present authors agree that the effect of pressure gradient upon skin friction coefficient at low acceleration levels appears to be over-estimated by the present procedure. However, we feel that the discrepancy between predicted and measured Stanton number shown in Fig. 9 may be somewhat attributable to the data. These data were taken in the Stanford tunnel which, as discussed in the subject paper, appears to have a tendency to resist laminarization. A comparison of predicted Stanton number with another low $K$ flow, for a periodically varying wall temperature which was also measured in the Stanford tunnel, is presented in reference [12] of the subject paper. Here the prediction showed considerably better agreement with the data than is shown in Fig. 9.

**Comprehensive Design of Axisymmetric Wind Tunnel Contractions**

P. H. ROTHÉ. The author is commended for his contribution to design procedures for axisymmetric contractions. The overall approach is an appropriate blend of design criteria, analysis, and judgement. The potential flow analysis and the design charts reflect the effects of wall curvature, but Stratford's criterion for boundary layer separation does not. A more general separation criterion might include the effects of wall curvature on turbulent mixing and shear stress. Bradshaw [13] has pointed out that the normal component of acceleration resulting from a streamline radius of curvature as large as 300 boundary layer thicknesses can perceptibly stabilize (or destabilize) turbulent boundary layers. In extreme cases a stabilized turbulent boundary layer can be re laminarized by normal acceleration alone.

Fortunately, the present analytical model appears to be conservative, or possibly insensitive, to the effects of wall curvature on turbulent mixing. At the narrow end of the nozzle, where streamline curvature will tend to stabilize the boundary layer and enhance the possibility of boundary layer separation, the author states that the pressure coefficient $C_p$ is moderate in practical cases. Further, the author argues compellingly that separation at the narrow end is a secondary design consideration in most cases. At the wide end, streamline curvature may be expected to destabilize the boundary layer and reduce the prospect of separation relative to the flat wall boundary layers treated by Stratford (authors reference [12]).

In Section VI of the paper the author concludes that the design ratio of nozzle length to inlet diameter decreases as contraction ratio increases if the criterion of fixed pressure coefficient at separation is employed. Would the author care to comment on the possible sensitivity of this conclusion, and of predicted nozzle length ratio in general, to variations in Stratford's empirical coefficient or to reasonable alternative models of separation?

Comparisons to data would be useful in order to verify the detailed analytical predictions. Has the author had an opportunity to apply this analytical method?

**Additional Reference**


GUNNAR HESKESTAD. As an investigator often resigned to the less sophisticated methods of designing flow contractions, mostly via the "plausible-shape" route, this reader is pleased to see a practical paper on contraction design. Although limited to axisymmetric contractions, the results offer at least a guide to the design of other contractions.

Given the contraction ratio, the remaining design task is to select the shape of the contraction, where the contraction length may or may not be subject to limitations of available space. The author has studied power-law contractions and selected the cubic for detailed study as the best overall performer. The nozzle contour is then fixed for every combination of contraction ratio, length ratio ($L/D_i$) and location of matching point ($X = x_L/L$). For the matched cubics, the author shows that the nondimensional pressure gradients at inlet and outlet ($dC_p/d(x/D_i)$ and $dC_p/d(x/D_o)$) are roughly proportional to the pressure coefficients at inlet and exit, being approximately $4 C_{p_{in}}$ and $4 C_{p_{out}}$, respectively. On this basis, the designer can assess the probable performance of a given design from Figs. 4-6. The author suggests using Stratford's criterion for boundary layer separation criterion, which appears appropriate. He works out an example which indicates that the pressure coefficient at the inlet, $C_{p_{in}}$, should be less than 0.30. A value of about 0.35 is perhaps conservative in most applications.

Separation at the exit, for some reason, is not a common problem in "intuitive" designs. The author simply assumes that 1) a nondimensional pressure gradient at exit of 0.24 presents no danger of separation for a turbulent boundary layer and 2) a laminar boundary layer would resist separation, since it is well energized in the contraction. It would be useful to the designer to have quantitative estimates of tolerable adverse pressures in the exit region; the following discussion is offered for that purpose.

