Prediction of filter expansion during backwashing

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Abstract The application of the Ergun equation to predict the expansion of filter media during backwashing is investigated. Fluidization data from the literature have been analyzed and the values $k_1 = 3.519$ and $k_2 = 0.266$ have been found to give a very good fit to the data in the range of Reynolds numbers of interest in filter backwashing. An empirical equation that is applicable over a wider range of Reynolds numbers than the Ergun equation is also developed. New experiments have been carried out with glass spheres, plastic spheres, silica sand, and crushed glass. The effect of particle shape on expansion behaviour is investigated. It is found that the influence of particle shape is larger than previously recognized. Furthermore, the shape effect depends on the Reynolds number based on the backwash velocity. The advantages, limitations, and range of applicability of each method of calculation are delineated.

Keywords Backwash hydraulics; filter backwash; filter media; filtration; fluidization; water treatment

Introduction Granular medium filtration is one of the most important unit operations in both water and advanced wastewater treatment. Effective backwashing of rapid filters is essential to long-term successful operation of the filters. Methods for backwashing may be classified roughly as follows (AWWA, 1999): (i) water fluidization alone, (ii) surface wash plus fluidized-bed backwash, (iii) air-scour followed by full-bed fluidization, (iv) simultaneous air scour and water backwash without fluidization. Fluidization principles and the prediction of fluidized-bed porosity are important in understanding and designing the first three types of backwashing systems. Furthermore, the cleaning procedure for dual and triple-media filters must include a final fluidization stage using water only so that the different media re-stratify.

This paper considers the hydraulics of particulate fluidization and explains a method for the prediction of the velocity–porosity relationship in a fluidized filter bed. The paper focuses on single-medium beds consisting of uniform grains. It should be noted, however, that if an accurate model for the expansion of a uniform bed is available, the expansion of a non-uniform bed can also be predicted (Fair et al., 1971). This calculation method can be extended to predict the total expansion of a multimedia bed. Furthermore, rational methods for predicting the stratification, intermixing, and possible inversion of different media during fluidization require the availability of an expansion model for uniform grains as a pre-requisite (Cleasby and Woods, 1975).

Theory Numerous equations have been proposed in the engineering literature for the expansion of granular media during particulate fluidization. Unfortunately, almost all of these equations are applicable to spherical particles only, and the majority of the few models proposed for non-spherical particles are empirical in nature and do not exhibit the influence of shape
explicitly. Such models are therefore specific to the particular materials studied by the investigators in question and do not have general applicability (Couderc, 1985; Dharmarajah and Cleasby, 1986; Di Felice, 1995).

A correlation that accounts for the effect of shape explicitly was proposed by Dharmarajah and Cleasby (1986) and it is probably the most widely used correlation for the prediction of filter expansion during backwashing (AWWA, 1999). They have adopted Blake’s (1922) definition of a modified Reynolds number $Re_1$,

$$Re_1 = \frac{\psi d_{eq} \rho V}{6\mu (1-\varepsilon)}$$  \hspace{1cm} (1)

and utilized a dimensionless group (a friction factor) $\varphi$ which appears to have been defined first by Richardson and Meikle (1961) to correlate fluidization and sedimentation data:

$$\varphi = \frac{e^3}{(1-\varepsilon)^2} \frac{\psi^3 d_{eq}^3 (\rho_p - \rho) g}{216 \mu^2}$$  \hspace{1cm} (2)

This group was denoted by $A_1$ by Dharmarajah and Cleasby. Here $V$ = backwash velocity based on the empty cross-section of the bed, $\varepsilon$ = porosity, $\mu$ = absolute viscosity, $\rho$ = density of the fluid, $\rho_p$ = particle density, $d_{eq}$ = equivalent diameter defined as the diameter of the sphere with the same volume as a filter grain, $\psi$ = sphericity defined as the surface area of the equivalent volume sphere divided by the actual surface area of the grain, and $g$ = gravitational acceleration.

