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The Application of Radioactive Tracers for Determination of Bed-Load Transport in Alluvial Rivers

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Radioactive isotopes have been applied for determining the transport rate of bed load in an alluvial river on the basis of: centroid velocity of the tracer particles, size and material-transporting width of mobile layer. These parameters were found by detailed measurements in the field. Computed values were produced on the basis of Engelund and Fredsøe's model on sediment transport (1976) and on the propagation of bed forms. When comparing measured and computed values, the difference was about 25%. Finally, the applicability of tracer methods for solving practical problems is discussed.

Introduction

This article describes the practicability of radioactive tracers in studies of sediment transport along the bottom of an alluvial stream in western Jutland. The investigations took place in the river Ansager, at the bridge at Lavborg east of Ansager, cf. key map in Fig. 1.

This locality was chosen because the river is alluvial and of suitable dimensions for a pilot study to allow injection and detection of tracer material. In the study area there was already a self-registering gauger, placed by The Danish Land Development Service. Moreover, monthly registrations of the transport rates of water and sediment have been made since 1970 by the Skalling Laboratory, Esbjerg, under the leadership of B. Hasholt, Geographical Institute, Univ. of Copenhagen (Hasholt 1972).

From Lavborg bridge and 500 m downstream there are two meander bends.

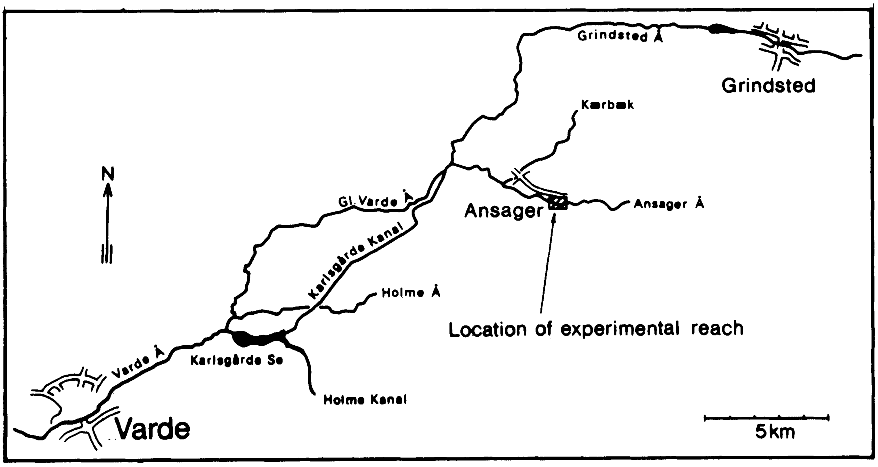

Fig. 1. Key map with location of test area.

Within this reach there are no traces of erosion, the river banks are covered with vegetation to the water surface so that prior to, or during the investigation period, lateral supplies to the river caused by erosion can be left out of account, cf. i.a. Hasholt (1974).

Experimental Methods

Spatial Integration Method

The sediment particle transport along the bottom q_b, by volume per unit width across which the transport takes place, can be deduced from the motion of the tracer particles along the bottom. Where the velocity of the tracer, expressed as the centroid movement of the longitudinal tracer distribution, equals the mean velocity of the bed particles u_t, the transport can be calculated from

$$q_b = u_t E$$

where E is the mean depth of tracer mixing.

The centroid velocity is found by the change of the position of the centroid, observed at surveys of the distribution at different times, t_1 and t_2, can be expressed by the following calculations

$$u_t = \left[\left(\frac{\int_0^\infty N_A dx\, X}{\int_0^\infty N_A dx} \right)_{t_2} - \left(\frac{\int_0^\infty N_A dx\, X}{\int_0^\infty N_A dx} \right)_{t_1} \right] \left(\frac{1}{t_2 - t_1} \right)$$

N_A is activity of tracer per unit surface area, X is distance from the origin.

For some time after being injected, the tracer will be inhomogeneously mixed with the bottom sediment. Therefore, the u_t and E values will change rapidly along with the vertical diffusion of tracer particles through the surface layers of the bed. Usable values of u_t and E are not obtained until the mixture has become completely homogeneous; subsequent variations must then be due to change in hydraulic conditions.

The applicability of the method both in steady and non-steady flows qualifies it for measurements in natural environments. Another advantage is that the tracer particles automatically integrate the effect of varying hydraulic conditions between two successive surveys. The method's theoretical background and applicability has previously been described by i.a. Courtois et Sauzay (1966), Crickmore (1967) and Rathbun et al. (1978).

