Development of method for X-band weather radar calibration

J. E. Nielsen, S. Thorndahl and M. R. Rasmussen

ABSTRACT

Calibration of the X-band LAWR (Local Area Weather Radar) is traditionally based on an assumed linear relation between the LAWR radar output and the rainfall intensity. However, closer inspection of the data reveal that the validity of this linear assumption is doubtful. Previous studies of this type of weather radar have also illustrated that the radar commonly has difficulties in estimating high rain rates. Therefore, a new radar–rainfall transformation model and a calibration method have been developed. The new method is based on nonlinear assumptions and is aimed at describing the whole range of rain intensities in a more comprehensive way for the LAWR system. The new proposed calibration method improves the LAWR QPE (quantitative precipitation estimate) accuracy by reducing bias and describing the temporal dynamics better for the vast majority of the observed rainfall. However, in heavy rainfall, the LAWR system still faces significant challenges in measuring the peak intensities accurately.

Key words | error filtering, LAWR, precipitation, radar QPE, weather radar calibration, X-band

J. E. Nielsen (corresponding author)
S. Thorndahl
M. R. Rasmussen
Department of Civil Engineering,
Aalborg University,
Sohngaardsholmsvej 57,
DK-9000 Aalborg,
Denmark
E-mail: *Jen@civil.aau.dk*

INTRODUCTION

The Local Area Weather Radar (LAWR) belongs to a small group of weather radars dedicated to urban drainage applications (Jensen & Overgaard 2002). Hence, this radar type focusses on high resolution in both time and space. By utilizing the shorter X-band waveband and a simpler scanning strategy, the LAWR radar can operate with temporal and spatial resolutions down to 100 m and 1 min. The downside of using the X-band wave is a higher atmospheric attenuation which significantly reduces the range of the radar compared to conventional C- and S-band radars (Einfalt *et al.* 2004).

Urban drainage applications based on modelling urban runoff rely on inputs of accurate rainfall observation. Traditionally, rain gauges have served this purpose. However, data from rain gauges may be insufficient either if gauges are located too far from the catchment or if the catchment is large and the point measurement from one gauge does not represent the rainfall for the whole catchment. Several authors have addressed the temporal and spatial resolution requirements for urban runoff, e.g., Schilling (1991), Quirmbach & Schultz (2002), Einfalt & Maul-Kötter (2002), Berne *et al.* (2004) and Einfalt *et al.* (2004). The required resolution is, in general, found to be high. Quirmbach & Schultz (2002) compared rain gauge and radar data as input for urban drainage runoff modelling. It was found that if the catchment is small and the rain gauge was near the catchment, the rain gauge outperforms the radar as input for the runoff model. However, with growing catchment size and growing distance between the rain gauges, the heterogeneities of the rainfall observed in the radar input became significant for the model result. The conclusion of the investigations was that distributed rainfall measurements from weather radars should be used if the rain gauge is located more than 4 km from the catchment or if the catchment is monitored by a gauge network with a density of less than 1 gauge per 16 km^2. Based on a German survey conducted among regional state environment agencies and regional water authorities in North Rhine-Westphalia, Einfalt & Maul-Kötter (2002) conclude that the desired resolution requirements are 100 m and 1 min for rainfall runoff models. Slightly less strict requirements are suggested by Berne *et al.* (2004). Based on quantitative investigations of the space–time scales of urban catchments and rainfall, Berne *et al.* (2004) concluded that depending on the catchment size, a spatial resolution of 1–3 km and a temporal

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resolution of 1–5 min are necessary for modelling runoff in urban catchments.

The LAWR system operates with a resolution comparable to the values suggested above. However, the accuracy of the rainfall estimate is in most applications more important than the data product resolution. For most conventional meteorological weather radars, the quantitative precipitation estimate (QPE) is based on the widely accepted radar equation and the Marshall–Palmer relationship between the transmitted power, reflectivity and the rain intensity (Marshall & Palmer 1948; Battan 1973). These relations are, however, not applicable for the LAWR radar, due to limitations in the system design. Instead, QPEs are based on empirical relations. Consequently, calibration is needed in order to obtain accurate and reliable rainfall estimates from the LAWR radar.

Previously, this empirical calibration has been investigated by, for example, Jensen (2002), Jensen & Pedersen (2005), Rollenbeck & Bendix (2006), Goormans & Willems (2008), Pedersen *et al.* (2008, 2010), Borup *et al.* (2009) and Thorndahl & Rasmussen (2012). The calibration approaches differ but can in general be grouped into two: static and dynamic calibrations. The static approaches aim at identifying an average transformation model between LAWR reflectivity and rainfall, whereas the dynamic (or transient) approaches, in general, make use of continuous adjustments against rain gauges.

A linear relationship between the LAWR radar output and rain intensity is the underlying assumption by nearly all previous calibration methods. This relation is stated by the manufacturer and is attributed to the logarithmic receiver in the LAWR radar system. Pedersen *et al.* (2010) deviate slightly from the linear assumption and use a multilinear regression analysis instead. In this approach, additional information from the observation rain gauges is used for the LAWR radar–rainfall transformation. By including event duration and average rain intensity information, it was possible to improve the LAWR radar performance. Unfortunately, this method is not useable in real time as the observed event duration and average rain intensity will be available only after the rainfall has ended. Moreover, the transformation is only applicable for radar pixels, where corresponding rain gauges exist, as the study does not support two-dimensional (2D) distribution of the transformation in the whole radar domain.

Thorndahl & Rasmussen (2012) investigated three different calibration approaches. The three methods all rely on the linear assumption, but differ on how the LAWR parameters are estimated in terms of different temporal data aggregation and optimization methods. One of the three methods, the *linear regression method* has become the most widely used method for calibrating LAWR radars. However, closer inspection of the data reveal that the validity of this linear assumption is doubtful, as illustrated in Figure 1. If event accumulations are compared (as pictured in the left part of Figure 1), the linear assumption seems plausible. However, if the intensities within these rainfall events are compared directly with the dimensionless LAWR radar output (DRO) (as pictured in the right part of Figure 1) a nonlinear relation becomes apparent.

