Discussion

A. S. Hall.[6] In Fig. 8 of the paper there is displayed a family of four-bar mechanisms, each of which satisfies the specified values of $d\phi/d\theta$, $d^2\phi/d\theta^2$, and $d^3\phi/d\theta^3$. Inspection of this drawing might suggest that possibly the trajectory of P_{bd} is a circle. If this were true, then the job of finding the different members of the family of four bars would be greatly shortened.

One method for showing that the trajectory of P_{bd} is a true circle depends on recognition of the nature of the curve containing crankpin 2 for the different members of the four-bar family. This curve is the "cubic of stationary curvature" (German "Kreisungspunktkurve") for motion of crank a with respect to lever c. One reference for a discussion of this curve is a book by R. Beyer.[7]

In order to outline the proof, let us first refer to the author's Fig. 1 for notation. Let the *directed* lengths from pole P_{ac} to pole P_{bd}, crankpin 2, and pivot 1 be represented, respectively, by the symbols ρ, r, and r_0. Now, since both the crankpin 2 and pivot 1 must lie on the afore-mentioned cubic of stationary curvature, the following equations must be satisfied

$$1/r = 1/K \cos(\lambda + \psi) + 1/L \sin(\lambda + \psi) \quad [40]$$

$$1/r_0 = 1/K \cos \psi + 1/L \sin \psi \quad [41]$$

where K and L are constants.

If we eliminate L between Equations [40] and [41], the result can be written

$$\sin\psi/r_0 - \sin(\lambda + \psi)/r = \frac{1}{K}\left[\frac{\sin\psi}{\cos\psi} - \frac{\sin(\lambda+\psi)}{\cos(\lambda+\psi)}\right] \quad [42]$$

Rearranging Equation [42] yields

$$r \sin\psi - r_0 \sin(\lambda + \psi) + \frac{r_0 r}{K}\left[\frac{\sin\lambda}{\cos\psi\cos(\lambda+\psi)}\right] \quad [43]$$

Now consider areas of triangles

$$\Delta P_{bd}P_{ac}2 - \Delta P_{bd}P_{ac}1 + \Delta 2P_{ac}1 = 0$$

or

$$\rho r \sin\psi - \rho r_0 \sin(\lambda + \psi) + r r_0 \sin\lambda = 0 \quad [44]$$

Dividing through by ρ yields

$$r \sin\psi - r_0 \sin(\lambda + \psi) + \frac{r_0 r}{\rho}\sin\lambda = 0 \quad [45]$$

Comparison of Equation [43] with Equation [45] shows

$$\rho = (K \cos\psi)\cos(\lambda + \psi) \quad [46]$$

This will be recognized as the equation of a circle in polar coordinates if we remember that, for the problem considered, angle ψ is a constant.

J. F. D. Smith.[8] The author has taken a long step forward in the analytical analysis of four-bar linkages. It will be quite helpful to the designer who already has the mechanism to analyze. Unfortunately, before the engineer analyzes the mechanism he first has to design it. For example, if it is desired to have some particular part of a machine go through a predetermined path, and it is known from experience that a four-bar linkage might give this path approximately, with the expectation that modifications to get the exact path could be supplied by some other means, the designer is confronted with the problem of first determining approximately the linkages to be used.

It is thus very important to have analyses such as this one by the author, but it is equally important to the designer to have advice on how to design the mechanism in the first place. It might be expected that a mathematical solution in each case to determine the proper linkage would be tedious. Rigorous graphical solutions, however, are available for a great many of the possibilities. Hrones and Nelson[9] give graphical solutions for the case where the driving crank makes a complete revolution and the follower crank oscillates. This discussion is principally to bring to the attention of those interested in the design of four-bar linkages that there is a book available which might be of some help in the primary design stages.

J. P. Vidosic.[10] This paper provides an excellent addition to that of Freudenstein. The collineation-axis approach along with the Taylor-series expansion of the crank-follower functional relationship permits of motion analysis and of approximate four-bar linkage synthesis in a relatively simple way. It is unfortunate, however, that the synthesis can satisfy only one phase of the mechanism at a time since location of the rotopole P_{ac} and its describing parameters e and f change with phase.

In Fig. 8 the numerical values of the fourth derivative of follower displacement with respect to crank displacement for each rotopole position are given. Equation [39] of the paper, on the other hand, to which reference is made, is the fourth derivative of follower displacement with respect to time. This may not be properly understood.

Again the author should be heartily complimented for a definite contribution in the area of mechanism synthesis. It is hoped he will continue to add to this area of engineering.

Author's Closure

The author is very grateful to Professor Hall for the proof concerning the nature of the trajectory of P_{bd}. The knowledge of the circular shape of the trajectory will considerably reduce the labor incident to a particular solution.

With regard to Professor Vidosic's comment that e and f change with phase, it should be noted that if the derivatives of ϕ with respect to θ have the proper values at a given position in the linkage travel, the changes of e and f with phase are approximately as they should be to produce the desired motion. Of course only four of the derivatives may be satisfied since there are only four degrees of freedom in the layout of the linkage. The practical result of this is that the difference between the desired motion of the driven member and the motion actually produced by the driven member increases with the displacement of the driving member from the design position. The difference, or error, may in many cases be only a small percentage for a 40-degree motion of the driving member on either side of the design position.

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[7] "Kinematische Getriebesynthese," by R. Beyer, Julius Springer, Germany, 1953.

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[9] "Analysis of the Four-Bar Linkage," by J. A. Hrones and G. L. Nelson, The Technology Press, Massachusetts Institute of Technology, Cambridge, Mass., and John Wiley & Sons, Inc., New York, N. Y., 1951.

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