

# Probabilistic modelling of combined sewer overflow using the First Order Reliability Method

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## ABSTRACT

This paper presents a new and alternative method (in the context of urban drainage) for probabilistic hydrodynamical analysis of drainage systems in general and especially prediction of combined sewer overflow. Using a probabilistic shell it is possible to implement both input and parameter uncertainties on an application of the commercial urban drainage model MOUSE combined with the probabilistic First Order Reliability Method (FORM). Applying statistical characteristics on several years of rainfall, it is possible to derive a parameterization of the rainfall input and the failure probability and return period of combined sewer overflow to receiving waters can be found.

**Key words** | combined sewer overflow, First order Reliability Method (FORM), Monte Carlo sampling, MOUSE, uncertainties, urban drainage modelling

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## INTRODUCTION

Hydrodynamic urban drainage models for load prediction of drainage systems are frequently used by consulting engineers to determine if the system in question maintains the requirements defined by the authorities. The purpose of modelling is mainly to determine the number of failures in an urban drainage system during a given period of time, i.e. to attach return periods to different occurrences in the system, e.g. surcharge, flooding, or combined sewer overflow to receiving waters. However, inputs (boundary conditions), parameters, model structure, etc. are encumbered with uncertainties causing model outputs to be uncertain which affects the reliability of the return periods.

Defining an occurrence of a combined sewer overflow as a system failure, the aim of the paper is to determine the system failure probabilities and return periods. To quantify these, standard approaches make use of simulation of design storms or long historical rainfall series in a hydrodynamic model of the urban drainage system. In this paper, an alternative probabilistic method, the First Order Reliability Method (FORM), is investigated. To apply this method, a long rainfall time series is divided in

rain storms (rain events) and each rain storm is conceptualized to a synthetic rainfall hyetograph by a Gaussian shape with the parameters rain storm depth and duration (Willems 2001; Thorndahl & Willems 2007). Using a hydrodynamic simulation model, the failure conditions for each set of variables are predicted. The method takes into account the uncertainties involved in the rain storm parameterization and uncertainties related to the measurement of the rain as well as the geographical variation. In addition to these input uncertainties, a number of hydrological and hydrodynamical variables are selected and handled stochastically. In order to validate the FORM approach the analysis is also conducted using Monte Carlo Direct Sampling (MCDS) and Monte Carlo Importance Sampling (MCIS).

FORM has been extensively applied within the area of structural engineering and building technology (Madsen *et al.* 1986; Ditlevsen & Madsen 1996; Melchers 1999), and to some extent within the area of groundwater and river modelling as well as water quality modelling (Sørensen & Schaarup-Jensen 1995, 1996; Schaarup-Jensen & Sørensen

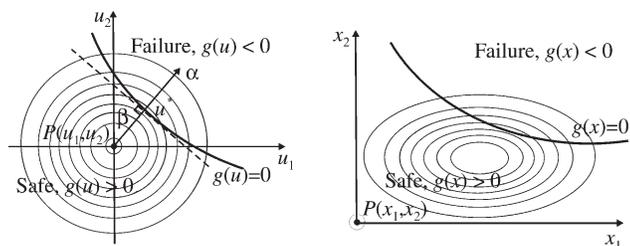
1996; Portielje *et al.* 2000), but as far as the authors know only in the context of urban drainage in Thorndahl & Willems (2007).

## METHODOLOGY

The concept of FORM is to find the probability of failure of a component in a given system. In the predefined probability distributions for each variable, the FORM algorithm searches for the combination of variable values which are most likely to cause failure of the system. This approach is unique compared to traditional long-term simulations of drainage systems as a parameterization of the rainfall input is conducted. Thus, it is possible to determine the frequency of combined sewer overflow (with uncertainty assessment) using much less computation time. Moreover, it is possible to add statistically based uncertainties to the rainfall input, which is traditionally difficult to apply to real measured rainfall input time series.