Several authors, for example Rollenbeck & Bendix (2006), Pedersen *et al.* (2008, 2010) and Thorndahl & Rasmussen (2012),

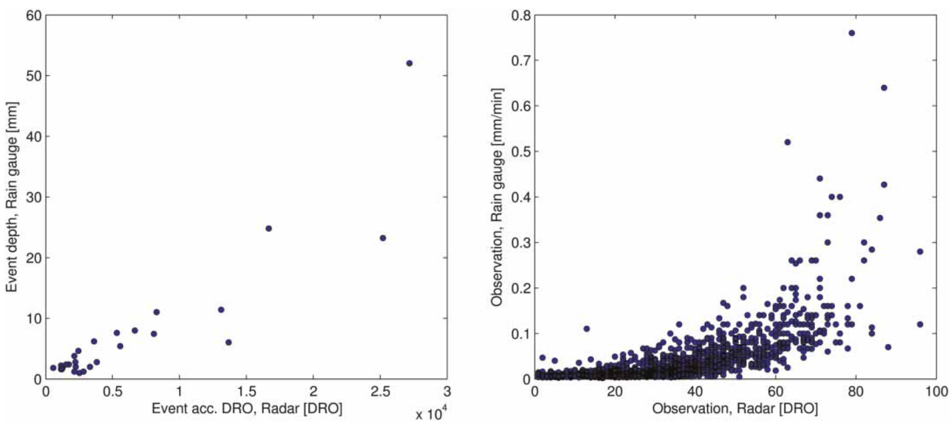

Figure 1 | Left: Event accumulated LAWR DRO (dimensionless radar output) vs. gauge measured event depth. Right: LAWR DRO vs. gauge observed rain intensity. The two plots represent the same dataset.

have illustrated that the LAWR has the tendency to underestimate high rain intensities. It is here important to mention that early LAWR radar versions suffered from saturation of the receiver close to the radar, which naturally reduces the capability of measuring peak rain intensities. However, this problem was eliminated by a system upgrade (Pedersen *et al*. 2010). Thus, peak underestimation found in newer LAWR radar studies is not caused by precipitation saturating the receiver. However, this underestimation could, in principle, be related to the linear assumption.

As a result of these findings, a new calibration method has been developed. Unlike previous solutions, the proposed method relies on nonlinear assumptions and aims at describing the whole range of rain intensities more comprehensively for the LAWR system. The performance of the developed calibration method is evaluated based on ground observations from rain gauges and compared with the performance of the *linear regression method* (Thorndahl & Rasmussen 2012), since this is currently the most widely used LAWR calibration method.

METHODS AND MATERIALS

The data and study area

The calibration method is tested and evaluated on data from the Hvidovre LAWR, located approximately 8 km southwest of Copenhagen. The radar covers the Capital Region of Denmark consisting of Copenhagen and most of the suburbs. The area is fairly densely populated and houses around 1.2 million inhabitants. Approximately half of the area covered within the quantitative range (15 km) of the Hvidovre LAWR is occupied by urban land use. Moreover, the terrain in the area is relatively flat.

A site map is provided in Figure 2 and the specifications for the radar are listed in Table 1. As pictured in Figure 2, the capital area is monitored with a fairly dense population of rain gauges, whereas 32 tipping bucket rain gauges are located within the quantitative range of the radar. The radar and gauge data used originates from the period July 1st to November 1st, 2010.

Although the working principle of the LAWR is similar to conventional meteorological weather radars, the LAWR is operated differently. While conventional weather radars mostly scan the atmosphere at different elevation angles with a narrow, torch-shaped radar beam, the LAWR scans the atmosphere continuously with a large vertical opening angle of $\pm 10^{\circ}$. Consequently, the LAWR system does not contain information about the vertical reflectivity profile of the atmosphere. Partly filled sample volumes will occur in the case of low lying precipitation at longer ranges as the

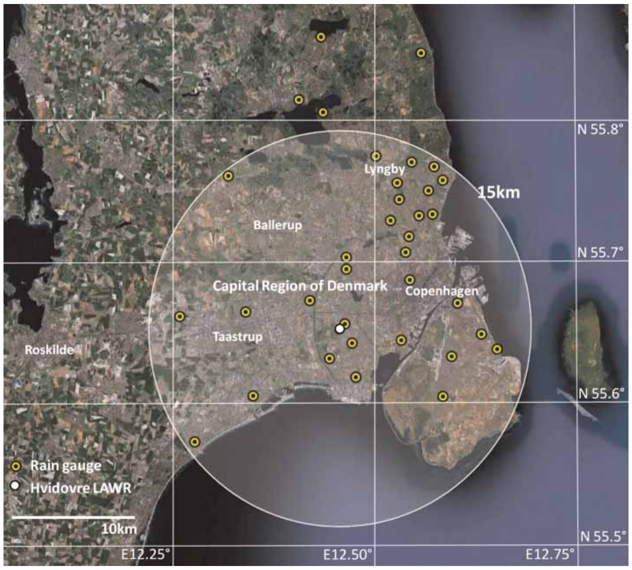

Figure 2 | Sitemap of the study area. Background map (Google maps, 2012).

Table 1 | Technical specifications of the LAWR (Pedersen *et al*. 2010)

Hvidovre LAWR	X-band (Furono 1525)
Frequency	9.41 GHz
Wave length	3.2 cm
Emission power	25 kW
Angular resolution	0.95° azimuth
Vertical resolution	$\pm 10^{\circ}$
Rotation speed	24 rpm
Scanning elevation	0°
Temporal resolution	1 or 5 min
Data products	500×500 m, max. range: 60 km
	250×250 m, max. range: 30 km
	300×300 m, max. range: 15 km
	100×100 m, max. range: 15 km
Quantitative range approx.	15 km
Data resolution	255 classes

sampling volume increases significantly with the range from the radar. Furthermore, the LAWR uses radar waves in the X-band region, which are affected by atmospheric attenuation. This combination of the large vertical integration and the attenuated X-band waves explains the relative short quantitative range of the LAWR of approximately 15 km. Beyond this range the radar is still able to detect rainfall qualitatively, but the QPE accuracy decreases rapidly from around this range (Pedersen *et al.* 2010; Thorndahl & Rasmussen 2012). Similarly, conventional C-band weather radars are able to detect precipitation at ranges up to 240 km, whereas the quantitative range is typically considered approximately 100 km.

In order to account for the increasing sampling volume and the attenuation in the LAWR output, the received radar signal S is adjusted by two corrections (Jensen 2010; Pedersen *et al.* 2010):

$$\mathrm{DRO}(r) = S(r) \cdot \gamma_{\mathrm{ATT}}(r) \cdot \gamma_{\mathrm{VOL}}(r) \tag{1}$$

where DRO is the radar output, S is the received signal, r is the radial distance from the radar, γ_{ATT} is the attenuation correction and γ_{VOL} is the volume correction. Both Pedersen *et al.* (2010) and Jensen (2010) provide detailed processing schemes of the LAWR system.

The attenuation correction is performed within the LAWR radar software (Jensen 2010; Pedersen *et al.* 2010):

$$\gamma_{\mathrm{ATT}} = 1 + \frac{\alpha \cdot \sum_0^r S_{(r)}}{C} \tag{2}$$

where r is the radial distance from the radar, α and C are correction constants ($\alpha = 1$ and $C = 1{,}200$).