Consider first a turbulent boundary layer near the exit. Prior to the adverse pressure region, the boundary layer has experienced a pressure history roughly analogous to the pressure history of a boundary layer approaching the minimum-pressure-point of a circular cylinder in a free stream. At supercritical Reynolds numbers, the boundary layer on the cylinder probably becomes turbulent near the minimum-pressure point (say, at 80 degrees from the forward stagnation point) and separates about 122 degrees from the forward stagnation point (Fig. 152 of reference [14] for a Reynolds number of $2.12 \times 10^6$). Using Stratford's separation criterion (equation (11)), where $x$ represents the arc length from 80 to 122 degrees and $dC_p/dx$ is approximated $C_{p_{in}}/x$, the nondimensional pressure rise from the minimum pressure point to separation, $C_p$ is estimated at 0.47 based on the velocity at the minimum-pressure point, which corresponds closely to the pressure rise obtained experimentally, 0.50. An analogous calculation for the exit flow of the contraction, where $x$ is taken as $x_L$ and $dC_p/dx$ approximated as $4 C_{p_{in}}/D_i$ (author's approximation), yields a pressure rise to separation of $C_p = 0.53$ (for Reynolds numbers of order 100). Consequently, values $C_p$ to 0.53 might be tolerable at the exit for turbulent boundary layers, a value which is so high that none of the configurations in Figs. 4-6 is likely to induce turbulent separation.
Next consider a laminar boundary layer at the exit. Stratford's criterion for separation of a laminar boundary layer (12) is:

\[ C_p \left( \frac{dC_p}{dz} \right)^\infty \approx 0.0076 \]

which, for the current application, may be written:

\[ C_p \frac{dC_p}{dz} \approx 0.0076 \]

Solving for \( C_p \):

\[ C_p = \frac{0.078}{(\pi/2)} \approx 0.0076 \]

The length \( x \) is the distance from the virtual origin of the boundary layer. To establish a realistic value for this distance in the current adaptation, the subcritical flow over a circular cylinder is considered (Fig. 152 of reference [13] for a Reynolds number of 1.06 x 10^6). The average pressure gradient between the minimum-pressure point and separation for this case is \( dC_p/d(z/R) \approx 0.48 \), where \( C_p \) is based on the velocity at the minimum-pressure point and \( R \) is the cylinder radius. The pressure rise to separation, \( C_p \), is 0.09. Stratford's criterion is written:

\[ C_p \left( \frac{dC_p}{dz} \right)^\infty \approx 0.0076 \]

Solving for \( C_p \):

\[ C_p = \frac{0.078}{(\pi/2)} \approx 0.0076 \]

The case \((L + A)/D_i = 1.04\), or \(L/D_i = 0.86\) for a contraction ratio of 16 are 0.8, 1.0, 1.3, and 0.83. The ratios for a contraction ratio of 16 are 0.8, 1.0, 1.3, and 1.9 through the same \(L/D_i\) range. Although the precise magnitudes of these ratios are not indicative of whether laminar separation will occur or not (because of the several assumptions employed), they do indicate a tendency to laminar separation increases as the length ratio decreases. For sufficiently high, but still common, Reynolds numbers, however, a laminar boundary layer will undergo transition to a turbulent one well ahead of the laminar separation point. Therefore separation at the exit appears to be possible only for the low Reynolds number flows.

Finally, the relevance of the exit-velocity uniformity as a performance measure will be considered. For illustration, consider a contraction ratio \( CR = 9 \) and selection of \( C_{pi} = 0.35\). The designer has the only remaining option of varying the length of the contraction, \( L/D_i \), and observes from the design charts that \( C_{pe} \) consequently \( \frac{\delta}{U \delta} \approx 0.35 C_{pe} \) decreases as \( L/D_i \) increases. He has ruled out separation at the exit, but may want to achieve the uniform flow as best possible function of \( L/D_i \). This exit where flow distortions cease to be important. Knowing the decay curve of \( U \) downstream of the exit \( \exp \left( \frac{-7.5z}{D_i} \right) \), he can calculate \( L/D_i \) as function of \( L/D_i \). The case \( L/D_i = 1.25 \) \( (C_{pe} = 0.35) \) corresponds to \( \frac{\delta}{U \delta} = 0.0076 \), a very slight distortion. However, he will achieve the same slight distortion with \( L/D_i = 1.04 \) or \( L/D_i = 0.85 \) \( (C_{pe} = 0.35) \) or \( L/D_i = 0.75 \) \( (C_{pe} = 0.35) \).