By using the data for fluidized spheres published by Wilhelm and Kwauk (1948) and Loeffler (1953), they carried out a regression analysis to obtain the coefficients in a power series with $\log(Re_1)$ as the independent variable and $\log(\varphi)$ as the dependent variable. The result was the following piecewise correlation:

For $Re_1 < 0.2$: $\varphi = 3.01 Re_1$  \hspace{1cm} (3a)

For $Re_1 > 0.2$: $\log \varphi = 0.56543 + 1.09348 \log Re_1 + 0.17979 (\log Re_1)^2 - 0.00392 (\log Re_1)^4 - 1.5 (\log \psi)^2$  \hspace{1cm} (3b)

The term $1.5 (\log \psi)^2$ in the second equation was obtained using the experimental data obtained by Cleasby and Fan (1981) and Dharmarajah (1982) with non-spherical particles. While this correlation represents the spherical particle data with a very good overall accuracy, it has a number of shortcomings. First, the calculation of either the porosity or the velocity when the other one is given requires tedious and error-prone iterative computations. Second, it contains a discontinuity at $Re_1 = 0.2$. Furthermore, two different porosity values are possible in a vicinity of the discontinuity (see Figure 1). This leads to an ambiguity in the application of the correlation.

Based on the same data used by Dharmarajah and Cleasby plus the data by Wen and Yu (1966) and Hartman et al. (1989), the following alternative expression has been developed in this work and will be presented here (see Figure 2):

$$\log \varphi = 0.565013 + 1.157034 \log Re_1 + 0.12866 (\log Re_1)^2 + 0.02195 (\log Re_1)^3 - 0.008 (\log Re_1)^4$$  \hspace{1cm} (4)

This equation has the following advantages over Eq. 3a–b: (i) It is simpler to use (with Eqs. 3a–b, it is not clear a priori which equation is to be used when $Re_1$ is not known.), (ii) It is continuous (the discontinuity in Eq. 3 is not physically reasonable), (iii) It is slightly more accurate. With the 540 distinct measurements (displayed in Figure 2) in the range
While Eq. 4 has the mentioned advantages, it retains two shortcomings of Eq. 3a–b: It lacks a theoretical basis and it is an implicit equation that must be solved iteratively. A method of correlation with a physically plausible basis was proposed by Fair and Hatch (1933). Fair and Hatch assumed that the flow is laminar during fluidization and the Blake-Kozeny equation is applicable to fluidized beds. With this assumption, they derived an equation that can be written as follows:

\[ \varphi = k Re_1 \]  

The constant \( k \) in Eq. 5 is known as the Kozeny’s constant. It is notable that Fair and Hatch used the value \( k = 4 \) for fluidized beds and \( k = 5 \) for fixed beds. Akgiray and Saatç (2001) considered the application of the Ergun equation, instead of the Blake-Kozeny equation, to fluidized beds. Equating the head loss predicted by the Ergun equation to the buoyant weight of the medium, the following was obtained:

\[ \varphi = k_1 Re_1 + k_2 Re_1^2 \]  

They presented an exact explicit solution of this equation giving porosity as a function of superficial velocity:

\[ \varphi = k_1 Re_1 + k_2 Re_1^2 \]
where $Q$ and $R$ are intermediate quantities calculated as follows:

$$Q = \frac{12k_1Re}{\psi^3Ga} \quad R = \frac{3Re}{\psi^3Ga} \left[ k_2Re + 6k_1 \right]$$

The Reynolds and Galileo numbers appearing in these equations are defined as follows:

$$Re = \frac{\psi d_{eq} \rho V}{\mu} \quad Ga = \frac{d_{eq}^3 \rho (\rho_p - \rho) g}{\mu^2}$$

Akgiray and Saatç (2001) noted that the values of one or both of the constants $k_1$ and $k_2$ may be different for fixed and fluidized beds and suggested the use of the values $k_1 = 4.17$ and $k_2 = \frac{1}{6}$ (instead of the fixed-bed value $k_2 = \frac{1.75}{6}$) for fluidized beds. The following section reports the results of a comprehensive analysis of fluidization data from the literature to determine the best values of $k_1$ and $k_2$ for fluidized filter beds.

**Optimization of the Ergun Equation for Filter Backwash**

Figures 2 and 3 display the fluidization data published by Wilhelm and Kwauk (1948), Loeffler (1953), Wen and Yu (1966), and Hartman et al. (1989). Only the data for $\varepsilon < 0.90$ were included, as the data for higher porosities deviated from the general trend of the data shown in these figures. An empirical equation based on Loeffler’s wall-effect data was employed to correct for wall effects (Dharmarajah and Cleasby, 1986). The data in Figure 3 have been analyzed in this work to determine the optimal values of $k_1$ and $k_2$ for fluidization.