The attenuation correction is performed on the raw scan line measurement including ground clutter echoes. Consequently, the attenuation correction has to be performed in the pre-processing within the LAWR software.

The volume correction is implemented as an exponential function within the LAWR software (Jensen 2010; Pedersen *et al.* 2010) and is a result of the empirical LAWR system design:

$$\gamma_{\mathrm{VOL}} = \frac{1}{A \cdot \exp(B \cdot r)} \tag{3}$$

where A and B are volume correction constants ($A = 1$ and $B = -0.0006\ \mathrm{km}^{-1}$).

Ideally, geometric corrections should be based on the radar equation; however, this is not possible due to limitations in the system design as the absolute scale of DRO is unknown.

The volume correction (Equation (3)) can be applied directly in the pre-processing of the LAWR radar data within the LAWR software. However, for this study, the distance correction is performed in the radar data calibration as an integrated part of the post-processing and transformation of the LAWR DRO into rain intensities. Thorndahl & Rasmussen (2012) found this approach most beneficial, although the model parameters will depend on the distance from the radar.

The LAWR system does not provide Doppler measurement, as most modern weather radars. Therefore, the radial speeds of detected targets are not available for clutter removal routines. Instead, the LAWR system uses dry weather clutter maps for the ground clutter detection and removal.

The last step in the LAWR data processing is the transformation of the data from polar to Cartesian data products with the resolutions and ranges specified in Table 1. The Hvidovre LAWR produces 5-min temporal averaged measurements based on 120 scan rotations, and the 500 m resolution is used.

The study area contains a dense concentration of tipping bucket rain gauges, which are all part of the national Danish network of rain gauges, managed by the Danish Water Pollution Control Committee (DWPCC) (Madsen *et al.* 1998; Mikkelsen *et al.* 1998). Location of the rain gauges is presented in Figure 2 and a summary of the gauge dataset is presented in Table 2. As illustrated by Table 2, the data period contains both a significant number of events and accumulated rain depth. Most of the gauges have captured a precipitation volume equivalent to half of the annual rainfall in Denmark.

The temporal resolution of the rain gauge time series is 1 min and a single tip measured by the gauge corresponds to 0.2 mm of precipitation. In the Danish network of rain gauges, an event is defined as a period of precipitation with at least 1 hour of dry weather prior to and after the event.

Table 2 | Summary of the rain gauge dataset with number of detected events and measured accumulated precipitation

Gauge No.	Radial distance from radar (km)	No. of recorded events	Accumulated precipitation (mm)
30,184	23.2	63	313.2
30,191	17.0	85	324.4
30,201	22.8	78	363.2
30,208	14.7	53	263.4
30,218	13.9	84	390.4
30,222	9.40	83	335.0
30,231	12.3	76	297.6
30,232	11.2	56	205.6
30,233	10.9	63	265.6
30,234	11.6	74	282.4
30,235	12.9	64	271.6
30,236	14.2	62	281.2
30,237	14.2	68	307.8
30,242	18.2	77	285.4
30,307	0.61	76	350.6
30,309	5.67	64	314.0
30,313	9.49	84	314.2
30,314	4.92	80	319.8
30,316	14.8	78	317.4
30,317	3.23	59	311.6
30,318	1.46	78	333.8
30,319	3.95	83	372.6
30,321	4.75	57	157.6
30,325	9.09	72	301.8
30,326	7.96	73	321.0
30,348	11.1	80	263.8
30,351	9.04	74	290.2
30,352	9.60	70	272.0
30,353	12.5	47	113.2
30,381	6.74	78	355.4
30,383	2.40	89	340.6
30,386	7.45	60	285.0
30,388	12.5	69	308.0
30,395	8.51	47	218.0
30,451	14.3	94	364.8

The LAWR calibration

The *linear regression method* introduced by Thorndahl & Rasmussen (2012) is used as reference for the evaluation of the new calibration method. The method uses event accumulated rain depth as aggregation of the rain gauge and LAWR data. Based on linear regression between the event rainfall depths and the accumulated LAWR DRO in the corresponding radar pixels, the gauge/radar ration (β-value) is estimated for each of the rain gauges in the calibration method. Based on the linear assumption, the β-value is then used to transform the LAWR DRO to rain intensity:

$$i = \beta \cdot \mathrm{DRO} \tag{4}$$

where i is the rain intensity, β is the gauge/radar ratio and DRO is the LAWR output.

The distance dependency of β (volume correction) is described by an exponential function in the method, and the function is estimated by fitting the β values as a function of the range. This relation is termed the β-model (Thorndahl & Rasmussen 2012):

$$\beta(r) = C_1 \cdot \exp(C_2 \cdot r) \tag{5}$$

where C_1 and C_2 are model parameters. C_1 and C_2 are purely empirical calibration parameters and vary from one LAWR to another.

The new calibration method replaces the linear assumption in Equation (4) with an exponential relation:

$$\begin{aligned} &\mathrm{DRO} > 0{:}\, i = a \cdot \exp(\mathrm{DRO} \cdot b) \\ &\mathrm{DRO} = 0{:}\, i = 0 \end{aligned} \tag{6}$$

where a and b are empirical model parameters.

The model parameters (a and b in Equation (6)) are estimated in the method by an exponential function fit between the LAWR DRO and the observed rain intensity for each of the calibration gauges. This yields a set of a and b parameters for each calibration gauge. However, similar to β these sets of parameters were found range dependent as a result of the increasing sampling volume. Therefore, empirical range dependencies for each of the model parameters are implemented by two additional exponential functions:

$$i = C_{1a} \cdot \exp(r \cdot C_{2a}) \cdot \exp(C_{1b} \cdot \exp(r \cdot C_{2b}) \cdot \mathrm{DRO}) \tag{7}$$

where C_{1a}, C_{2a}, C_{1b} and C_{2b} are model parameters.

One of the major pitfalls in the approach is that the calibration and the transformed radar QPE become highly sensitive to errors in the radar data. Abnormal high LAWR DRO values will give unrealistic translated rain intensities, due to the exponential transformation and it is therefore important that the radar data are quality controlled before being used in this model.

PRE-PROCESSING AND ERROR FILTERING

Radar data are potentially affected by multiple sources of errors. An extensive literature review on error sources in radar meteorology such as attenuation, ground clutter, abnormal beam propagation, variability of the Z–R relation, precipitation drift, etc. is provided by Villarini & Krajewski (2010).

