Scale-up of well performance for reservoir flow simulation

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ABSTRACT: The method of Ding for scaling-up in the near-well region is evaluated on a variety of two- and three-dimensional problems, including cases with partially penetrating wells and some with non-vertical wells. The method is found to work well for all cases tested although accuracy is in general lower for 3D cases and possibly for wells producing at constant bottom hole pressure. The computational effort involved in the scaling-up can be minimized by use of a reduced computational domain with only a slight degradation of the results. Both well index and modified horizontal transmissibility are required for satisfactory results, but use of modified vertical transmissibility appears to be unnecessary.

KEYWORDS: Scale-up, well performance, simulation

INTRODUCTION

The scaling-up of small-scale reservoir heterogeneity has become increasingly important with the advent of methods to describe this geological variability. For single-phase flow, a variety of methods for scale-up of permeability are now well established and the limitations of each are reasonably well understood (Durlhoksf et al. 2000; Renard & de Marsily 1997). One of these limitations is in the vicinity of wells. There the general character of the flow is likely to be radial, whereas most conventional scale-up techniques assume an approximately linear flow regime.

In particular, for reservoir simulation on a coarse grid, attention must be paid to the calculation of the well index (more correctly called the completion connection factor). This quantity relates the production rate to the difference between the pressure in the well and the pressure in the grid block in which the well is situated:

\[ q = \frac{WI}{\mu} (P_{gb} - P_{wf}) \]  

Use of the standard Peaceman (1983) formula to calculate \( WI \) may not be appropriate for two reasons. First, the formula involves the permeability of the grid block, and this may have been calculated by scaling-up under an assumption of linear flow. Second, the derivation of this formula assumes that the grid block (strictly speaking, a large region around the well) is homogeneous. Matheron (1967) has shown that no homogeneous equivalent medium exists for radial flow.

A local grid refinement around the well is one possible way of avoiding scale-up. However there may be a significant computational overhead associated with the extra grid blocks and the non-neighbour connections, particularly in problems with many wells. In addition, small grid blocks in the high flow rate region around the well can pose severe convergence problems for the simulator. As a result local grid refinement may not be a practical solution for many cases.

PREVIOUS WORK

The problem of scale-up in the near-well region has already been addressed by a number of authors. Soetawinata et al. (1997) calculate an effective permeability for the grid block containing the well using a combination of arithmetic and harmonic means based on radial flow rather than linear flow. Chen et al. (1995a, b) calculate a pseudo-skin by matching a coarse grid simulation to an analytical solution for pseudo-steady state flow with effective permeabilities obtained by power law averaging. Dupouy et al. (1998) describe a method for calculating the coarse grid \( WI \) as the sum of the fine grid values of \( WI \) adjusted for different grid block size (equivalent radius) and partial penetration. These essentially analytical methods are very rapid and relatively easy to use. However, they are not really satisfactory as any analytic approach for heterogeneous reservoirs involves approximations and there will always be some cases where it will not work.

Methods based on a fine grid numerical simulation have the potential to be generally applicable for heterogeneous reservoirs. The method of Chen et al. (1995a, b) can be generalized by replacing the analytical calculation of the pressure field with a fine grid numerical simulation. Lin (1995) proposes an alternative method in which the coarse grid \( WI \) is calculated directly from the solution of a fine grid flow simulation that has been run until pseudo-steady state conditions are reached. The motivation for Lin’s rather complex formulation comes from theoretical calculations for partially perforated wells in homogeneous reservoirs.

The most promising numerical method is that of Ding (1995). First, he calculates the pressure field for steady state incompressible flow on the fine grid model. The coarse grid is then overlain on the fine grid and the pore-volume weighted average pressure within each coarse grid block \( P_{gb} \) is found. The fine grid fluxes across each interface between coarse grid blocks are also summed to give the equivalent coarse grid fluxes.
These average pressures and coarse grid fluxes are then used to calculate both a coarse grid $WI$ and modified transmissibilities between the grid block containing the well and its neighbours. The relevant equations are:

$$WI = \frac{\mu q}{P_{gb} - P_{wf}}$$

and

$$T_{eq} = \frac{\mu F_{eq}}{\Delta P_{gb}}$$

The advantage of this method is that it models the effect of heterogeneity both on the flow from the well grid block into the well and on the flow into the well grid block from the surrounding grid. However, in validating his method, Ding published results mainly for two-dimensional problems. Also, his method involves solution of the steady-state pressure equation on the full fine grid, which is computationally expensive and probably unnecessary.

