

## **Eutrophication Models for Lakes**

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In the years 1970-1977 eutrophication models have been developed and applied for about 15 shallow Danish lakes. The structure of these models are discussed. An example of the calibration of the model on data from Lake Gyrstinge in the Suså catchment area is shown, and the use of the model for calculating ecological consequences of changed hydraulic retention times is demonstrated. The dynamic eutrophication model is compared to the Dillon-Rigler version of the input-output model. Limitations of both types of models are discussed.

### **Introduction**

Eutrophication models of lakes are developed for several purposes. On one hand the models are used as research tools to establish a further understanding of complex interactions between physical/chemical/biological processes and to provide guidelines for fruitful laboratory and field experiments. On the other hand models are used as management tools to provide the necessary background for decisions on the abatement of eutrophication problems.

The allocation of resources towards development of models in different countries depends on the general type of waste water planning policy which is used in the country. In some countries a Best Technical Means, BTM, policy is put forward. In this case the decisions are based on standards of quality for the

discharged water, and the demand for models to describe the processes in the receiving water body is low. In other countries a receiving water quality policy is put forward, i.e. a set of quality standards for the receiving water concerned is outlined. This set of standards is the background for defining the standards of quality for the discharged waste water. The definition of the discharged standards is based on a calculation of biological/chemical consequences of proposed standards by means of some sort of calculation tool, which may be a simplified tool, e.g. a surface load ratio or more advanced tool, e.g. a eutrophication model.

In Denmark the receiving water approach is preferred. A primary reason is, that this approach is cost-efficient. A given quality of a receiving water body is achieved by a lesser amount of resources by this policy than by the BTM policy.

### **Development of Lake Models**

Internationally the work on lake models have been concentrated on two model types. One of these types is the input-output oriented models, which based on measurements of loadings of a lake with phosphorus and nitrogen, calculate some lake parameters (e.g. retention coefficient) on a statistical/empirical basis. The values of these parameters are then used for calculating consequences of changed loadings on concentrations of the substances in view. The classical models of this type are the models of Vollenweider (1969) and the later extended versions by Dillon and Rigler (1974). Generally the models are limited in their scope by several factors.

Firstly, the models are stationary, i.e. they cannot take yearly variations of loadings and parameter values into account. Secondly, in many cases the models do not describe the biological variables which are of main interest, and what is very important, do not take into account the influence of biological processes on the model parameters.

Thirdly, because of the lack of description of biological processes important interactions between several phytoplankton limiting factors, light, phosphorus, nitrogen are not taken into account.

The advantage of some of the simplified versions of these input-output models is that a »rule of thumb« calculation based on these models can be carried out in a few hours.

The other type of models, which has been developed internationally is the eutrophication model which is dynamic, i.e. includes time dependent variations and takes important biological/chemical processes into account. Pioneers of these developments are Chen (1970), Di Toro et al. (1971), and Scavia et al. (1976). The first two authors have used their models in a management context, whereas Scavia et al. primarily have developed a research orientated modelversion.

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In the nordic countries work on ecological models was initiated at the Askö Laboratory in Stockholm (Sjøberg et al. 1972). The scientists of this laboratory emphasized modelling of the Baltic ecosystems, however, the problems of these ecosystems were approached by the same basic methods as eutrophication models for lakes. In Sweden an eutrophication model for lake Norrviken has been developed by I. Ahlgren (1974). This model has been used for describing the consequences of a removal of waste water discharge from lake Norrviken and especially the role of the sediments in the transition period. The model developed by Ahlgren is one of the few existing models which takes into account the yearly succession of the three groups of phytoplankton species (diatoms, green, and bluegreen).

In 1976 a work on lake modelling has been initiated in Finland by the National Water Board. The lake in view is Lake Päijänne and the EPA-ECO model, which has been used for lake investigation in USA, is applied on the lake.

In general the main emphasis on eutrophication modelling for lakes among the nordic countries has been placed in Denmark. In Fig. 1 are outlined the developments and interactions of the research- and management-orientated lake models in Denmark.

The work on research-orientated lake models has primarily been catalysed by the Joint Lake Research Project (JLRP) in which several basic and applied research institutions are cooperating to gain understanding of lake eutrophication problems and possibly to model a number of complex aspects of the eutrophying processes. The work in this project started in 1972 and a number of publications have been the result (Joint Lake Research Project Reports 1974-1976, Gargas 1976, Jørgensen et al. 1976.) The work is continuing, and among other the emphasis is now on the general description of vertical gradients of phytoplankton biomass, nutrients, and dissolved oxygen.

