}$$

The expression for viscous torque given in equation (6) is obtained by substituting equation (34) into equation (40).

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DISCUSSION

R. J. Gross[2]

In Appendix I of this paper, Bernoullis' equation (equation (15)) is used to derive equation (17) which implies a negative radial pressure gradient in the potential flow region. However, the linear momentum equation gives a positive radial pressure gradient. A positive gradient can be seen to be correct as an element of fluid moving in a circular path requires a larger pressure on its outer surface then on its inner surface to keep it moving in a circular path. This leads one to suspect that Bernoullis' equation is not valid for this potential flow region. Since Bernoullis' equation is valid only for a streamline or for an irrotational flow field and the velocity distribution given by equation (12) describes a rotational flow, Bernoullis' equation is not appropriate. The result is that the pressure gradient dp/dx given by equation (19) is positive rather than negative and another explanation must be given to explain why oil in the tapered bearing flows from the small end to the large end of the roller or against an opposing pressure.

The velocity distribution after equation (31) should be attributed to reference [3] rather than reference [2].

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Author's Closure

The comment of Dr. Gross concerning the sign of the pressure gradient dp/dx is correct. However, if this change is made the revised equations (29) and (30) can no longer be solved if the present boundary conditions are used. The reason for this is if the pressure gradient along the boundary layer (cup surface) is positive (increasing) then the axial velocity u must be decreasing. But since the boundary condition at $x = 0$ is $u = 0$ this is impossible. Therefore, if the positive pressure gradient along the cup surface is to be applied to the proposed model then another hypothesis is required to explain the initial velocity and thickness of the boundary layer at the entry region of the bearing. A study of the cage flange rotation at the small end side of the bearing may provide this needed boundary condition. If such a boundary condition could be developed the fundamental equations in the paper would still apply. The only changes would be in the values for the flow proportionality factors given in Fig. 5.

The author has one additional comment to make concerning the viscous torque formulation. In the derivation of equation (40) δ_1 was assumed to be a constant. If δ_1 is considered a function of R_1, which it really is, then equation (40) becomes,

$$T = 4\pi\rho\nu^{1/2}\omega^{3/2}(\sin \alpha)^{7/2}R_0^4 \int_1^{R_w/R_0} \frac{R_1^4}{mR_1 + b} dR_1$$

if $\delta_1 = f(R_1)$ (Fig. 5) is approximated with a series of straight lines. If one evaluates this integral it also suggests that the boundary condition $\delta_1 = 0$ at $R_1 = 1$ $(x = 0)$ is not appropriate since the viscous torque would then become infinite if the present boundary conditions are used.

Therefore, it is the author's opinion that additional refinements must be made to the proposed oil flow model. In particular, the boundary conditions at the entry region must be re-evaluated based on consideration of the cage flange rotation.

The author would like to thank Dr. Gross for the very constructive criticism of the model presented in this paper as I am sure it will eventually lead to a better understanding of the oil flow phenomenon in a tapered roller bearing.