Durlofsky et al. (2000) also concluded that Ding's method was the most promising. They extended Ding's method to 3D and solved the pressure equation only on a limited region of the fine grid around each well. They tested the method on a number of examples with satisfactory results, though only vertical wells were treated. In extending the method to 3D, a natural question is whether or not the vertical transmissibilities between the well block and its neighbours, as well as the horizontal ones, should be modified. Durlofsky et al. do not address this point.

**OBJECTIVES OF THE PRESENT STUDY**

The present study aims to evaluate Ding's method on a variety of 2D and 3D problems, including cases with partially penetrating wells and some with non-vertical wells. The use of a reduced computational domain, and the need to introduce vertical transmissibility modifiers, are also investigated. Although much of our work was completed before we became aware of the work by Durlofsky et al., our results are complementary to theirs and our test cases are different in character in that we use a realistic reservoir geometry rather than simple rectangular domains.

**METHODS**

Each test begins from a fine grid model with a heterogeneous permeability distribution. A coarse grid is superimposed on the fine grid and a scaled-up permeability is calculated for each coarse grid block and each flow direction using the method of Alabert & Corre (1991). This is similar to the method of Warren & Price (1961).

For the fine grid simulations and the conventional coarse grid simulations, well indices (connection factors) for vertical wells are calculated using Peaceman's formula (1983). For horizontal and deviated wells, they are calculated from a generalization of this formula described in Appendix B.

Two separate implementations of the Ding method have been made. In the first, a single-phase flow simulation with production from a single well is run on the full fine grid model for a few time steps, until a pseudo-steady state has been reached. The pressure and flux distributions are then output by the simulator and read by a post-processing program that calculates the coarse grid well indices and modified coarse grid transmissibilities according to Ding's formulae. This procedure is repeated for each well in the model. In the second implementation, for each well, the pressure equation for single phase, incompressible flow is solved on a limited region of the fine grid model corresponding to the coarse grid blocks in which the well is completed and a buffer zone around them. The buffer zone is one coarse grid block thick in all directions, including above and below, but does not extend beyond the boundaries of the reservoir. Constant pressure ($P = 1$) is imposed on the lateral boundaries of the region and at the well ($P = 0$), with no flow allowed through the upper and lower boundaries. Note that our reduced computational domain is larger than that used by Durlofsky et al. (2000): they treat each completed coarse grid layer separately with a region just one coarse grid layer thick. The first implementation is used in all the cases discussed below except for those in the final section entitled 'Reduced computational domain'.

**SYNTHETIC TEST CASES**

**2D cases**

We first ran some test cases in 2D using the simple reservoir model shown in Figure 1. The reservoir is square, has no vertical relief and is drained by a single well at its centre. To avoid two-phase flow effects the reservoir contained no mobile water and the bubble point pressure of the oil was very much lower than the external atmospheric pressure.
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Table 1. Reservoir parameters for the synthetic test cases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir thickness (ft) and porosity</td>
<td>20, 0.2</td>
</tr>
<tr>
<td>Permeability (mD) (outside central region)</td>
<td>200</td>
</tr>
<tr>
<td>Well production rate (STB/D)</td>
<td>4000 (2D); 2667 or 1667 (3D)</td>
</tr>
<tr>
<td>Vertical/horizontal permeability ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial reservoir pressure (psi)</td>
<td>4000</td>
</tr>
<tr>
<td>Oil viscosity (cP) and formation volume factor</td>
<td>0.51, 1.08</td>
</tr>
<tr>
<td>Oil, water and rock compressibility (psi⁻¹)</td>
<td>6.9 × 10⁻⁶, 2.83 × 10⁻⁶, 3.66 × 10⁻⁶</td>
</tr>
<tr>
<td>Connate water saturation</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean central region permeability by layer (3D case) (mD)</td>
<td>200, 200, 50, 100, 300, 300</td>
</tr>
<tr>
<td>Variance (log K) by layer (3D case)</td>
<td>0.6, 0.3, 0.1, 0.3, 0.6, 0.6</td>
</tr>
<tr>
<td>Range used in all layers (3D case) (grid cells)</td>
<td>10</td>
</tr>
</tbody>
</table>

less than the reservoir pressure. Some further details are given in Table 1. Various different heterogeneous permeability distributions were inserted in the central region comprising 21 × 21 fine grid blocks around the well. A coarse grid model was constructed by replacing the entire central region by a single grid block.