### The First Order Reliability Method

The present paper does not present the specific details of the FORM algorithm. For further details see Thorndahl & Willems (2008). In a model setup with  $i$  random variables the limit of the failure space is also called the failure function and is defined as an  $i$ -dimensional surface. In FORM this multidimensional failure surface is approximated by a hyperplane in a standard normal space. The point on the hyperplane in which the failure probability is the highest (corresponding to the vector of variable values which is most likely to occur) is labelled the design point ( $\mathbf{x}^*$ ). In Figure 1 a theoretical example of a two-variable



**Figure 1** | Example of failure surface, design point and contours of the joint probability density function in FORM with two variables. Left: Failure surface and linear approximation in a standard normal space. Right: Failure surface in the real space.

FORM analysis is shown. As two variables are applied, the failure surface is approximated by a line. Failure is defined whenever the maximum water level in the combined sewer overflow structure exceeds the overflow crest level:

$$g(\mathbf{x}) = H_{\text{crit}}(\mathbf{x}) - H_{\text{max}}(\mathbf{x}) = 0 \quad (1)$$

$H_{\text{max}}(\mathbf{x})$  is the maximum water level in the overflow structure, and  $H_{\text{crit}}(\mathbf{x})$  is the crest level.  $\mathbf{x}$  is a vector of random variables. From this, a value of the failure function  $g$  smaller than zero corresponds to failure (overflow). The failure probability,  $P_f$ , within one rainfall event in the observation period  $P_t$  is defined as (Melchers 1999):

$$P_{f,\text{FORM}} = P(g(\mathbf{x}) \leq 0) = \Phi(-\beta) \quad (2)$$

$\Phi$  is the standard normal distribution function, and  $\beta$  is the Hasofer & Lind reliability index, which is the minimized distance perpendicular from the linearized failure surface (the point with the highest joint probability density) to the origin in a standard normal space (cf. Figure 1 left).  $\beta$  represents the point with the largest failure probability, given the probability distributions of  $\mathbf{x}$ . FORM is based on standard normal independent variables, and all variables in the standard normal space ( $\mathbf{u}$ ) are transformed into in the real space ( $\mathbf{x}$ ) using inverse transformation. From an initial guess of  $\mathbf{u}$ , a transformation to the  $\mathbf{x}$ -space is performed and the MOUSE model is evaluated with these values. FORM is based on an iteration procedure in which new values of  $\mathbf{u}$  are calculated until convergence of  $\beta$  and  $\mathbf{x}$  (or  $\mathbf{u}$ ) is obtained. The gradient vector of the failure surface is found using a central finite difference approximation for every variable. This means that the model must be executed twice for every variable. The return period of the event corresponding to the design point is calculated by:

$$T = (P_f \cdot E / P_t)^{-1} \quad (3)$$

$E$  is the number of rainfall events in a given period of time  $P_t$  and the number of failures per time period  $f_p$  (most often in years) is calculated by:

$$f_p = T^{-1} \quad (4)$$

A disadvantage of FORM is that it requires good initial guesses of variable values (especially if more than two

variables are implemented). The algorithm often finds a local minima on the failure surface instead of the global one.

Therefore, FORM is tested using a Monte Carlo direct sampling (MCDS) technique. Random values are sampled from the standard normal distribution and transformed to the real space as explained above. The failure probability is then calculated (Melchers 1999):

$$P_{f,MCDS} = \frac{1}{N} \sum_{n=1}^N I(g(\mathbf{x}) \geq 0) \quad (5)$$

$N$  is the total number of simulations, and  $I$  is an indicator function ( $I = 1$  if failure and  $I = 0$  if no failure). Simulations are performed until convergence on  $P_f$ . This approach can be used to validate the linear approximation of FORM, as the whole variable space is simulated. The MCDS approach samples the whole variable space, but in order to test the linear approximations in FORM it can be advantageous only to sample around the design point ( $\mathbf{u}^*$ ) with a specified standard deviation ( $\sigma$ ), using a Monte Carlo Importance Sampling (MCIS) technique (Sørensen 2004):

$$P_{f,MCIS} = \frac{1}{N} \sum_{n=1}^N I(g(\mathbf{x}) \geq 0) \frac{f_U(\sigma \cdot \mathbf{u}_n + \mathbf{u}^*)}{f_U(\mathbf{u}_n)} \sigma^n \quad (6)$$

$f_U$  is the joint density function of the standard normal density functions.