In the same period work on management-orientated models have been carried out, and the research and management work has been mutually inspiring. The work on management models was implemented in 1970 on Haderslev Dam (VKI 1972). The aim of these initially very simplified models was to use the information obtained from a survey of Haderslev Dam to calculate the consequences of a proposed waste water treatment on concentrations of phosphorus and nitrogen.

In the years from 1973 to 1976 major surveys of approximately 20 Danish lakes have been carried out, and in these surveys have been included comprehensive measurements of chemical and biological statevariables and processes and of loadings with nitrogen and phosphorus and hydraulic properties.

The results of these surveys have been used by N. Nyholm to establish an eutrophication model for a shallow lake, i.e. a lake where complete vertical mixing can be assumed. A comprehensive documentation of this model is given in a VKI report (1978). Generally this model has been successful in simulating the measured values of state variables and processes for the lakes in view, and it is

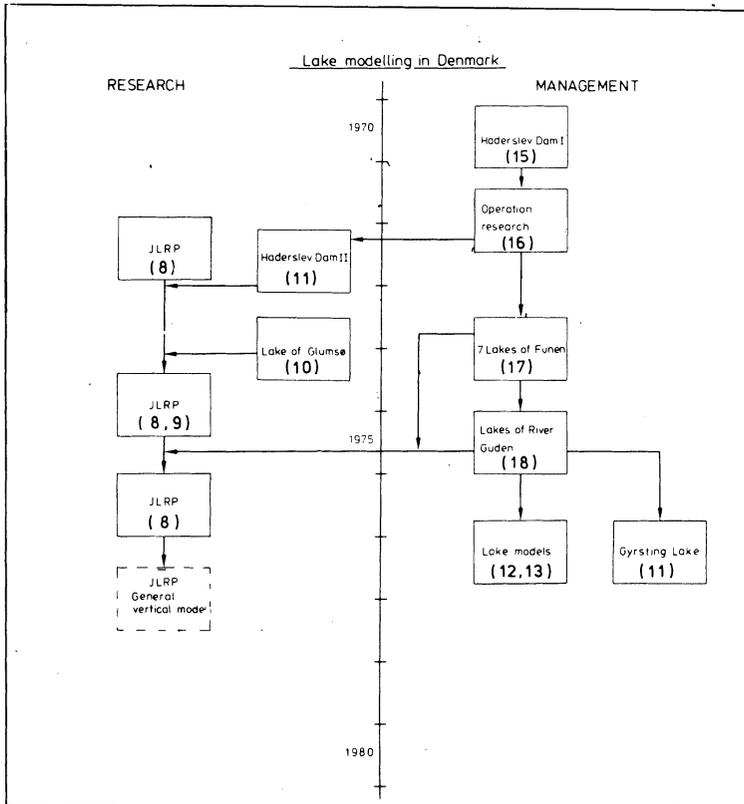


Fig.1. Development of research and management lake models in Denmark.

important to note that a reasonable agreement between simulated and observed values have been obtained by changing only very few (3-4) parameters within narrow intervals from lake to lake. However, there are limitations in the applicability of the model. Firstly, the simulation uses time averages of loadings and light intensities over weeks, i.e. short time fluctuations cannot be reproduced. Secondly, the model is limited in its possibilities for reproducing the diatoms in early spring. Thirdly, the model generally gives better results for lakes with a low hydraulic retention time.

The latest work in Denmark on management lake models has been carried out for lake Gyrsting, and the background and results of this work will be outlined in the following.

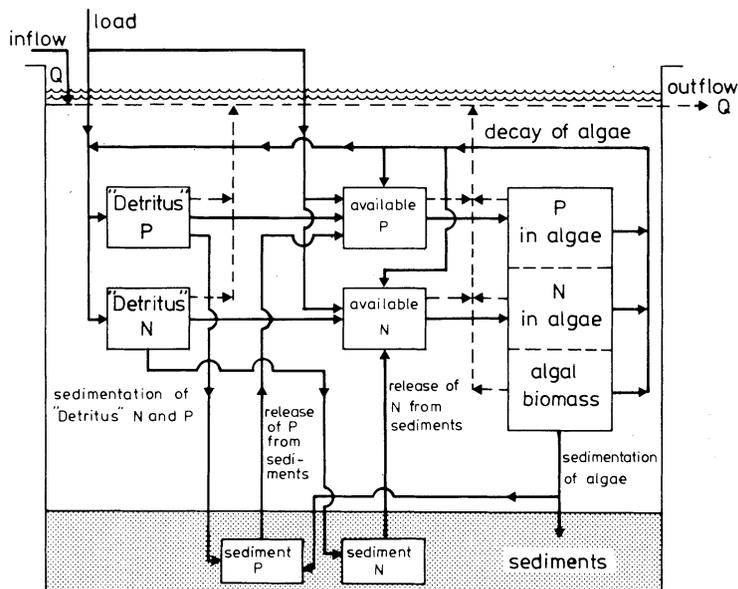


Fig.2. The structure of the eutrophication model for a shallow lake.