Radioactivity- and Tracer Amounts

The amounts of radioactivity M (μCi) and of tracer P (kg/Ci) necessary for carrying through a tracer test can be calculated according to the procedure described by Courtois et Sauzay (1970).

Tracer Size

The tracer particles to be applied under the given hydraulic conditions are to fulfill the following 3 criteria:

1) All exposed particles should be movable, minimum Q, i.e. that the minimum discharge and the resulting bottom shear stresses must be able to move the tracer particles exposed for motion, according to Shield's criterion.

2) All particles in motion should be movable along the bottom, maximum Q, i.e. the particles must not be suspended. This is achieved by means of the criterion $w > 0.8\ U_f'$ see Engelund and Fredsøe (1976).

3) All particles should enter the normal transport in a natural way, i.e. the tracer size found according to 1) and 2) is to correspond to a main fraction of the natural bottom sediment.

In the calculations to fulfill these criteria previous measurements of the hydraulic parameters were used (B. Hasholt, personal communication) as also the duration curve for similar watercourses in the area elaborated by the Danish Land Development Service. The resulting tracer size was 500-595 μm (1.00-0.75Φ).

In the laboratory test and in the present investigation natural sand from the river Ansager was used. After sieving, the selected fraction was labelled with Cr-51 at the Isotope Centre, Copenhagen, after a process developed by Petersen (1960).

Calibration

In the laboratory a calibration was made of the detector equipment by reproducing the geometrical conditions existing in the field.

The solid scintillation counter's calibration constants α and f_o are determined to known amounts of tracer dispersed throughout sand layers of various thickness to calculate the following response function

$$f = f_0 e^{-\alpha E}$$

The calibration- and calculation procedure resulted in the following values, Table 1.

Table 1 – The resulting calibration and calculation data

isotope	α (cm^{-1})	f_o (cps/μCi/m^2)	P (gr/Ci)	M (Ci)	tracer size (Φ)
Cr-51	0.149	1.4	650	1.0	1.00-0.75

The duration of the test was estimated to be about 60 days, *A'* to roughly 2,000 m^2, and *E* to approximately 15 cm. For estimation of the values *A'* and *E,* data were applied from a previous tracer investigation with fluorscent tracer carried out at the same place, see Hasholt (1972).

Field Methods

Injection Procedure

The dosing of the tracer was made as an δ-injection through a tube placed on the bottom with 704,0 gr radioactive sediment particles with an activity level of 515 mCi. All later comparable activity measurements have been converted to time of injection after correction for background radiation and decay.

Detection

The longitudinal distribution of the tracer was measured at selected times by means of a solid scintillation detector connected with a BASC. For each measurement the detector was placed on the bottom to ensure identical geometrical measuring conditions. Control poles were established at fixed intervals along the river from a point near the place of injection, which made it possible to record the activity in exactly the same places at different surveys. At each survey the control wooden stakes were connected across the river by a steel wire furnished with 20-cm marks. In these lines activity variations with time could be recorded very precisely. The background radiation was measured in the same lines and found to

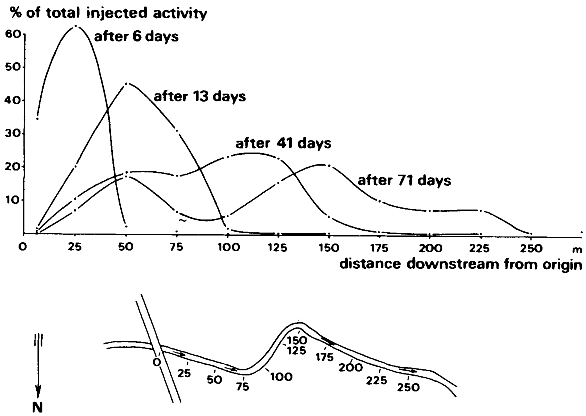

Fig. 2. The Longitudinal distribution of tracers in the river Ansager Å.

be up to 100% higher in the more fine-grained parts than in the relatively coarse-grained midstream sand areas; the water depth, however, was of minor importance for the natural background radiation. The mean activity in the single profile lines was then found by numerical integration of the lateral tracer distribution. The individual points represent mean values determined by monitoring the complete width at each measuring section.