### Sensitivity measures

Using FORM, it is possible to define two different sensitivity measures to determine the relative sensitivity of every variable regarding the model output (Melchers 1999):

- The  $\alpha$ -vector is a unit normal vector to the failure surface at the design point, cf. Figure 1, left.  $\alpha_i^2$  is a measure of the percentage of the total uncertainty associated with the stochastic variable  $i$ . The sum of all  $\alpha_i^2$  equals 1. Total uncertainty refers to the total uncertainty implemented in this analysis. There might be other uncertainties which is not included.
- The omission sensitivity factor ( $\zeta_i$ ) determines the relative importance of the failure probability by assuming that the stochastic variable  $i$  is fixed, i.e. it is considered deterministic (Madsen 1988):

$$\zeta_i = \frac{1}{\sqrt{1 - \alpha_i^2}} \quad (7)$$

### Setup of the MOUSE model and randomization of variables

The commercial urban drainage model MOUSE 2005 (DHI 2005) features advanced hydrological and hydraulic simulations of a complete urban catchment and drainage system. The model setup applied in this paper is based on a well calibrated setup of the Frejlev catchment in the northern part of Denmark, as described in Schaarup-Jensen et al. (2005), Thorndahl et al. (2006), Thorndahl & Schaarup-Jensen (2007). The choice of variables in this paper is also based on these references. The model is divided in two sub models, the surface runoff model and the pipe flow model.

The hydrological part of the surface runoff sub model is governed by two parameters: (1) the *hydrological reduction factor* ( $\varphi$ ) determining the part of the impervious area contributing to the runoff and (2) the *initial loss* ( $i$ ) which is the hydrological loss due to wetting and filling of terrain depressions. These two parameters are considered global variables, i.e. the same value is implemented for every catchment. The *hydrological reduction factor* is also used to implement an error term on the rainfall input, see paragraph: *Conceptualization of rainfall input*. The flow routing on the surface can be modelled in different ways using the MOUSE model. In this paper the *Time Area model* is applied. This is based on a constant *concentration time* on the surface (DHI 2005). Values of this variable are sampled from a uniform distribution based on Thorndahl et al. (2008). All variables and distributions are presented in Table 1. The geometrics of the drainage system are based on technical maps from the Municipality of Aalborg. In the model setup these values are kept deterministic. Parameters related to the loss of energy are made stochastic, i.e. the friction loss in pipes (*the Manning number*) and the headloss in outlets from manholes. The pipes in the drainage system are of different materials with different roughness, e.g. plastic, smooth or normal concrete. The Manning number is considered a global variable, and therefore these are drawn fully correlated and normally

**Table 1** | Variables and chosen probability distributions

No.	Variable	Distribution	Parameters
1	Rainfall event duration, $t_d$ (min)	2-comp. exp.	$\beta_1 = 160, \beta_2 = 50, p = 0.57$
2	Rainfall event depth, $d$ (mm)	2-comp. exp.	$\beta_1 = 7.0, \beta_2 = 1.8, p = 0.20$
3	Error on rainfall event depth, $E_d$ (mm)	Normal	$\mu = 0, \sigma = 0.48$
4	Water level error, overflow structure, $E_{Hmax}$ (m)	Normal	$\mu = 0.002, \sigma = 0.003$
5	Hydrological reduction factor, $\varphi$ (-)	Normal	$\mu = 0.49, \sigma = 0.23$
6	Initial loss, $i$ (mm)	Uniform	$x_{min} = 0, x_{max} = 0.001$
7	Surface concentration time, $t_c$ (min)	Uniform	$x_{min} = 1, x_{max} = 10$
8	Manning number, $M$ ( $m^{1/3}/s$ )		
	Smooth concrete	Normal	$\mu = 85, \sigma = 5$
	Normal concrete	Normal	$\mu = 75, \sigma = 5$
	Rough concrete	Normal	$\mu = 68, \sigma = 5$
	Plastic	Normal	$\mu = 80, \sigma = 5$
9	Headloss factor, $K_m(-)$		
	Round edged outlet	Uniform	$x_{min} = 0, x_{max} = 0.5$
	Sharp edged outlet	Uniform	$x_{min} = 0.25, x_{max} = 0.75$