In fact, the smallest length ratio, \( L/D_i = 0.75 \), which has the largest value \( \frac{\delta}{U \delta} \) produces the smallest length \( L + A \). Therefore, the exit-velocity uniformity per se, as assessed via \( \frac{\delta}{U \delta} \), is not a meaningful performance measure. No performance penalty appears associated with large values \( \frac{\delta}{U \delta} \) contrary to the view expressed in the paper.

Additional Reference


Author's Closure

Reply to P. Rothe

The comment of P. Rothe on the streamline-curvature effects on turbulence and separation criteria is highly appropriate, as these effects may be important in many situations. Although the nature of the curvature effects on turbulence is well recognized today, their inclusion, in a quantitative manner, into a simple separation criterion (e.g. Stratford's) has not yet been accomplished. Fortunately, it seems that, at the inlet end, the omission of curvature effects from Stratford criterion makes it somewhat conservative, rather than optimistic. This is not too serious a drawback, as some small margin of safety is advisable to compensate for possible inaccuracies in operation of the nozzle contour.

The conclusion that \( L/D_i \) decreases with \( CR \) for fixed values of \( C_{pi} \) and \( C_{pe} \) is independent of the separation criterion. However, since the inlet-end Reynolds number and details of the long acceleration in the nozzle change with \( CR \), the prescribed values of \( C_{pi} \) and \( C_{pe} \) vary slowly with \( CR \).

The sensitivity of the predicted nozzle length ratio \( L/D_i \) to small changes in separation criteria may be judged from the sensitivity of \( L/D_i \) to small changes in \( C_{pi} \) and \( C_{pe} \). The design equation (10) and equations (9b) and (9a) may be linearized to give:

\[ \frac{\Delta L}{L_i} = \left( \frac{F_{pi}}{F_{pi}^\alpha - 1} \right) \frac{1 - X_o}{1 + X_o} \]  

(14)

for fixed \( C_{pi} \) and small changes in \( C_{pe} \)

\[ \frac{\Delta L}{L_i} = -\left( \frac{G_{pi}^{1/\alpha}}{G_{pi}^\alpha - 1} \right) \frac{4/3 X_o}{1 + X_o/3} \]  

(15)

where subscript zero refers to the original nozzle. Equations (14) and (15) are very good approximations in the range \( C_{pi} > 0.1 \), \( C_{pe} < 0.1 \), \( 0.4 < X < 0.7 \), \( 0.77 < F_{pi}/F_{pi}^\alpha < 1.3 \) and \( 0.83 < G_{pi}^{1/\alpha} < 1.2 \).

The author has not yet had the opportunity to apply the procedure to the design of an asymmetric nozzle. It has been successful.
The discussion of G. Heskestad may be divided into three parts: a summary of the proposed design procedure, discussion of the exit separation criterion and a discussion of the appropriateness of exit velocity uniformity as a meaningful performance measure.

The reply will be directed to the latter two parts. G. Heskestad analyzes the exit portion of a nozzle by analogy to the pressure distribution on a circular cylinder. His order-of-magnitude laminar analysis is interesting, but has the disadvantage of including too many approximations to give a clear indication of whether separation will, in fact, occur. It offers only the suggestion that by sufficient lengthening of the nozzle, separation may be avoided. However, such conclusions may be drawn directly from Fig. 7 and 10, which show that both $C_p$ and $dC_p/dx$ decrease rapidly with increasing $L/D_i$. Instead, one may attempt to perform a different analysis similar to that which led to the inlet separation criterion equation (12), and which is more closely related to the problem at hand than the analysis proposed by the discusser.