Figure 2 shows results from a case in which the permeabilities were drawn at random from a log-normal distribution with a relatively large standard deviation. No correlation between permeabilities in adjacent grid cells was imposed. In the conventional coarse grid simulation, the bottom hole pressure is significantly over-estimated by comparison with the fine grid simulation. This error is almost entirely eliminated when Ding’s method is used. If only the well index calculated from Ding’s method is used (the simulation labelled ‘coarse grid/no upscaled transmissibilities’ in Fig. 2), the agreement with the fine grid is not so good, but it is still substantially better than the conventional coarse grid.

Similar results (not shown) were obtained for several other permeability distributions, including correlated random fields and sand-shale models.

3D cases, vertical wells

The same model was then used for some 3D test cases. The reservoir was divided into 6 layers, each 3.3 ft thick. In the central region, horizontal permeability distributions in the six layers were generated independently with a geostatistical simulation technique. Thus there was no vertical correlation in the model. However, the horizontal permeability distributions had a correlation length of 10 grid blocks. The geostatistical parameters are summarized in Table 1.

Two cases were run with a vertical well. In the first case, the well was completed only in layers 1, 3, 4 and 6. An oil production rate of 2667 STBD was imposed. Coarse grid models with 1, 2 and 3 equally thick layers were constructed. In the second case, the well was completed only in layers 1 and 2 (the upper two layers), a rate of 1667 STBD was imposed and a coarse grid model was used. Only thick layers were used. Ding’s method was applied to calculate coarse grid well indices and horizontal (but not vertical) transmissibilities between the central coarse grid block in each coarse grid layer and its neighbours.

Figure 3 shows that, as for the 2D cases, use of Ding’s method results in good agreement between the coarse and fine grid results, though the agreement is not as good as in the 2D cases. Use of the Ding well index only, without the modified transmissibilities, does not give such good agreement but is still a significant improvement over the conventional coarse grid simulations. Changing the vertical transmissibilities was found to have virtually no effect on the bottom hole pressures. This is because the reservoir is very thin and the ratio of vertical to horizontal permeability is 0.1. Hence most flow occurs horizontally and the effect of vertical transmissibility on well performance is negligible.

3D case, horizontal and deviated wells

For the next case, a horizontal well is placed in layer 4 of the 6-layer synthetic model. The well was orientated parallel to the x-axis, completed in 15 fine grid blocks (out of the 21 in the central region) and centrally placed with respect to both the x and y directions. For this case, a coarse grid with 3 × 3 × 3 blocks covering the central region was used, so the horizontal well is completed in 3 coarse grid blocks. The Ding method was applied to calculate the 3 coarse grid well indices and both horizontal and vertical transmissibilities between the 3 coarse blocks containing the well and their neighbours.

The results are shown in Figure 4. The Ding method again gives a substantial improvement over the conventional coarse grid results but, perhaps surprisingly, neither the modified horizontal nor vertical transmissibility multipliers have much effect. The vertical transmissibilities have little effect because the reservoir is so thin. Most flow into the horizontal well will be from the sides rather than above or below. Inspection of Figure 4 shows that the horizontal transmissibilities have a slightly greater effect on the well BHP than the vertical transmissibilities. The horizontal transmissibilities are less important than in the case of a vertical well because, in the horizontal plane, the flow into the well is approximately linear. The flow regime is thus consistent with that used in the upscaling of the absolute permeability, and little local adjustment of the horizontal transmissibility is necessary.