### System Analysis

The state variables, processes and forcing functions for the eutrophication model of a shallow lake is shown in Fig. 2.

The state variables are phytoplankton-biomass, phytoplankton-nitrogen, phytoplankton-phosphorus, dissolved inorganic phosphorus, dissolved inorganic nitrogen, »detritus« nitrogen, »detritus« phosphorus.

The recycling of matter is described by the processes: phytoplankton growth, uptake of nitrogen and phosphorus in phytoplankton biomass, decimation of phytoplankton, sedimentation of phytoplankton and detritus, mineralization of detritus, and release of nutrients from sediment to phosphorus.

A comprehensive presentation of process kinetics is given in VKI 1978a. As important factors can be mentioned that the phytoplankton growth rate is related to intracellular concentrations of nutrients. Furthermore the release of nutrients from sediments is a function of the gross sedimented amount of nutrients. This implies that the model can describe a new state of equilibrium but not the length and course of the transient period.

The forcing functions for the model are light intensity of surface, water temperature, loadings with nitrogen and phosphorus, and water flow. The forcing functions are smoothed by appropriate averaging. Furthermore, in the version of

the model used for Lake Gyrstinge the yearly variation of depth and volume of the lake is taken into account.

The state variables, processes and forcing functions are combined as a set of differential equations, one for each state variable:

$$\frac{dx}{dt} = \text{input} + r + D x$$

where:

$x$  = concentration of state variable

$r$  = sum of reactions

$D$  = hydraulic dilution

The equations are then solved by appropriate numerical methods to produce yearly variations of all state variables and processes.

### **Calculation Example – Lake Gyrstinge**

An important problem in the Danish water management is the allocation of water for different uses. In the later years intensive use of ground- and surface water for drinking water supply and agricultural purposes has produced biological problems in Danish fresh water systems. The Suså catchment area in the southern part of Zealand has been chosen by the Danish Committee of Hydrology (DKH) to serve as a subject for field studies and investigation of the consequences of increased water consumption. In one of the lakes of the system, Lake Gyrstinge, measurements of variations of state variables and processes, and loadings with nutrients have been carried out in 1976 as part of these pilot investigations (VKI 1978b). Important lake data area (1976):

Surface area	2.4 km <sup>2</sup>
Volume	9.7·10 <sup>6</sup> m <sup>3</sup>
Mean depth	4.0 m
Max. depth	7.9 m
Mean discharge	5.3·10 <sup>6</sup> m <sup>3</sup>
Mean hydraulic retention time	1.8 year

The measurements on the lake have been used for calibrating the shallow lake eutrophication model. Fig. 3 and Table 1 show comparisons between simulated and observed values of state variables, processes and massflows.

As can be seen from Fig. 3 and Table 1, the agreement between simulated and observed values of primary production, secchi depth, and accumulated amounts of nitrogen and phosphorus in sediments is satisfactory. However, the compari-