The following are concluded: the Ergun equation cannot fit the fluidization data very accurately in the entire range of Reynolds numbers for which experimental data are available. Furthermore, the values of $k_1$ and $k_2$ depend on the range of data used in their determination. Considering the fluidization data in the range of Reynolds numbers of interest in filter backwashing ($-2.0 < \log Re_1 < 2.0$), the following values were obtained: $k_1 = 3.519$ and $k_2 = 0.266$ ($r^2 > 0.996$ with 474 measurements). These values give an excellent agreement with the mentioned data in the range $-2.0 < \log Re_1 < 2.0$. The mean error in predicted porosity values is 3.70% (compared to 3.41% and 3.35% obtained by Eq. 3 and Eq. 4, respectively). Figure 3 displays the curves predicted by the Fair-Hatch equation (i.e. Ergun equation with $k = k_1 = 4.17$ and $k_2 = 0$) and the modified Ergun equation (with $k_1 = 3.519$ and $k_2 = 0.266$).
It is important to note that the Fair-Hatch equation can be found in many widely used textbooks such as those by Foust et al. (1960), Geankoplis (1993), and McCabe et al. (2001), although these texts do not cite the work of Fair and Hatch (1933). These authors arrive at this equation by neglecting the second term in the Ergun equation, and therefore they take \( k = k_1 = 150/36 = 4.17 \), i.e. the value obtained by Ergun (1952) for fixed beds. Figure 3 shows that neglecting the second term in the Ergun equation leads to a very poor agreement with the fluidization data at high Reynolds numbers. Furthermore, for small Reynolds numbers, the use of the fixed-bed value of \( k_1 = 4.17 \) gives friction factors larger than those experimentally observed. Finally, with the explicit equation given above (Eq. 7), there remains no need to neglect the second term in the Ergun equation. Eq. 7 is very simple and facilitates manual calculations. To conform to the non-spherical particle data by Cleasby and co-workers, the constants \( m_1 = k_1 10^{g(y)} \) and \( m_2 = k_2 10^{g(y)} \) (in which \( g(y) = -1.5(\log y)^2 \)) should be used instead of \( k_1 \) and \( k_2 \), respectively, in Eq. 7. The validity of this shape term, however, is examined in detail later in this paper.

### Experimental

Fluidization experiments have been carried out with glass spheres of two different sizes (1.11 mm and 3 mm), plastic spheres of three different sizes (1.97 mm, 2.48 mm, and 2.87 mm), five sieved fractions of silica sand, and four sieved fractions of crushed glass. The sand utilized in the experiments is the sand currently used in all the municipal water treatment plants in Istanbul. Sand and crushed glass fractions were obtained by double sieving followed by 1 minute of manual sieving such that the change in weight during the latter was less than 1% for each fraction. Equivalent diameters have been measured by counting and weighing 200 grains of each fraction. Densities were measured by a water-displacement technique. The properties of the media are summarized in Table 1. Water was the fluidizing medium. A 40 cm high column with an internal diameter of 38 mm was employed. Porosities were calculated from bed weight, bed height, and density values. Sphericities of the sand and crushed glass fractions were determined by measuring the fixed-bed head loss for each fraction and using the Ergun equation (with the fixed-bed values \( k_1 = 4.17 \) and \( k_2 = 0.29 \)). The values reported in Table 1 are the mean values for several measurements (such as the 18 measurements in Figure 4). The validity of this method of determining sphericity was checked by using the fixed-bed head loss data for spheres (see Figure 4). Fixed-bed head loss data were collected by ensuring that there was no accumulation of bubbles within the bed (which would lead to increased head loss values and thereby could totally invalidate the