The daily maintenance and operation of the applied Hvidovre LAWR is performed by the LAWR manufacturer DHI. It is therefore expected that the radar measurement is significantly free of stationary errors such as heading errors, timing errors, etc. However, it is possible that the radar measurement contains transient errors. In general, it is difficult to cope with many of the transient error types, because the information needed for correction is not available for the LAWR radar system as the LAWR is non-polarized without Doppler. However, empirically based filtering techniques can be applied for the LAWR system and, to some extent, correct errors in the individual radar images.

Four different error types have been identified in the present LAWR dataset: sectors with partial beam blockage, near radar ‘blindness’, insufficient clutter removal and noise in the form of individual spurious echoes and radial echo lines.

Beam blockage and ‘blindness’ near the radar

Blocking or partial blocking of the radar beam by nearby obstacles is a common problem for most weather radars. The problem can be reduced by careful site planning when the radar is installed. However, as it is the case for the Hvidovre LAWR, compromises have to be made for the location. Rollenbeck & Bendix (2006) handled partial beam blockage for a LAWR in a mountainous region by use of a terrain model for describing the blockage fraction. The situation for the Hvidovre LAWR is different; here, the terrain is flat and the beam blocking is caused by urban obstacles, such as buildings. In order to evaluate the beam blocking, DRO data are accumulated and presented in Figure 3 for the investigated data period (July 1st to November 1st, 2010).

Hvidovre LAWR is clearly affected by near radar ‘blindness’ and beam blockage in several sectors as Figure 3 illustrates. In these areas the radar is not able to detect the precipitation sufficiently. Hence, including rain gauges from these areas will affect the calibration negatively.

Therefore, the rain gauges are categorized with regard to their location in blocked or unblocked areas. This categorization is performed by evaluating the accumulated DRO of the rain gauge pixels in the radar image. The accumulated DRO in the gauge pixels is compared with the average radial scan line of the accumulated image in Figure 3, as illustrated in Figure 4.

The average accumulated scan line decreases with the range except for the closest 2.0 km. This is due to the fact that the data at this stage are not range corrected. The ‘blindness’ close to the radar is most probably caused by strong ground clutter echoes near the radar. When the radar receives strong ground echoes only a small amount of signal is available for the rain detection, as the ground clutter is subtracted in the data processing. Consequently, rain gauges closer than 2.0 km are discarded from the calibration dataset. Similarly, rain gauges located more than 15 km from the radar are also discarded from the calibration dataset, as this range is considered as the maximum quantitative range for the LAWR system.

If the accumulated DRO in the gauge pixel is significantly higher than the average accumulated scan line at the same range, this might indicate that the gauge pixel is affected by ground clutter. However, if the gauge pixel accumulated DRO is significantly lower, it might indicate that the pixel is located in a partial beam blocked area. However, due to heterogeneities in the precipitation distribution, it cannot be expected that the average signal level of a gauge pixel will match the mean signal level of the accumulated scan line exactly. The remaining rain gauges are therefore divided into two groups based on the statistics of the gauge pixel residuals from the average accumulated scan line. The residuals are log-normal distributed which is supported by a Lilliefors test with a standard significance

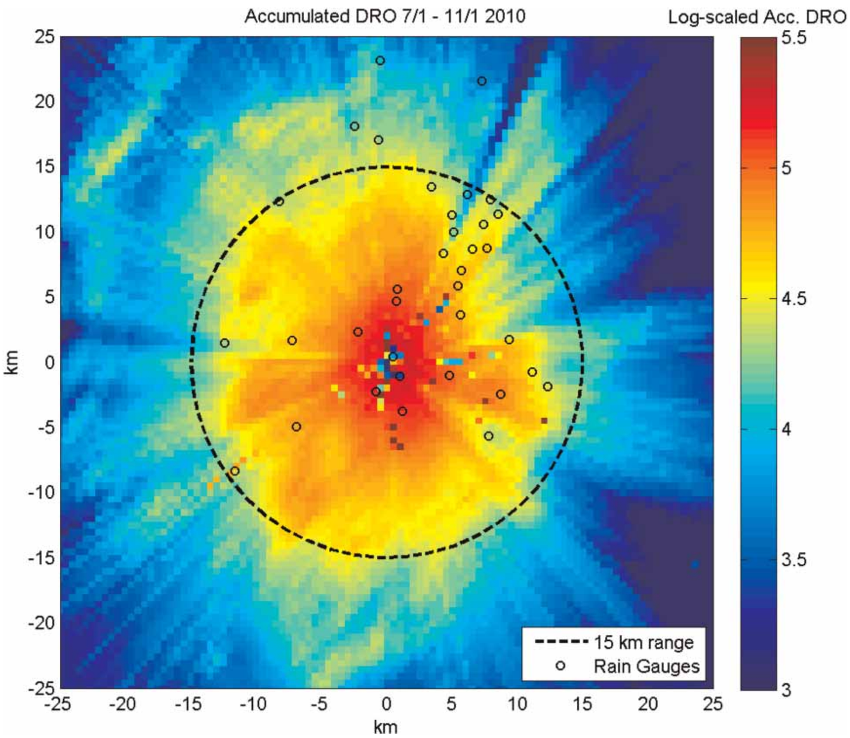

Figure 3 | Accumulated LAWR DRO from July 1st to November 1st, 2010, in a logarithmic scaled colour scheme. The circles indicate the locations of the rain gauges. The dashed line indicates the 15-km radar range. Please refer to the online version of this paper to see this figure in colour: http://www.iwaponline.com/jh/toc.htm.

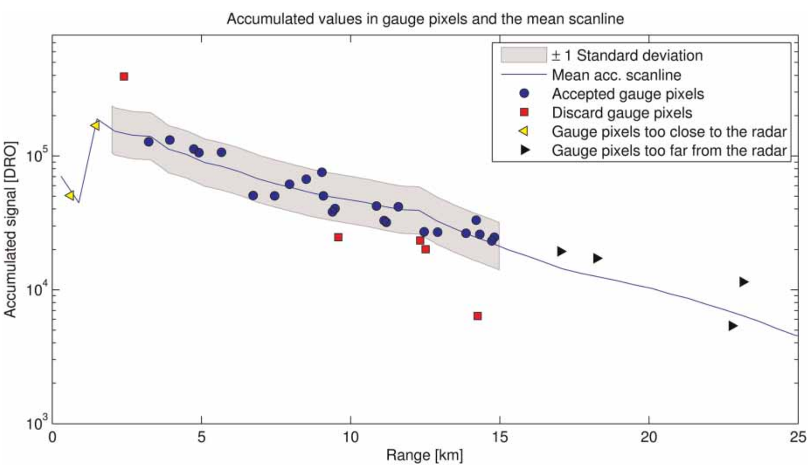

Figure 4 | Comparison of the accumulated DRO signal in the gauge pixels and the average accumulated scan line.

level of 5% on the log-transformed residuals. A threshold of ±1 log transform standard deviation of the residuals is thereby used to discard significant outliers from the remaining calibration rain gauges. Consequently, only gauges marked as 'accepted' are used for the LAWR calibration (Figure 4).