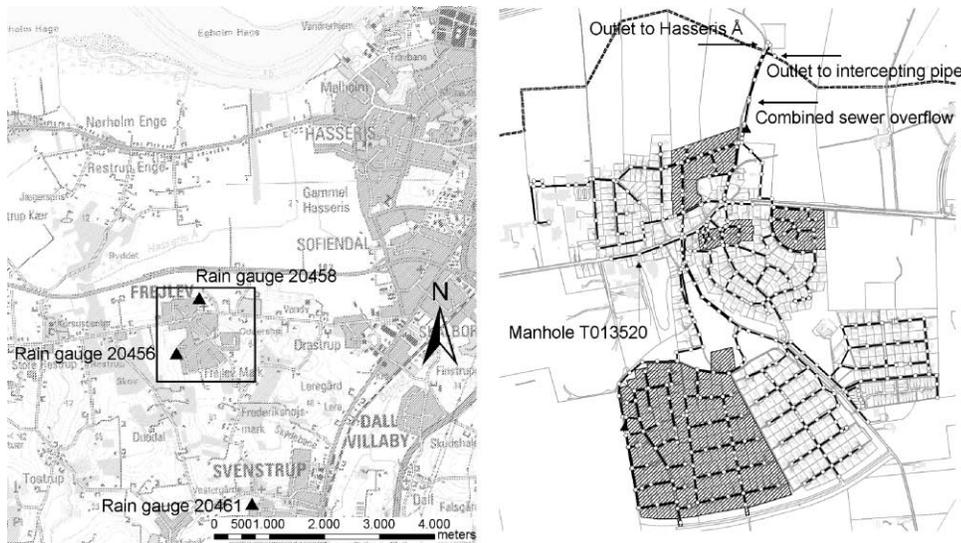
distributed. Values of the *headloss factor* are also drawn fully correlated depending on whether the outlet is round edged or sharp edged, but as no preferences are given to the distribution of the variable, a uniform distribution is applied.

### Conceptualization of rainfall input

In Thorndahl & Willems (2007) the rainfall event duration  $t_d$  and the rainfall event depth  $d$  are parameterized by two-component exponential distributions using 18 years of data from the Svenstrup rain gauge (no. 20461, Figure 2). It is shown that these two variables are the most important in modelling of combined sewer overflow. This is reasonable, at least in the Frejlev catchment, as a large inline retention basin is located just upstream from the overflow structure. This pipe basin fills up slowly and smoothes out the hydrographs, neglecting the peaks. Therefore it is primarily the runoff volume (the area under the hydrograph, given by the rainfall depth and duration) which induces the overflow. On the contrary Thorndahl & Willems (2008) shows that rain intensity peak values for different aggregation levels are decisive for modelling of surcharge and flooding in manholes.

By sampling correlated values from the exponential distributions it is possible to generate synthetic rain storm

events with a truncated Gaussian shape (Willems 2008). As this synthetic event is obviously a simplification of the real events, Thorndahl & Willems (2008) investigated the errors in the maximum water level prediction ( $E_{Hmax}$ ), introduced by this conceptualization. It was found that the errors could be parameterized by a normal distribution, in which the data was transformed with a Box-Cox transformation to account for heteroscedasticity (non-constant variance). The error introduced is of minor magnitude due to the overflow's primary dependence of the runoff volume and not the peaks. Thus, an event with more than one peak does not necessarily induce a larger error. One of the advantages of modelling with synthetic rainfall events is the possibility to implement uncertainties on the rainfall input. Two types of rainfall input uncertainty are considered in this paper. The first is the uncertainty introduced by not applying a geographical variability over the catchment. This uncertainty is implemented implicitly within the hydrological reduction factor (Figure 3), as the scatter around the regression line obviously is due to imperfectly uniform distributed rainfall events. The scatter is fitted to a normal distribution, cf. Table 1. The second type of rainfall input uncertainty is the uncertainty introduced when using a rain gauge which is not located within the catchment.