<table>
<thead>
<tr>
<th>Particle type</th>
<th>Density ( \rho_p ) (g/cm³)</th>
<th>( d_{eq} ) (mm)</th>
<th>Sphericity, ( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass balls</td>
<td>2.519</td>
<td>1.11</td>
<td>0.998</td>
</tr>
<tr>
<td>Glass balls</td>
<td>2.532</td>
<td>3.00</td>
<td>0.984</td>
</tr>
<tr>
<td>Plastic balls</td>
<td>1.171</td>
<td>1.97</td>
<td>0.983</td>
</tr>
<tr>
<td>Plastic balls</td>
<td>1.193</td>
<td>2.48</td>
<td>0.983</td>
</tr>
<tr>
<td>Plastic balls</td>
<td>1.180</td>
<td>2.87</td>
<td>0.958</td>
</tr>
<tr>
<td>Silica sand 0.70 ( \times ) 0.84</td>
<td>2.640</td>
<td>0.84</td>
<td>0.757</td>
</tr>
<tr>
<td>Silica sand 0.84 ( \times ) 1.00</td>
<td>2.639</td>
<td>1.02</td>
<td>0.738</td>
</tr>
<tr>
<td>Silica sand 1.00 ( \times ) 1.19</td>
<td>2.641</td>
<td>1.11</td>
<td>0.706</td>
</tr>
<tr>
<td>Silica sand 1.19 ( \times ) 1.41</td>
<td>2.628</td>
<td>1.40</td>
<td>0.714</td>
</tr>
<tr>
<td>Silica sand 1.41 ( \times ) 1.68</td>
<td>2.629</td>
<td>1.65</td>
<td>0.723</td>
</tr>
<tr>
<td>Crushed glass 0.84 ( \times ) 1.00</td>
<td>2.486</td>
<td>0.96</td>
<td>0.413</td>
</tr>
<tr>
<td>Crushed glass 1.19 ( \times ) 1.41</td>
<td>2.494</td>
<td>1.24</td>
<td>0.442</td>
</tr>
<tr>
<td>Crushed glass 1.41 ( \times ) 1.68</td>
<td>2.496</td>
<td>1.48</td>
<td>0.430</td>
</tr>
<tr>
<td>Crushed glass 2.00 ( \times ) 2.38</td>
<td>2.499</td>
<td>2.08</td>
<td>0.415</td>
</tr>
</tbody>
</table>
results). For the five different sizes of balls, the calculated sphericities were between 0.96 and 1 (Table 1). Considering that the Ergun equation was derived by means of curve-fitting to a large amount of data with varying degrees of experimental error, these results are judged to be quite satisfactory. Sphericities of the sand fractions were found to be in the range 0.70–0.76. These values are consistent with the typical values reported in the literature (Fair et al., 1971; AWWA, 1999). It was visually observed that the crushed glass particles were highly non-spherical and the low sphericity values measured (0.41–0.44) are consistent with this observation. In analyzing the fluidization data, it is assumed that $\psi = 1$ for the balls; the $\psi$ values in Table 1 are used for the fractions of sand and crushed glass.

**Results and discussion**

The fluidization data obtained with glass and plastic spheres are shown in Figure 5. It should be noted that these data were not included in the nonlinear regression analysis previously carried out to determine the best values of $k_1$ and $k_2$. The agreement with the Ergun equation is excellent, the mean error in the predicted $\log\phi$ values being only 1.66%. When the Dharmarajah-Cleasby correlation is employed, the mean error is 1.68%, i.e. the two correlations have similar accuracy in this range of Reynolds numbers. This is consistent with the earlier conclusion that the two correlations have about the same accuracy in the range
This statement was based on the fluidization data compiled from the literature and displayed in Figure 3. The data for the five sand fractions are displayed in Figure 6. The two curves were obtained using the Dharmarajah-Cleasby correlation with $\psi = 0.7$ and $\psi = 1.0$. (Note that the sphericities of the sand fractions were between 0.71 and 0.74.) The data for the four fractions of crushed glass are displayed in Figure 7. Again, the two curves were obtained using Equation 3 with $\psi = 0.4$ and $\psi = 1.0$. (The sphericities of the crushed glass fractions were between about 0.41 and 0.44.) These results show that the effect of shape (i) depends on Reynolds number, and (ii) is stronger than that predicted by the term $-1.5(\log\psi)^2$ suggested by Dharmarajah and Cleasby. The deviation of the non-spherical particle data from the trend-line for spheres is larger at higher Reynolds numbers.

Summary and conclusions
The application of the Ergun equation for the prediction of filter expansion during backwashing is considered. Fluidization data from the literature have been analyzed to determine the best values of the constants $k_1$ and $k_2$. The following are concluded: the Ergun equation cannot fit the trend of the data over the entire range of Reynolds numbers for which fluidization data are available in the literature, and the constants $k_1$ and $k_2$ depend on the range of data employed in the regression analysis. Considering the range of Reynolds numbers of
interest in filter backwashing, the following values have been obtained from spherical particle data: \( k_1 = 3.519 \) and \( k_2 = 0.266 \) (\( r^2 = 0.99631 \) with 474 data points). New experiments have been carried out with glass and plastic spheres of five different sizes to verify the accuracy of the proposed calculation method. The agreement of the Ergun equation with the new data is very good.

Additional experiments have been carried out with carefully sieved fractions of silica sand and crushed glass to ascertain the effect of shape on expansion behaviour. The non-spherical particle data presented here (Figures 6 and 7) cover a range of about 20% to 100% media expansion. It has been verified that the non-spherical particle data fall below the curve for spheres on the friction factor versus the Reynolds number diagram. It is found that this shape effect depends on the Reynolds number and is stronger than documented previously. Further work is needed to model the expansion of non-spherical media.

References