Noise removal

A vast number of suspicious echoes are observed in the radar dataset. The phenomenon is not a particular problem for the LAWR system, but may occasionally occur.

However, for the Hvidovre LAWR it occurs quite often. Two distinctive types of noise are observed in the data: individual spurious echoes and radial lines of echoes as presented in Figure 5. The individual spurious echoes are most likely to be found in the area close to the radar and may originate from insufficient ground clutter removal. The urban surroundings of the radar might be the reason for this. If the clutter from the surroundings varies slightly over time, it is difficult to suppress the ground clutter sufficiently by stationary clutter maps. The radial lines of high DRO readings occur in almost all directions in the radar data. The exact source of this noise is unknown, but may come from other microwave emitters in the area.

In order to remove or at least reduce these two types of noises from the radar data, a correction routine is developed and applied on the data prior to the calibration. The routine compares each individual cell in the radar data with a surrounding window of radar cells (5 × 5 cells), and if the examined cell is five times higher than the mean of the neighbouring cells, the values of the examined cell are corrected to the mean value. The thresholds were found to be reasonable in a series of tests of the correction routine and by inspecting the radar dataset. It was found that the precipitation was never detected with a DRO reading higher than 200, therefore an additional constraint is added so that if the window contains cells with readings larger than 250 DRO, these cells are excluded from the mean value calculation in order to make the routine more efficient. Even though the correction is rather simple and pragmatic, the routine was efficient and removed most of the noise, as illustrated in Figure 5.

RESULTS

Figure 6 illustrates an example of a regression form (the *linear regression method*) and an exponential fit form (the new *exponential method*) is illustrated for calibration gauge 30317. The *linear regression method* compares event rain depth with event accumulated DRO, whereas the *exponential method* compares 5-min rain intensities with DRO,

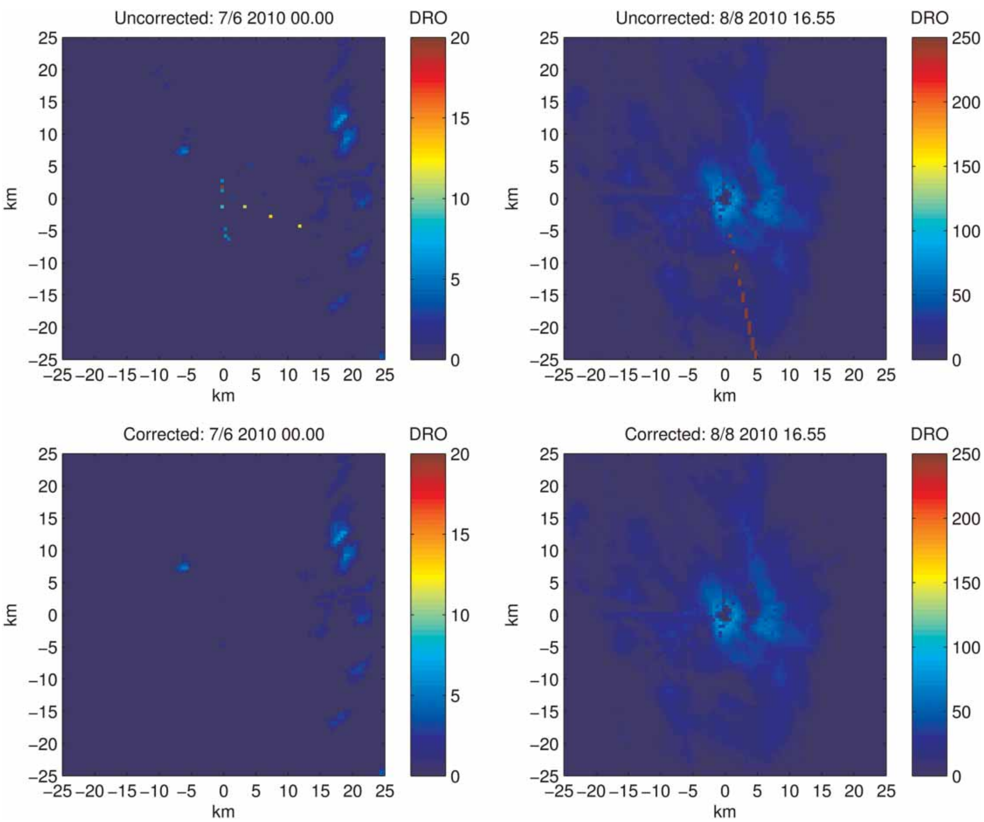

Figure 5 | Example of noise removal. Left: Individual suspicious echoes. Right: Radial line of strong echoes.

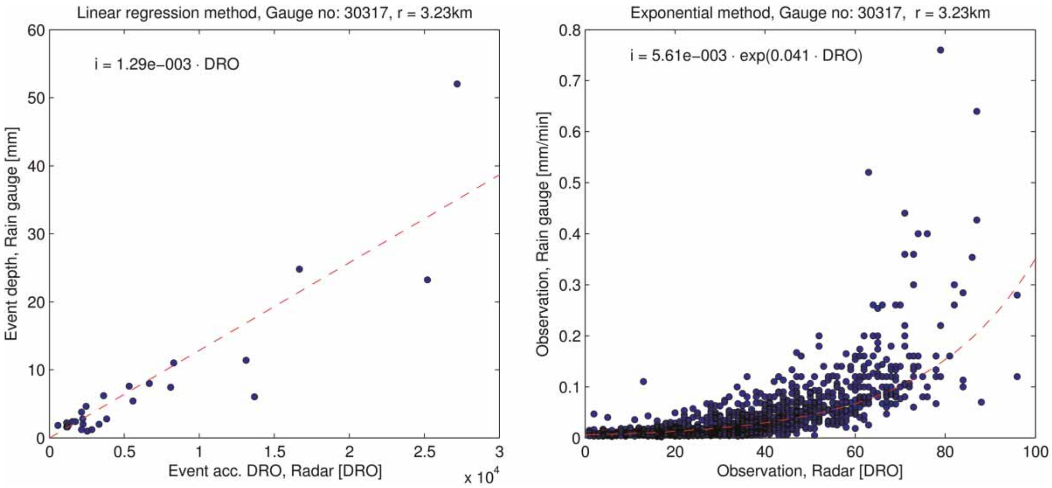

Figure 6 | Left: Linear regression of event rain depths and accumulated DRO. Right: Nonlinear relation and fit of exponential relation between DRO and rain intensity.

hence more data points exist for the exponential fit. The scatter might appear larger for exponential fit, due to the larger amount of data points. However, the nonlinearity between the LAWR DRO and the gauge measured rain intensity is obvious despite the scattered data.