To this end one may use the laminar version of Stratford’s criterion (as presented in reference [14])

$$C_p \left[ (x-x_B) \frac{dC_p}{dx} \right]^2 \approx 0.01$$

(16)

where

$$C_p = 1 - \frac{U}{U_{max}}$$

and

$$x_B = x_{max} - \int_0^{x_{max}} \sqrt{(U/U_{max})} \, dx$$

is the artificial origin such that the momentum thickness of a flat-plate flow starting at $x_B$ just matches at $x = x_{max}$ (the pressure minimum) the momentum thickness of the flow under consideration. The equation may be approximated for cubic shapes by taking: (1) $C_{ps} = 0.8 C_p$ and (2) $(D_i dC_p/dx)_{max} = 4 C_p$, leading to

$$C_{ps} \approx 0.092 \left( \frac{x - x_B}{D_i} \right)^{3/2}$$

(17)

Estimating the ratio in the parenthesis as $0.8 - 1.0$, we obtain the separation criterion for the exit to be $C_{ps} = 0.09 - 0.11$. In the turbulent case the maximum permissible value of $C_{ps}$ is much higher, and we agree with the discusser’s estimate of $C_{ps} \approx 0.5$ (at $Re = 0(10^6)$).

The last part of the discussion concerned the use of flow uniformity at the nozzle exit plane as one of the design criteria. This point is well taken: one should probably always follow the nozzle by a short piece of straight duct ahead of the test section to reduce the exit nonuniformity through a fast exponential decay (tenfold reduction in a duct $0.3 D_i$ long). Then, the only exit-end criterion which sets the value of $C_{pe}$ is flow separation; in the case of a very large tunnel with boundary layers that are turbulent at all air speeds of interest, a significant saving in overall nozzle length may be achieved. However, the laminar case calls for some caution. A design to the limit of performance $C_{pe} \approx 0.1$ implicitly assumes a high level of manufacturing precision. Even very small departures from the prescribed cubic shape (or waviness of the surface due to welding, etc.) can cause sizable excursions in $C_p(x)$, and may cause local separation bubbles and transition. Thus, a more conservative criterion, say, $C_{pe} = 0.06$ should be used. On the other hand, designing a nozzle for exit nonuniformity of 0.35 percent (as in the example used by G. Heskestad) is not very practical. It should be realized that this really means that the flow on the centerline and at the wall is allowed to differ only $\pm 0.175$ percent from the mean velocity. Such non-uniformity can probably not even be detected. A more practical requirement is $L/D_i = 1$, percent, i.e. $C_{pe} = 0.01$. For the example worked out by the discusser (CR = 9, $C_{ps} = 0.35$) one finds that for $L/D_i = 1.0$, $C_{pe}$ equals 0.026 and $L/D_i = 1$ percent; for $L/D_i = 0.85$, $C_{pe}$ equals 0.002 and $L/D_i = 2.17$ percent. If the shorter nozzle is to give the same flow uniformity it must be followed by a straight duct 0.105 $D_i$ (0.035 $D_i$) long. Thus, prescribing $C_{pe} = 0.06$ rather than 0.026 results in a shortening of the total nozzle from $L/D_i = 1.0$ to $L/D_i = 0.85$, a substantial length reduction.

Finally, in most wind tunnels one usually does not place test models so that they extend to the nozzle-exit plane, and thus a small portion of the test-section is always available for non-uniformity decay. Taking a length of 0.05 $D_i$ giving a decay of 33 percent, one can arrive at the following reasonable upper bound on $C_{ps}$ for nozzles which are not followed by a special straight section for non-uniformity decay.

$$C_{pe} = \frac{1}{\sqrt{0.35}} \frac{1}{0.01} = 0.043$$

This gives a 1 percent nonuniformity at a distance of $z/D_i = 0.05$ inside the test-section.

In summary, in wind tunnel applications where laminar boundary layers are expected, a good practical choice for $C_{ps}$ is probably in the range $0.04 - 0.06$ (which is the range we are currently using). In very large tunnels where boundary layers are expected to be always turbulent, one may use considerably larger values.

Additional References


ERRATA

p. 228 last line $x/D_i$ instead of $x/D_t$

p. 229 line 2 and 5 $x/D_i$ instead of $x/D_t$

Fig. 9 ordinate $x/D_i$ instead of $x/D_t$

p. 231 equation below equation (12) missing exponent

\[ \left( \frac{U}{U_{max}} \right)^{1/3} \]