Finally, a case was run with a deviated well cutting through the reservoir at an angle of 85°. The well is orientated in the x-z plane, centrally placed with respect to the z direction, and intersects 6 fine grid blocks in different columns as it cuts through the 6 layers (exactly one per layer). It is not centralized
in the \( x \)-direction so that it cuts 4 coarse grid blocks in two different columns in the coarse grid with \( 3 \times 3 \times 3 \) blocks covering the central region.

Results for this case are shown in Figure 5. The Ding method works well, mainly due to the well index but with some contribution from the modified horizontal transmissibilities. The vertical transmissibilities again have essentially no effect for the reasons discussed above.

**REAL FIELD TEST CASE**

**Reservoir model**

Part of an actual geostatistical model of a mixed carbonate platform reservoir situated offshore West Africa (Biver et al. 1994; Dupouy et al. 1998) was used for the remaining tests. Several details of the original model were modified for the purposes of this research study. Figure 6 shows a top view of the grid and the location of three wells. Table 2 lists some of the principal properties of the model.

The porosity and permeability fields in the model were constructed using a fairly complex procedure. First, probability field simulation techniques were used to create a distribution of facies within the model, and to assign petrophysical classes within each facies. Within each petrophysical class, porosity values were assigned using a sequential Gaussian algorithm. Horizontal permeability was obtained from porosity by a
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Another case was run with the vertical wells shifted slightly to lie in fine grid blocks adjacent to the corners of the corresponding coarse grid blocks (positions shown by the open circles in Fig. 6: the dotted lines show the corresponding coarse grid blocks). Figure 9 shows that the error made using the conventional coarse grid model is less in this case than in the first case. The error made with Ding's method is about the same as before and is still an improvement on the conventional coarse grid. The introduction of the modified horizontal transmissibilities does have a small beneficial effect but introduction of the vertical ones does not.

The somewhat larger error for the Ding method for some of the wells in these two cases has no obvious explanation and may simply be related to the particular permeability distribution present. The minimal effect of modifying the vertical transmissibilities may again be ascribed to the small reservoir thickness (compared to its horizontal extent). It should be noted that results for wells producing at constant bottom hole pressure may be less good than for those producing at constant rate because the latter case is likely to be closer to pseudo-steady state. In principle, wells producing at constant bottom hole pressure may require the calculation and input of time varying well indices and well block transmissibilities.

Non-vertical wells
A third case was run where the 3 vertical wells were replaced by 3 horizontal and deviated wells in approximately the same locations (solid lines in Fig. 6). Wells P1 and P2 are not strictly horizontal but each lie entirely in one layer of the fine grid. Well P3 is deviated and cuts through several fine grid layers.

Results for this case are shown in Figure 10. Ding’s method gives a much better match to the fine grid results than does the conventional coarse grid. The introduction of the modified vertical and horizontal transmissibilities does not significantly improve the results.

REDUCED COMPUTATIONAL DOMAIN
Figures 11–13 show results obtained with a reduced computational domain for three cases: the synthetic case with horizontal wells, the field case with vertical wells and the field case with non-vertical wells. In all three cases, the results are not quite so good with the reduced computational domain, though the Ding method remains an improvement over the conventional coarse grid model except for well P1 in the last case (Fig. 13), where the coarse grid errors are already small.

CONCLUSIONS
We have demonstrated the importance of scaling-up well performance as well as grid block permeability when modelling heterogeneous reservoirs. Ding’s method of scaling-up both

Vertical wells
In the first case, the wells are vertical and are located as shown by the solid circles in Figure 6. Note that the wells are only completed in certain layers (Table 2). Figure 8 shows the total field production rate as a function of time for this case. The results obtained with the conventional coarse grid model are very different from those obtained on the fine grid. This large error is substantially reduced when Ding’s model is used, though an error of about 5% remains, larger than in the synthetic cases. The improvement brought by Ding’s method is entirely due to the modified well indices: introducing modified transmissibilities (horizontal and/or vertical) has virtually no effect on the results.

Deterministic $\varphi - \log(k)$ relationship for each petrophysical class. A constant ratio of vertical to horizontal permeability was applied within each facies. For each of the first three steps, a different correlation ellipsoid was used. Lateral and vertical variations in the proportions of facies and petrophysical classes were also modelled. The resulting permeability fields therefore exhibit a considerable variety of structure, as illustrated in Figure 7.