During the period Nov. 2, 1977 – Jan. 12, 1978, four surveys were made of the longitudinal tracer distribution. On the basis of the values found, expressed in per cent of total tracer amount, the centroid velocity of tracer distribution was determined, see Fig. 2.

As appears, the distribution after 6 and 13 days differs from the one found after 41 and 71 days. Thus the shape of the distribution curve changes after second survey. When looking at the river pattern it is seen that after 13 days the tracer particles farther downstream pass the first bend of the river downstream of the injection point. The different hydraulic conditions in the meander bend compared with the straight reach, influence the sediment transport resulting in higher particle velocity after 41 and 71 days, Fig. 2, cf.i.a. Hooke (1974).

Results

Centroid Velocity – u_t

The centroid position of the tracer distribution at each survey has been calculated after the previously indicated expressions for spatial integration method, Table 2.

The centroid velocity just after injection should be neglected, as all tracer particles must have entered the bottom transport over the whole effective cross section i.e. be removed from the injection point and mixed with the mobile layer.

Table 2 – Mean velocities of tracer particles

time after injection	centroid position	distance downstream	time	centroid velocity
6 days	18.5 m	18.5 m	6 days	3.1 m/day
13 days	53.1 m	34.6 m	7 days	4.9 m/day
41 days	90.0 m	36.9 m	28 days	1.3 m/day
71 days	134.2 m	44.2 m	30 days	1.5 m/day

The homogeneous mixing between radioactive and inactive sediment particles was completed after 14 days, as no activity could be recorded at the injection point after 13 days, and only about 3% of the total injected tracer was found within the first 10 m from the injection point.

The relatively high centroid velocities after 6 days and 13 days show a higher transport rate of sediment particles in the surface of the mobile layer until a complete mixing between active and inactive sediment particles has taken place in the whole sediment layer participating in the active bed-load transport.

Mobile Layer – E

By applying radioactive tracers it is possible to determine the mixing depth of tracer particles. When detecting two or more radioactive distributions of same injected activity *A*, the detected activity at the surface depends on the depth of tracer mixing as both degree of absorption and detection geometry will change. Thus there is a relation between measured surface activity in mobile layer and mixing depth. This relation is function of the characteristics of the detector, isotope used, the detection geometry adopted and the vertical distribution of radioactive particles beneath the detector, see Fig. 3.

By calibration of the detector and according to principles described by Courtois et Sauzay (1966 and 1970) and Caillot (1971), the thickness of the mobile layer has been determined according to the following response function

$$N = T_M \frac{f_0}{\alpha}(1-e^{-\alpha E})$$

where T_M is the true distribution of concentration with depth. This will often differ from the found mean concentration T_N expressed by the ratio total activity/thickness of mobile layer.

$$T_N = \frac{\int_0^E T_M(z)\,dz}{\int_0^E dz} = \frac{A}{E}$$

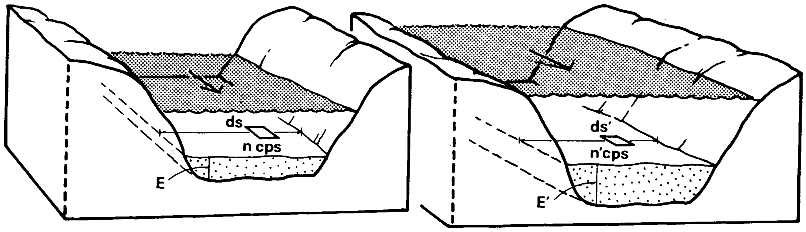

Fig. 3. Determination of mixing depth on the basis of the »countrate balance method« in two situations with identical activity in motion, where E and E' are mean thickness of mobile layer, and N and N' are total count.

$$\beta = \frac{T_M}{T_N} \Rightarrow T_M \equiv \beta \frac{A}{E}$$

$$N = \beta \frac{A}{E} \frac{f_0}{\alpha} (1 - e^{-\alpha E})$$

$$\frac{N}{A} \frac{1}{\beta} \frac{\alpha}{f_0} E \equiv 1 - e^{-\alpha E}$$

As appears from above, β is a function of E and varies according to the vertical distribution of tracer particles. Core samples of bottom sediment showed that a rectilinear, or parabolic vertical tracer distribution was common; with a parabolic distribution, E can thus be expressed as $E = 2.5\ z_m$, where z_m is the mean depth of tracer mixing. In practice, for values of E between 10 and 20 cm, β is put equal to 1.15 and with lower values for mixing depth $\beta = 1.05$, E is then found by iteration, cf. Table 3.