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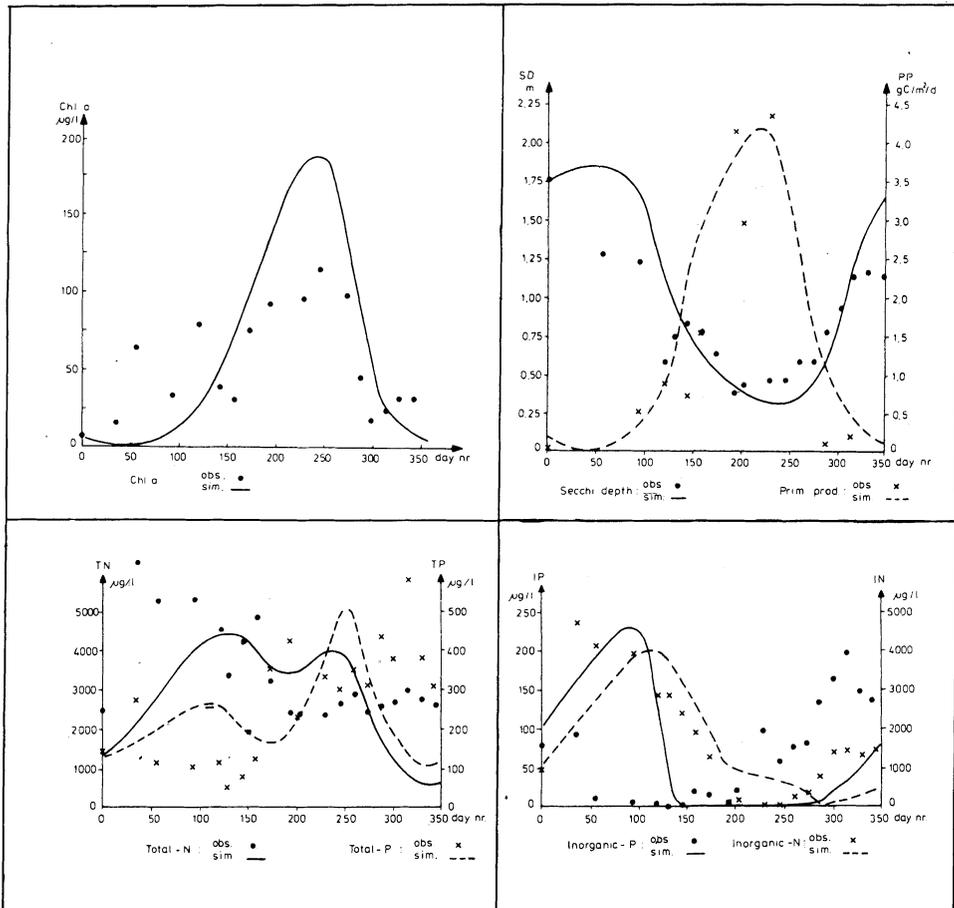


Fig.3. Simulated and observed values of statevariables and processes.

Table 1– Comparison of simulated and observed accumulated mass-flows.

	Primary production g C/cm <sup>2</sup> /year	Accumulation g/m <sup>2</sup> /year	
		N	P
observed	461	20.0	1.70
simulated	513	21.0	1.76

sons of the other state variables show, that the model is unable to reproduce the variations of nutrient and chlorofyll-a in early spring. Hypothetically, this can be explained by the fact that specific growth characteristics of the early spring diatoms have not been taken into account in the model. Furthermore, a lack of agreement is seen for the autumn concentrations of inorganic phosphorus. This lack of agreement shows that a further development of process kinetics is still necessary.

However, to obtain some preliminary estimates of the consequences of changes hydraulic retention times and loadings a number of calculations have been carried out by means of the calibrated model.

Three types of calculations are carried out:

- I *Loading* – increased input of water with constant concentrations of nutrients.
- II *Flushing* – increased input of water with zero concentrations of nutrients.
- III *Pumping* – decreased output of water.

Fig. 4 shows calculation results as chlorofyll-a concentrations in relation to hydraulic retention time.

It can be seen from Fig. 4 that the calculated phytoplankton biomass is only moderately sensitive to changes in loadings/retention times in the space near the existing or the reference situation. An increased sensitivity is seen for a decreasing retention time, and for retention times less than 1 year pronounced changes in phytoplankton biomass can be expected.

The consequences of changed retention times/loadings estimated by the eutrophication model can be compared to the estimations obtained by an input-output model. An example of this model type is the Dillon-Rigler equation:

$$\frac{dC}{dt} = \frac{L}{z} (1-R) - \frac{C}{T} \quad (1)$$

where:

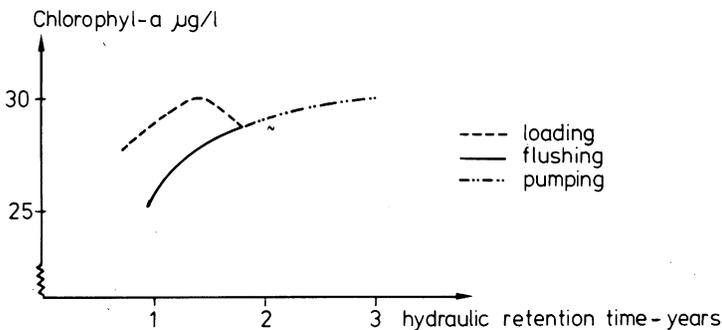


Fig.4. Calculated chlorofyll-a in relation to hydraulic retention times.

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- $C$  = the concentration,  $\text{g/m}^3$
- $L$  = loading in  $\text{g/m}^2/\text{year}$
- $z$  = mean depth in m
- $R$  = retention coefficient, dimensionless
- $T$  = hydraulic retention time, year.

A steady state solution of Eq. (1) gives:

$$C \equiv \frac{L T}{z} (1-R) \quad (2)$$

Data for the variables in the equations are shown in Table 2.