The gauge/radar ratio (regression slopes or β-values) are plotted as a function of the radar range in Figure 7. As the plot demonstrates, the gauge/radar ratio (β) follows the exponential function (Equation (5)) fairly well for the investigated data period (July 1st to November 1st, 2010). A summary of the estimated model parameters are listed in Table 3.

Similarly, Figure 8 illustrates the distance dependency of the parameters in the exponential calibration method. As the result demonstrated, both variations of a- and b-parameter (Equation (6)) are fairly well described by the empirical relations in Equation (7). A summary of the estimated model parameters are listed in Table 4.

Three-hour accumulated LAWR QPE

The performance of the two calibration methods and DRO transformation models is compared based on 3-hour LAWR QPE and gauge observations. The result hereof is presented as scatter plots in Figure 9, where corresponding data from all the applied rain gauge data are plotted. Ideally, the data points should be located along the bisector (dashed line), but the scatter is considerable for both methods and models. The *linear regression method* tends to perform QPE slightly better for 3-hour rain depths larger than 10 mm. Simultaneously, this method almost consistently overestimates observations smaller than 10 mm. This overestimation is not present for the *exponential method*, whereas the underestimation of observations larger than 10 mm is slightly larger.

A summary of regression slope, Nash–Sutcliffe efficiency index (NSE) and root mean squared error (RMSE) for the two

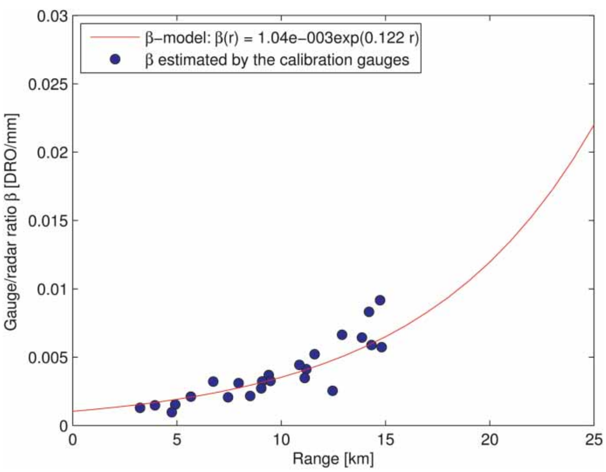

Figure 7 | Range dependency of β in the linear regression calibration method (Equation (5)).

Table 3 | Summary of the parameters estimated by the linear regression calibration method

The linear regression method Parameter	Value
C_1	1.04×10^{-3} mm/DRO
C_2	0.122 km^{-1}

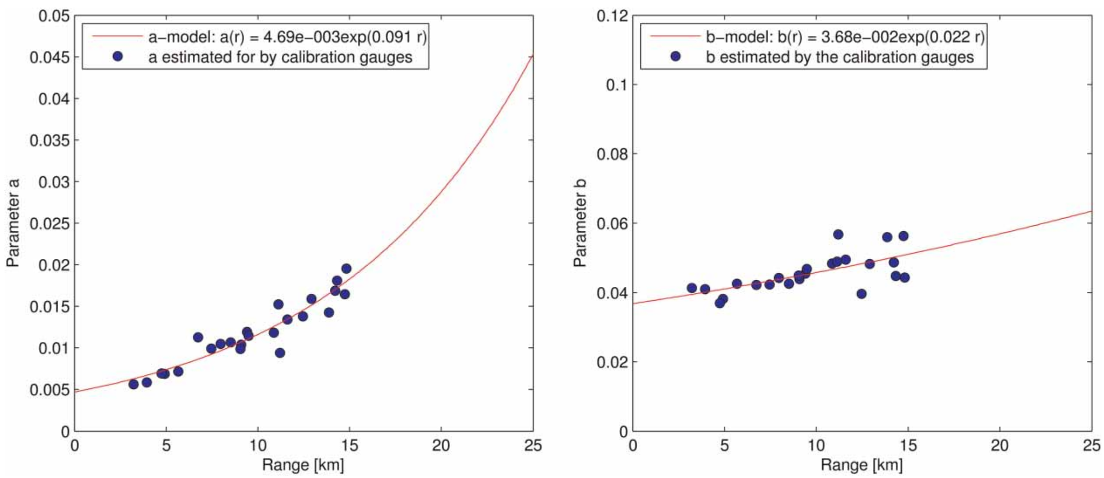

Figure 8 | Left: Distance model for parameter *a*. Right: Distance model for parameter *b*.

calibration methods and models are all presented in Table 5. If all 3-hour rain depths are included in the performance measures, the *linear regression method* performs best. However, this is only because the *linear regression method* tends to both underestimate and overestimate the gauge observations. This gives a better overall performance for all the examined measures, although the method is actually only performing well in the midrange of the observations. In contrast, the *exponential method* is performing equally for observations less than 10 mm. Consequently, the *exponential method* outperforms the linear regression approach, if the performance is evaluated on observations less than 10 mm (Table 5). Since observations less than 10 mm represent the vast majority of the observations (94%), the *exponential method* provides the best estimates most of the time.

Table 4 | Summary of the parameters estimated by the exponential calibration method

The exponential method Parameter	Value
C_{1a}	4.69×10^{-3} mm/min
C_{2a}	0.091 km^{-1}
C_{1b}	3.68×10^{-2} min/DRO
C_{2b}	0.022 km^{-1}

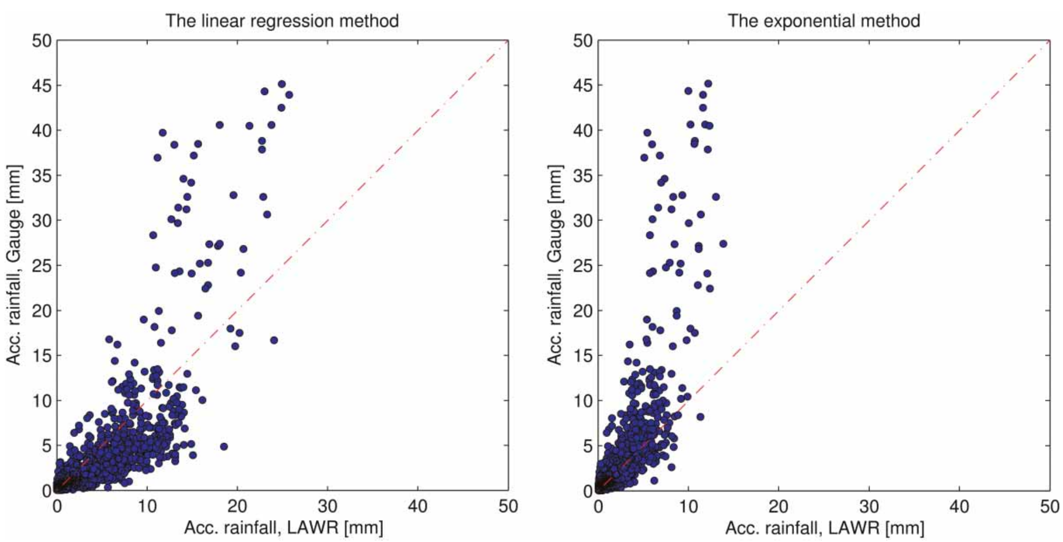

Figure 9 | Three-hour LAWR QPE vs. gauge observations from accepted gauges. Left: The linear regression approach. Right: The exponential model approach.