Table 3 –Determination of depth of tracer mixing on the basis of total-counts.

intervals	N	E
after 13 days	$3.17\ 10^5$ cps	15.9 cm
after 41 days	$3.05\ 10^5$ cps	16.7 cm
after 71 days	$3.07\ 10^5$ cps	16.5 cm

From calculations based on core samples of the specific vertical activity gradient in per cent it appears that the dispersion of tracer particles in the uppermost 4.5 cm is more than twice as high as the next 10 cm below. This indicates a max. height of 4.5 cm of the ripple forms, which are moving more rapidly than the dune forms,

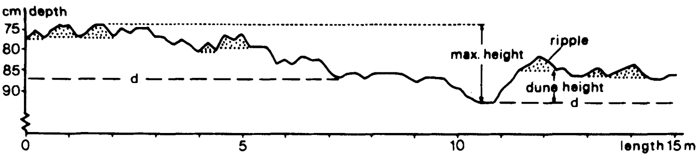

Fig. 4. Longitudinal bottom profile, length 15 m. The configuration refers to the bed forms, dunes and ripples, which were determined after Yalin's criteria (1977), cf. Thomsen (1978).

and consequently have a higher dispersion rate. On the basis of activity distribution in core samples, the highest bed-forms can be estimated to be approx. 15 cm which correspond to the thickness of the mobile layer.

For comparison, a survey was made after 37 days of a 15 m longitudinal bottom profile at 0.20 m-interval, see Fig. 4.

It appears that the maximum height of the bottom topography is about 18 cm, which is in good accordance with the mobile layer determined on the basis of the »countrate balance method«.

The Material-Transporting Width – B

The material-transporting width is normally defined as the width in the transversal river profile over which material transport can be observed. In the present test the width is determined on the basis of the lateral occurrence of tracer particles. Decisive for the size of the width is exclusively the local morphological conditions along the bottom zone of the single profiles. Thus the width ranged from 3.05 to 9.90 m along the 300 m long investigated reach. Therefore B is calculated as an arithmetically weighted average value of the single lateral tracer distributions. For the interval between 13 and 41 days in Fig. 2, B was found to be 5.44 m, and for the next interval from 41 to 71 days, B was found to be 5.40 m.

Bed-Load Transport – Q_B

The amount of sediment transport along the bottom measured by means of tracer particles does not indicate the transport through a single cross-section, but the average transport within the reach containing the tracer cloud.

As previously mentioned, the transport conditions for all tracer particles exposed in the surface layer differ from the natural conditions throughout the first 13 days after injection. Thus workable, representative values cannot be obtained before after this period when the hydraulic conditions have made the tracer particles quasi-mobile.

The calculations are based upon determination of how the centroid position of the tracer varies with time, and that the tracer behaves in the same way as the natural bottom sediment. Thus u_t defines the mean velocity of the bottom sediments and the transport in weight per unit time Q_B follows from

$$Q_B \equiv \rho_s u_t E B$$

where the density of sediment particles ρ_s = 2.65 t/m^3, Table 4.

Table 4 – The bed-load transport on the basis of computed and measured values. The indicated values of the discharge Q are average diurnal mean values.

interval	Q_B	Q	Q_{BF}	Q_{BT}
13-41 days	3.19 t/day	2.22 m^3/s	3.2 t/day	4.06 t/day
42-71 days	3.65 t/day	2.58 m^3/s	4.1 t/day	5.99 t/day

The theoretical calculation of the bed-load transport Q_{BT} was made according to Engelund and Fredsøe's sediment transport model (1976). In this, θ' was calculated on the basis of measurements in the field. Moreover the bed-load transport was calculated on the basis of propagation of the bedforms Q_{BF}, Hansen (1966). For the other procedures and the correlation with the average diurnal mean discharges, see Thomsen (1978). The transport rates are indicated in Table 4 above.

Discussion

It is demonstrated that the transport of bed-load in a river can be determined by means of radioactive tracers. The advantage by applying this technique is that the transport takes place over a longer period, and gives more details as to how the transport distributes over the cross-section.