In all cases, the central well (P1) is first produced for 5 years at constant bottom hole pressure. Then it is closed and the other two wells (P2 and P3) are opened and produced for a further 5 years, also at constant bottom hole pressure. The reservoir pressure is above bubble point at all times and there is no mobile water present.

Table 2. Reservoir parameters for the field case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fine grid blocks</td>
<td>$36 \times 20 \times 60 = 43,200$ (30,294 active)</td>
</tr>
<tr>
<td>Size of fine grid blocks (m)</td>
<td>$75 \times 200 \times (c/3)$</td>
</tr>
<tr>
<td>Vertical well completions (fine grid layers)</td>
<td>$1-5$ and 46–50 (P1), 45–49 (P2), 3–9 (P3)</td>
</tr>
<tr>
<td>Geometric mean of permeability (mD)</td>
<td>$1.0$ (horizontal), $0.1$ (vertical)</td>
</tr>
<tr>
<td>Average porosity and total pore volume (m$^3$)</td>
<td>$0.08, 1.03 \times 10^8$</td>
</tr>
<tr>
<td>Initial pressure (bar)</td>
<td>$405$</td>
</tr>
<tr>
<td>Oil viscosity (cP)</td>
<td>$0.23-0.34$</td>
</tr>
<tr>
<td>Number of fine grid blocks per coarse block</td>
<td>$4 \times 4 \times 4$</td>
</tr>
</tbody>
</table>

Fig. 6. Real field test case, top view.
well index and near-well transmissibilities was selected as the most appropriate method from the literature. We have evaluated this method for a range of 2D and 3D test cases, both real and synthetic, including cases with partially penetrating wells and non-vertical wells.

Ding’s method works well for all cases tested although accuracy is in general lower for 3D cases and possibly for wells producing at constant bottom hole pressure. The computational effort involved in the scaling-up can be minimized by use of a reduced computational domain with only a slight degradation of the results. Both well index and modified horizontal transmissibility are required for satisfactory results but the use of modified vertical transmissibility appears to be unnecessary.

![Fig. 7. Real field test case, permeability fields in selected layers.](image1)

![Fig. 8. Field case, vertical wells.](image2)

![Fig. 9. Field case, vertical wells in corners of coarse grid blocks.](image3)

![Fig. 10. Field case with non-vertical wells.](image4)

![Fig. 11. Synthetic case with horizontal well, reduced computational domain.](image5)
The authors thank TotalFinaElf Exploration plc and TotalFinaElf SA for permission to publish this research.

APPENDIX A: SYMBOLS

- $F_{eq}$: equivalent coarse grid flux
- $T_{eq}$: equivalent transmissibility
- $h$: formation thickness
- $k_x$, $k_y$, $k_z$: permeability in the $x$, $y$, and $z$ directions
- $P_{wf}$, $P_{gb}$: well flowing bottom hole pressure and grid block pressure
- $q$: volumetric production rate at bottom hole conditions
- $r_w$, $r_o$: well bore radius and equivalent radius of the grid block
- $s$: skin factor
- $WI$: well index or connection factor
- $G$: grid block size
- $\mu$: fluid viscosity

APPENDIX B

For a non-vertical well, we use a generalization of the Peaceman (1983) formula to compute the well index (connection factor):

$$W_I^2 = W_{I_x}^2 + W_{I_y}^2 + W_{I_z}^2,$$

where $W_{I_x}$, $W_{I_y}$, and $W_{I_z}$ are the well index that would be calculated with the Peaceman formula for a well penetrating the grid block in the $x$-, $y$-, and $z$-directions, respectively. The well index for a well penetrating the grid block in the $x$-direction and of length $b_x$ is given by:

$$W_{I_x} = 2\pi \sqrt{b_x b_x \left[ \ln \left( \frac{r_w}{r_o} \right) + 1 \right]^{-1}},$$

where $r_{ocx}$ is given by:

$$r_{ocx} = \frac{b_x^{1.5} + (k_x/k_z)^{1.5} \Delta z^{1.5}}{(k_x/k_z)^{1.5} + (k_x/k_z)^{1.5}} (B3)$$

and similar formulae apply for $W_{I_y}$ and $W_{I_z}$.

REFERENCES


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