Table 2 - Input-output data for Lake Gyrstinge.

	$L$ $\text{g/m}^2/\text{year}$	$C_{in}$ $\text{g/m}^3$	$C_{out}$ $\text{g/m}^3$	$1-R$	$z$ m	$T$ year
$N$	27	12.4	3.2	.26	4.0	1.83
$P$	2.4	1.1	0.29	.26	-	-

Based on the values in Table 2 the consequences of changed retention times on the output concentrations of nitrogen and phosphorus can be calculated. In Fig. 5. is shown a comparison between output concentrations calculated by the eutrophication model and by the Dillon-Rigler equation.

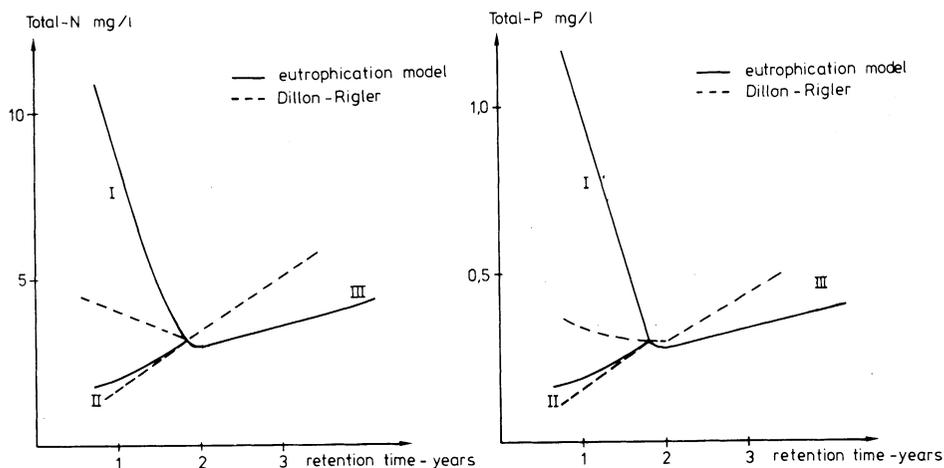


Fig.5. Comparison between eutrophication model and Dillon-Rigler equation.

From Fig. 5 can be seen that the agreement between output concentrations is relatively good for case II (flushing). However, in case I (loading) and case III (pumping) the agreement is not good. This lack of agreement is expected in view of the fact that the Dillon-Rigler equations do not take changes in the phytoplankton limitation situation into account, and these changes are pronounced for Lake Gyrstinge.

## **Discussion**

In relation to the above mentioned work a discussion on the applicability of eutrophication models versus input-output models and on the need for further development of the eutrophication model is carried out.

### **Eutrophication model >< input-output model**

The advantage of a simplified input-output model is the possibilities for making »rule of thumb« estimates of a lake behaviour based on existing information within a few hours. However, simple input-output models like the Dillon-Rigler equation often give results which are far from realistic, especially when the models are used for simulating changes which are far from the calibration situation. Furthermore, the models do not calculate the biological consequences which are of main interest for the decision makers.

To give more realistic results it is necessary for the input-output models to work with time-varying input and include more variables, and the necessary effort is comparable to the work needed for making calibrations with an eutrophication model.

The point is, that the eutrophication models give more well-founded and more comprehensive results than the input-output models, and that the resources needed for using the different types of models are almost equal. For these reasons the use of input-output models should be restricted to situations, where a »rule of thumb« estimate is sufficient for the decision making.

### **Further Development**

The work with eutrophication models for a number of Danish lakes including Lake Gyrstinge have shown some of the above mentioned limitations of the models. A further development of the model is necessary along several lines.

Firstly, an increased understanding and description of the light-adaptation mechanisms of the phytoplankton might be useful to increase the accuracy of simulating chlorofyll-a concentrations. For the time being the chlorofyll-a values are estimated from the calculated biomass in dry-weight by assuming a constant chlorofyll-a/biomass ratio.

Secondly, the problems of simulating the early spring maxima could possibly be overcome by including several groups of phytoplankton incl. diatoms. Furthermore, by including bluegreen algae a better simulation of late summer and autumn biomasses could be expected because of the possibility of taking phytoplankton group dependent growth and sedimentation properties into account. The disadvantages of this approach would be the relatively high increase of complexity of the model. However, an increase in complexity from the present 7 statevariables to 15 statevariables would still leave a relatively simple set of equations, which can be solved without excessive use of computer time.

For the time being the use of the Danish lake management models is limited to completely vertical mixed waters. A development towards a description of vertical gradients including dissolved oxygen is being carried out.

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