Moreover it is demonstrated that by means of a specific measuring procedure, radioactive tracers might reveal specific fluvial-morphological conditions such as the movement of sediment particles through a meandering river section compared with a straight reach, the material-transporting width of the river, the size of the mobile layer, and the impact of each parameter. The position of a single tracer particle at a given time tells the particle's history since it was added. Decisive for its position in the bottom layer is thus the frequency of the downstream movements, the distribution of grain sizes in the bottom sediment, and the hydraulic parametres during the observation period. The integrating effect of the method makes it well-suited on the unsteady flows which often occur in natural watercourses.

When applying the spatial integration method for determination of sediment transport, the depth at which the tracer is mixed up (mobile layer) must be known in order to determine the cross-section area through which the tracer transport is taking place. This depth is determined as the average depth over which the tracer is distributed. The actual mobile layer at a given place may differ from this value dependent on the dimensions of the bed-forms and the cross-section of the river. Determination of variations in thickness of the mobile layer on the basis of the distribution of tracers in undisturbed samples of bottom material would be very useful to elucidate the transport of sediment particles in different morphological parts of a river.

Future Application of Tracers

For studies of dispersion and transport of bed-load, radioactive tracers can give exact values and are reasonably easy to handle.

So far, it has not been possible to obtain parameters for determination of the dispersion processes in bottom sediments in natural alluvial rivers. Determination of the dispersion coefficient in natural rivers by means of tracer is hampered by the long tails appearing on the concentration distribution, due to the dead-zones which delay the tracer. The dispersion is a result of the uneven lateral distribution of the velocity combined with the transversal turbulent diffusion. Radioactive tracers make it possible to test the diffusion and dispersion theories for determination of the time scales *TL* (Lagrangian's time scale) and T_i expressing the time it takes particles to mix over the whole cross-section and the time they stay in the dead-zones, respectively.

The dispersion coefficient K_F determines the size of the mixing.

$$T_L = c_L(B_r^2/D\,U_f) \qquad T_i = c_{Ti}(f_r(B_r^2/D\,u_f))$$

$$K_f = c_K(U_f(B_r^2/D))$$ cf. Petersen (1977).

The dimensionless factors c_L, c_{Ti}, c_K and c_ε, are determined on the basis of experimental data from the tracer distribution curves. Further field studies would make it possible to unveil the natural restoring of recipients of a given type. For this purpose radioactive tracers offer a workable tool to supply details on bed-load and suspended transport in rivers.

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List of Symbols

A	injected activity	μCi
A'	surface area	m^2
B	material-transporting width	m
B_r	width	m
c_K, c_L, c_{Ti}, c_ε	dimensionless factors	
d	length of dune	m
D	water depth	m
E & E'	mean thickness of tracer mixing	m
f	detector reponse in cps for 1 μCi of the tracer uniformly distibuted at any depth z.	$cps/\mu Ci$ per m^2
f_o	detector response in cps for 1 μCi of the tracer uniformly distributed on the surface of the bed	cps/μCi per m^2
f_o and α	are calibration coefficients determined in the laboratory by finding the response function of the detector for a given isotope by reproducing the geometrical field conditions. The function is exponential and written as $f \equiv f_0 e^{-\alpha E}$ -	
f_r	friction factor	
K_F	dispersion coefficient	m^2/s
M	total calculated tracer activity	μCi
N & N'	total tracer activity in time units of desintegration. Index A per unit surface area	
n & n'	local tracer activity in time units of desintegration	
P	weight of tracer particles	kg/Ci
Q	water discharge	m^3/s
Q_B	total bed-load transport rate	t/day
q_b	bed-load transport rate in solid volume per unit width per second	$m^3/s/m$
Q_{BF}	total bed-load transport rate based on the propagation of the bed-forms	t/day
Q_{BT}	total bed-load transport rate based on the sediment transport model	t/day
T_L and T_i	time scales	sec
t_1 and t_2	time	
T_M	local vertical concentration distribution of tracer particles in the mobile layer	
T_N	mean vertical concentration distribution of tracer particles in the mobile layer	
U_f	shear velocity	m/s
$U_{f'}$	effective shear velocity	m/s
u_t	tracer velocity	m/s
w	particle fall velocity	m/s
X	distance from the origin	m
z	local depth of tracer mixing	cm
z_m	mean depth of tracer mixing	cm
α	coeffient determined by the used isotope and the bed material. α is constant under identical geometrical conditions	cm^{-1}
β	is a function of E and varies depending on the type of the vertical distribution of the tracer	
ρ_S	solid density	t/m^3

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