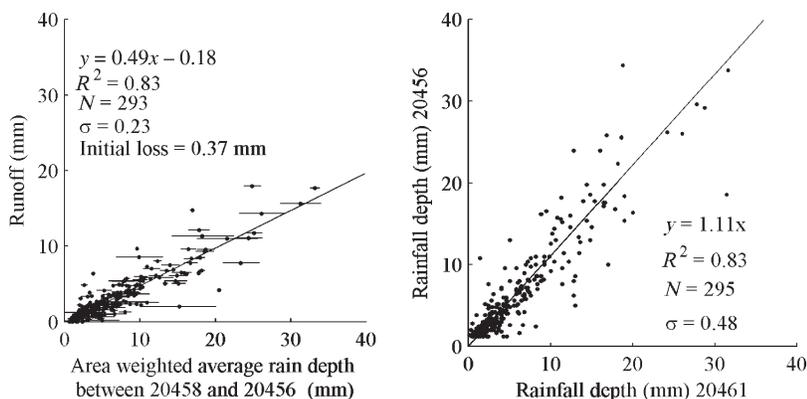
**Figure 2** | Left: The Frejlev catchment and surroundings. The black triangles are rain gauges. Right: close up of the Frejlev catchment. The hatching marks the areas with separate sewer system and the rest is a combined system.

The Svenstrup rain gauge (20461, Figure 2) is used for the parameterization of the rain as it is the longest of the local series. Using this gauge entails a small uncertainty due to its placement approximately 3.5 km from the centre of Frejlev. This uncertainty is investigated in Figure 3 (right), in which the rainfall depths from gauge 20461 are plotted against the depths from 20456, in a 9 year period. It is obvious that there is a small bias as well as some scatter. This error ( $E_d$ ) is implemented as an additional variable added to the synthetic rainfall depth  $d$ , as the scatter can be fit to a normal distribution. This variable clearly accounts

for some of the error in the geographical distribution of the rain fall input, thus this type of input uncertainty is treated as a lumped uncertainty.

## RESULTS

The first-order reliability method finds the design point ( $\mathbf{x}^*$ ), i.e. the set of variable values with the highest failure probability in terms of combined sewer overflow, corresponding to the values found in Table 2. It is seen that the rainfall event with the highest failure probability or smallest



**Figure 3** | Left: Calculation of the hydrological reduction factor and initial loss from an area weighted rain depth of two rain gauges. The runoff is calculated as the runoff volume per event divided by the impervious area. Horizontal lines indicate the individual values of the two gauges. Right: Event depth correlation between rain gauge 20456 (within the Frejlev catchment) and gauge 20461 (3.5 km from the centre of Frejlev).

**Table 2** | Results of the FORM analysis.  $x_i^*$  is the variable values in the design point and  $\alpha_i^2$  and  $\zeta_i$  are sensitivity measures (see the paragraph *Sensitivity measures*)

No.	Variable	$x_i^*$	$\alpha_i^2$	$\zeta_i$
1	Rainfall event duration, $t_d$ (min)	52.60	0.0186	1.0094
2	Rainfall event depth, $d$ (mm)	3.91	0.9163	3.4564
3	Error on rainfall event depth, $E_d$ (mm)	0.058	0.0095	1.0048
4	Water level error, overflow structure, $E_{Hmax}$ (m)	0.002	0.0001	1.0001
5	Hydrological reduction factor, $\varphi$ (-)	0.543	0.0553	1.0289
6	Initial loss, $i$ (mm)	0.000	0.0001	1.0001
7	Surface concentration time, $t_c$ (min)	5.49	0.0000	1.0000
8	Manning number (smooth concrete) $M$ ( $m^{1/3}/s$ )	85.1	0.0000	1.0000
9	Headloss factor (Round edged outlet) $K_m$ (-)	0.249	0.0000	1.0000

return period has a duration of 53 min. and a depth of 3.9 mm. The optimum of the hydrological reduction factor ( $\varphi = 0.54$ ) is somewhat larger than the mean value ( $\varphi = 0.49$ ). This is due to the large correlation with the rain depth. In order to maintain the same runoff volumes a small rain depth, which has the highest probability (cf. the exponential distribution), will cause a high value of the reduction factor. Examining the two sensitivity measures, the most important variable is by far the rainfall depth, which constitutes 92% of the total uncertainty. Subsequently, the rainfall duration and the hydrological reduction factor represent approx. 2% and 6% of the total uncertainty, respectively. The other variables are negligible in terms of combined sewer overflow. However, if this analysis was conducted on flooding in a specific manhole instead of overflow, the first and second order variables, concentration time, Manning number, and headloss is expected to be more important, as they are more decisive for the hydrograph peaks.

