reasonable life predictions for complex stresses. For the phase angle $\rho = 0$, the half-wave theory based on the Miner hypothesis gave the best predictions at high-stress levels, whereas the peak-stress theory based on the modified Goodman relationship gave the best predictions for low-stress levels. For the phase angle $\rho = \pi/2$, the peak-stress method based on the Gerber relationship gave the best predictions at high-stress levels, whereas either the half-wave method based on the Miner hypothesis or the peak-stress method based on the modified Goodman law gave satisfactory results at low-stress levels.

If it were necessary to select one method which gives either reasonable or conservative predictions at all stress levels, the peak-stress design method based on the modified Goodman relationship is recommended.

In the application of these conclusions it must be appreciated fully that the number of specimens used in the experimental investigation was small. Much more testing must be performed before these conclusions may be applied with a high degree of confidence.

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Discussion

H. T. Corten.[4] The authors are to be congratulated on their approach to the very difficult problem of estimating the fatigue life of members subjected to complex stress-time cycles.

As the authors have noted, statistically planned and analyzed data will be required before the precision of the design methods can be evaluated. A glance at Figs. 7 and 8 of the paper, however, gives the impression that the differences between the four methods are not very great. In fact, the scatter of the data appears to exceed the differences between the four methods in many instances. This situation is not new in studies of cumulative fatigue damage and may indicate that, with the present range of variables, the four different design methods are not separable, even when numerous specimens and statistical analysis of data are employed.

It is noted that only the fundamental and second-harmonic sine waves are employed in all the experiments. This means that there were approximately the same number of cycles of high and low stress. A study of the stress-time patterns in Fig. 1, and numerous other stress-time patterns for aircraft components[5,6,7] indicates that the number of cycles of moderate and low stress far exceed the number of high stress cycles. Further, experimental results obtained at the University of Illinois,[8] employing a two-stress-amplitude repeated block stress-time history, indicate that when the number of cycles at the high and low stresses are approximately equal, the influence of the cycles of low stress amplitude is quite small and difficult to measure. This is in agreement with the authors' results. However, when 10 per cent or less of the cycles are applied at the high stress, a condition that is closer to most measurements of service load spectrum, the cycles of moderate and low stress (even below the virgin fatigue limit) have a more pronounced influence of fatigue life than is readily measured.[8]

These considerations suggest the desirability of adding a higher harmonic wave (5th to 10th harmonic) of smaller or comparable amplitude to the existing stress history (fundamental and second harmonic), or of substituting a higher harmonic (5th to 10th harmonic), for the second harmonic. It is believed that either procedure would bring the resulting stress-time history closer to patterns measured in service and also make it possible to measure the influence of low and moderate stress.

A visual comparison of the S-N curves drawn in Fig. 6 and curves numbered 2 and 3 in Figs. 7 and 8 for $p = 0$ suggest that the fit of the data in Figs. 7 and 8 to the unaltered S-N curve would be as good as to curves numbered 2 and 3. This is simply another way of observing that the use of only the fundamental and second-harmonic stress-time pattern gives data from which it is impossible to differentiate between the different design methods.

Some of the reasons for making the initial experiments with only the fundamental and second harmonic are evident to the writer. It has become clear since the inception of these experiments that the more conclusive data will be obtained by employing the higher harmonic waves. It is to be hoped that future plans will include experiments with the higher harmonics present.

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