Table 5 | Performance of the two calibration methods

	Linear regression method	Exponential method
All observations		
Slope	0.91	1.68
NSE	0.57	0.47
RMSE	3.62	4.21
Observations <10 mm (94% of the observations)		
Slope	0.58	1.04
NSE	−0.37	0.64
RMSE	2.04	1.20

Rain intensity time series

The reason for the difference in performance between the methods has to be found within the capability of reflecting the rainfall intensities as pictured in the following three figures.

Figures 10, 11 and 12 illustrate three different rain events, where the radar QPE for both the *linear regression method* and the *exponential method* are compared with three corresponding gauge measurements at different ranges from the radar.

The rainfall event on September 12th, 2010 (Figure 10), is characterized as relatively widespread and long-lasting rainfall. The duration of the event was approximately 8 hours with an average rain depth of 9.23 mm recorded in the 35 rain gauges. It is clear that the *exponential method* outperforms *the linear regression method*. Close to the radar (top of Figure 10), the difference is relatively small. However, the *exponential method* still produces more accurate radar QPE for both the temporal dynamic of the event and total rainfall depth. At longer ranges, the improvements are even larger for the *exponential method*.

A similar pattern is observed for the rainfall event on August 8th, 2010 (Figure 11). This event was even longer lasting than the September event with a duration of approximately 24 hours and an average rain depth of 26.93 mm for the rain gauges. It is difficult to determine which method performed the best close to the radar, but the *linear regression method* has the tendency of overestimation except for the peak intensities. The *exponential method*

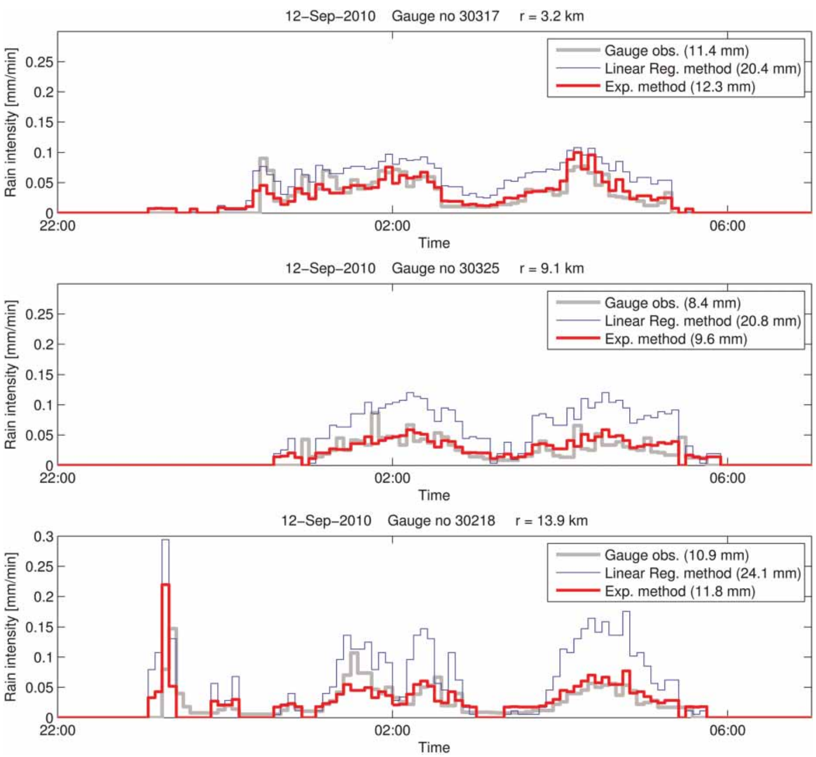

Figure 10 | Rain intensity time series from September 12th, 2010, for three selected rain gauges at varying distances from the radar. The event rain depth is listed in the key.

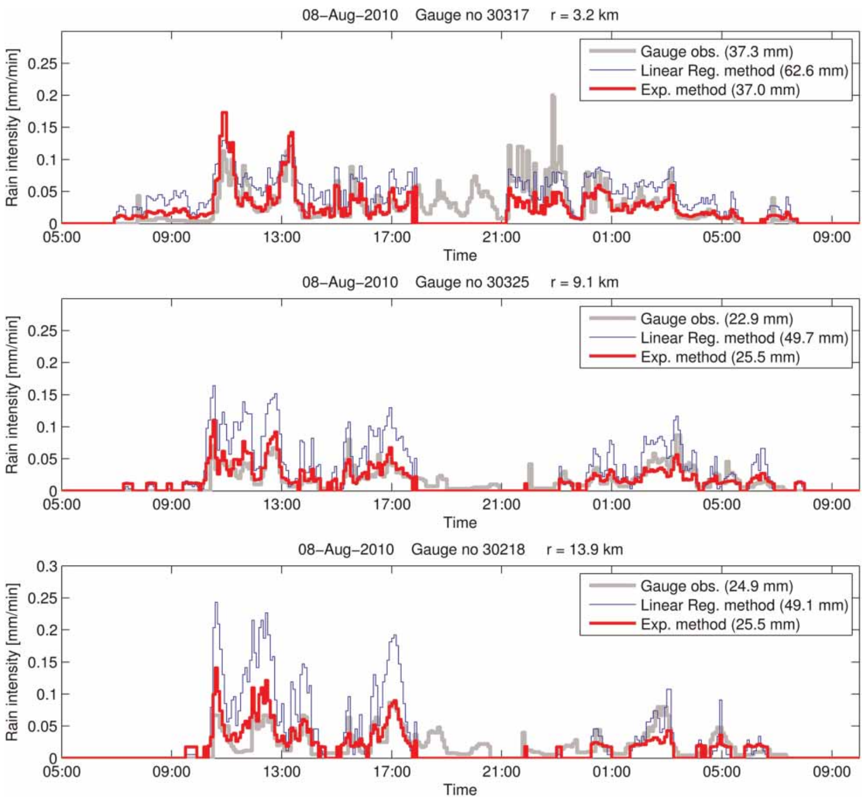

Figure 11 | Rain intensity time series from August 8th, 2010, for the three selected rain gauges at varying distances from the radar. Power failure of the LAWR occurs in the event. The rain depth of the events listed in the key is corrected for this disconnection.

overestimates the high rain intensities at the start of the event, while the higher rain intensities around 23:00 are underestimated. However, for the rest of the event, the *exponential method* captures the observation in the rain gauge fairly well, which also gives the more realistic LAWR QPE of the event rain depth. For the observations further away, the overestimation of *the linear regression method* is more severe, while the performance of the *exponential method* is similar or slightly better than at the shortest range.