During the analysis it was observed that FORM requires a good choice of initial values in order to find the design point as the global minimum. Especially for the two variables concerning the rainfall input, as small changes in the values cause a great change in the probability due to the exponential distributions.

## DISCUSSION

Using FORM, a failure probability of 0.105 (corresponding to 20.5 failures per year) is predicted (Table 3). Validating

the method applying the MCIS simulations, a failure probability of 0.108 (corresponding to 21.1 failures per year) is found. This indicates that the fit of the hyperplane to the nine-dimensional failure surface is a valid method of finding the failure probability, despite the small deviation. The MCDS, which is considered the most reliable method of the failure predictions (as the whole variable space is sampled), deviates insignificantly from MCIS.

One might consider using the second order reliability method (SORM) instead, which is based on a multidimensional second order polynomial approximation of the failure surface, but as the errors introduced by linear approximations in FORM are small, this is not of interest. Furthermore, convergence of SORM is empirically even more difficult compared to FORM, due to the numerical assessment of the second order derivatives.

Since 2004 the municipality of Aalborg has registered the number of combined sewer overflows and their durations in Frejlev. 22, 17, and 25 overflows were registered in the years, 2004, 2005, and 2006, respectively

**Table 3** | Failure probabilities etc. with FORM, MCDS, and MCIS

	FORM	MCDS	MCIS
Failure probability, $P_f$	0.1045	0.1098	0.1075
Return Period, $T$	0.0487	0.0464	0.0473
Failures per year, $f_p$	20.5	21.5	21.1
No. of iterations	13*	–	
No. of model simulations	247	3000	500
Simulation time (hours)	2.1	25	4.2

\*The number of iterations is very dependent on the initial values of  $\mathbf{u}$ . The simulation time of a traditional long-term simulation is approximately 10 hours.

(overflow events with less than 1 hour in between are counted as one). This is in the same order of magnitude as predicted with the three methods and the measurements can therefore not be used to accentuate if one of the methods predicts the failure better than the other.

Despite the consistency between the predicted failures per year and the observed, some uncertainty is still associated with conceptualisation of the rainfall events, as the rainfall events are treated individually. In reality two small rainfall events within a short span of time might induce an overflow which is not considered in the present. However examining the rainfall time series this problem is rather insignificant as the system concentration time is in the same order of magnitude as the minimum time between events. Thus, the runoff is at a minimum when another rainfall events starts. The problem might be more significant for larger catchments with larger concentration times. In that case more events should be combined e.g. using a Poisson process (Willems 2001).

The choice of variables and their distributions are indeed empirical. This will affect the results of this analysis. However, some indication of a good and representative choice of variables and distributions is present, as the three techniques predict in the same order of magnitude as observed.

The return periods in this paper are only associated with the rainfall variables, i.e. all other variables are kept fixed in time. One might consider if a return period should be added to some other variables as well, e.g. the hydrological reduction factor, as this might also vary in time. This is, not investigated in the present paper.

Thorndahl & Willems (2008) showed that the method presented is very applicable in prediction of surcharge and flooding as well, but it is beyond the limits of this paper to describe this further. Nevertheless, this represents a potential alternative to simple design methods such as synthetic rain generation based on intensity–duration–frequency (IDF) curves or Chicago Design Storm (Kiefer & Chu 1957).

## CONCLUSION

It is concluded that the presented conceptualization of the rainfall input in an urban drainage model without crucial

affects on the modelling accuracy, can be used as an alternative to traditional long term predictions of combined sewer overflow. The First Order Reliability Method has been validated methodically using Monte Carlo Direct Sampling and Monte Carlo Importance Sampling, showing similar results. Thus, the simplifications in FORM are negligible in terms of predicting occurrences of combined sewer overflow. Moreover, it is observed that both FORM as well as the Monte Carlo sampling methods predict the number of overflows per year in the same order of magnitude as observed. Using FORM it is possible to reduce the simulation time to approximately 20% of the simulation time using traditional long-term simulations (Table 3).

The prediction of combined sewer overflow is shown to be very dependent on the rainfall input variables and to some extent on the hydrological surface variables, which are also the variables that contain the highest level of uncertainty. However, the variables that are governing the temporal flow variations in both surface runoff model and pipe flow model are shown to be negligible in prediction of overflow.

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