ERRATA

The paper "Surfactant Effects on Fluid-Elastic Instabilities of Liquid-Lined Tubes: A Model of Airway Closure," by D. Halpern and J. B. Grotberg, published in the JOURNAL OF BIOMECHANICAL ENGINEERING, Vol. 115, August 1993, pp. 271–277, contained the following errors.

It recently came to our attention that the continuity Eq. (3), given in Halpern and Grotberg [3] has a term missing. The continuity equation is important in the derivation of the evolution equations for thin liquid films coating flexible tubes that are studied in [3]. Equation (3) is meant to be a leading order approximation to the exact continuity equation in cylindrical coordinates:

$$\frac{\partial w}{\partial z}+\frac{\epsilon}{r}\frac{\partial}{\partial r}(ru)=0 \tag{1}$$

where r and z are the radial and axial coordinates, u and w are, respectively, the radial and axial components of velocity, and $\epsilon=1-b/a\ll 1$ is the dimensionless unperturbed film thickness. The exact form of the kinematic boundary condition at the air liquid interface, $r=1+\epsilon(h(z,t)-1)$, is given by

$$\frac{\partial h}{\partial t}=u_s-w_s\frac{\partial h}{\partial z} \tag{2}$$

where the subscript s denotes that the variable is evaluated at the air-liquid interface. Alternatively, Eq. (2) can be written in terms of the axial flow rate, Q_w:

$$\frac{\partial h}{\partial t}=\frac{1}{1+\epsilon(h-1)}\left((1+\epsilon\eta)\frac{\partial \eta}{\partial t}+\frac{\partial Q_w}{\partial z}\right) \tag{3}$$

where Q_w is given by

$$Q_w=\epsilon^{-1}\int_{1+\epsilon(h-1)}^{1+\epsilon\eta} wrdr \tag{4}$$

Since the film is initially thin, a new coordinate system (y,ζ) is introduced where $y=\eta(z,t)+\epsilon^{-1}(1-r)$ and $\zeta=z$. To leading order in ϵ, Eq. (1) becomes

$$\frac{\partial w}{\partial \zeta}+\frac{\partial \eta}{\partial \zeta}\frac{\partial w}{\partial y}-\frac{\partial u}{\partial y}=0 \tag{5}$$

The middle term in the above equation is missing from Eq. (3) in Halpern and Grotberg [3]. However, the evolution Eq. [7], in [2] is still correct because it was derived in terms of the axial flow rate, i.e. using Eq. (3) above. To leading order in ϵ, the axial flow rate is given by

$$Q_w^a=\int_0^Y wdy=-\frac{Y^3}{3}\frac{\partial p}{\partial \zeta}+\frac{Y^2}{2}\frac{\partial \bar{\sigma}}{\partial \zeta} \tag{6}$$

where $Y=1+\eta-h$, and both the pressure, p, and the surface-tension, σ, are independent of y. We then substitute the above approximation into Eq. (3) to get Eq. (7) in the original paper [3], without making any further expansions in powers of ϵ:

$$\frac{\partial h}{\partial t}=\frac{1}{1+\epsilon(h-1)}\left((1+\epsilon\eta)\frac{\partial \eta}{\partial t}+\frac{\partial Q_w^a}{\partial \zeta}\right) \tag{7}$$

The subtlety here is that z should be placed with ζ in Eqs. (5)–(7) of [3] without producing any errors in the final evolution Eq. (7). This method has also been used in similar problems by Gauglitz and Radke [1] for rigid tubes and Halpern and Grotberg [2] for flexible tubes.

If we use the approximate form of the continuity Eq. (2), and express the kinematic boundary condition, (2), in terms of the approximate axial flow rate, $Q_w^a=\int_0^Y wdy$, then we obtain the following equation:

$$\frac{\partial h}{\partial t}=\frac{\partial \eta}{\partial t}+\frac{\partial Q_w^a}{\partial \zeta} \tag{8}$$

The above equation is the asymptotic limit, as $\epsilon\to 0$, of Eq. (7) in [3]. As explained in [1, 2, 3], additional small terms that represent the azimuthal curvature of the interface need to be kept in the evolution equations in order to predict closure.

Therefore the results shown in [3] are correct despite the omission of an important term in the expression for continuity.

Cited References

1 Gauglitz, P. A., and Radke, C. J., "An Extended Evolution Equation for Liquid Film Breakup in Cylindrical Capillaries," *Chem. Eng. Sci.*, Vol. 43, 7, 1988, pp. 1457–1465.

2 Halpern, D., and Grotberg, J. B., "Fluid-Elastic Instabilities of Liquid Lined Flexible Tubes: Airway Closure," *J. Fluid Mech.*, Vol. 244, 1992, pp. 615–632.

3 Halpern, D., and Grotberg, J. B., "Surfactant Effects on Fluid Elastic Instabilities of Liquid Lined Flexible Tubes: A Model of Airway Closure," JOURNAL OF BIOMECHANICAL ENGINEERING, Vol. 115, 1993, pp. 271–277.