The third rainfall event is also from August, specifically August 14th, 2010 (Figure 12). The event was a more severe event and released an average of 65.85 mm of precipitation in approximately 8 hours. This implies that this event contains significantly higher rain intensities than the previous, which also changes the picture of the LAWR performance. None of the methods is able to capture the high observed rain intensities regardless of time and range. The methods perform quite similarly close to the radar. The *exponential method* is slightly more dynamic during the observed high rain intensities, but none of the methods is able to perform QPE similarly close to the observations, when the observed rain intensity is higher than approximately 0.1 mm/min.

The *linear regression method* performs a better estimate of the event rain depth for rain gauge 30325 (Figure 12). However, this is achieved at the expense of the temporal dynamics of the event. Periods of low observed rain intensities are overestimated and the periods of high intensities are underestimated. Although the exponential model misses the event rain depth, this model is at least fairly close to the observations at the beginning and end of the event, where the observed rain intensity is lower than approximately 0.1 mm/min.

Both methods have problems in describing the dynamics of the observations in the rain gauge furthest away (gauge 30218). The *linear regression method* is both overestimating and underestimating, while the *exponential method* is underestimating most of the time. Even though *the linear regression method* produces radar QPE for the total rain depth closer to the observation, it is also clear that this does not necessarily mean that this method is describing the dynamic of the event sufficiently.

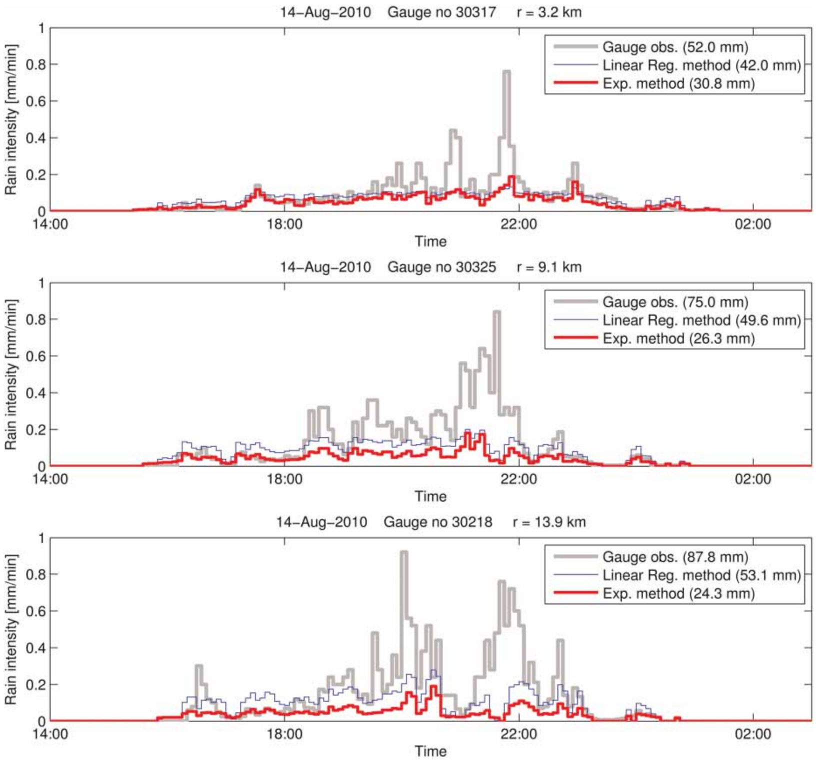

Figure 12 | Rain intensity time series from August 14th, 2010. The event rain depth is listed in the key.

DISCUSSION AND CONCLUSION

The results illustrate that the proposed *exponential method* performs better compared to rain gauge observations than *the linear regression method*. The *linear regression method* attaches great importance to precipitation events with large rain depth, due to the event-based aggregation of the calibration method. Consequently, *the linear regression method* is performing more convincing radar QPE for the event rain depth of the larger events. However, as the results also demonstrate, this does not imply that the *linear regression method* is performing realistic LAWR QPE of the temporal dynamics of the rainfall event. From the 3-hour accumulations pictured in Figure 9, it is easy to get the impression that combining the two methods could be an obvious solution as this probably would give a better overall outcome of 3-hour accumulation. However, this cannot be recommended. Although, the *linear regression method* gives larger 3-hour accumulations, the method and model is not able to perform a trustworthy description of the rainfall dynamics in any range of the 3-hour accumulations.

The proposed *exponential method* does not weight events with large rainfall depth higher, since the parameters are estimated based on rain intensities on a much smaller time scale. For the LAWR system, the *exponential method* yields LAWR QPEs which are more accurate, less biased, and describe the temporal dynamics better for the vast majority of the observed rainfall.

The advantage of this specific type of weather radar is in the detection of light precipitation, and the radar system performs best under these conditions, as the results illustrate. At the same time, the results also show that the LAWR system faces significant challenges under heavy rain. The general impression of the LAWR radar performance on more extreme events is that the radar with the linear transformation model underestimates the peaks, although the total rainfall depth is realistic. The more or less extreme events presented in Figure 12 show the same pattern; moreover, the exponential model did not perform that much better.

The direct reason for this is difficult to identify based on this work, but it is clear that the radar system is challenged even at 5-min mean rain intensities of 0.1 mm/min with a small dependency on the range. It is, however, most likely that the difficulties in measuring heavy precipitation are due to limitations within the radar processing rather than the calibration method or the radar QPE transformation model. It is obvious that if these challenges are not overcome, the application of the radar is limited. A recommended approach could be dynamic adjustments of the LAWR estimates. Applying the most recent ground observation for adjustment of the radar estimate on a shorter time scale could very well significantly improve the effects of varying sensitivity in the LAWR. The proposed nonlinear calibration method will in that effort work as a better starting point, as the method and model describes the first part of the rain intensity range. Therefore, the observations will mainly be needed to adjust the higher intensity range. In comparison, if the starting point for the dynamic adjustment is the *linear regression approach*, observations are needed for adjustments of the whole rain intensity range.

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