

Uncertainty assessment of monthly water balance models based on Incremental Modified Fuzzy Extension Principle method

M. Nasserri, B. Zahraie, A. Ansari and D. P. Solomatine

ABSTRACT

In this paper, a new method, namely Incremental Modified Fuzzy Extension Principle (IMFEP), is proposed for uncertainty assessment of conceptual water balance models. IMFEP is based on a new modification of fuzzy extension principle using fuzzy approximate. The most important feature of the IMFEP method lies in its realistic superposition of convex fuzzy membership functions of model inputs at different fuzzy α -cuts. To evaluate the IMFEP method, four other fuzzy-based approaches have been used to assess the uncertainties in simulating monthly water balance in basin scale and their results are compared with IMFEP. These approaches, one based on simple fuzzy mathematics, Vertex method, UNcertainty Estimation based on local Errors and Clustering (UNEEC) and Modified Fuzzy Extension Principle (MFEP) have been previously used for uncertainty estimation of water models. The nonlinear monthly water balance models calibrated for the two basins in Iran and France and their outputs with the five aforementioned methods have been compared. For both basins, IMFEP and MFEP methods have shown the best performance followed by UNEEC and Vertex methods (however, the differences in the underlying assumptions of the UNEEC method have to be taken into account). It can be concluded that the IMFEP method shows strong performance of uncertainty propagation in all evaluated fuzzy α -cuts.

Key words | fuzzy reasoning, mathematical fuzzy operator, modeling uncertainty estimation, monthly water balance, UNEEC

M. Nasserri (corresponding author)
School of Civil Engineering,
University of Tehran, Tehran,
Iran
E-mail: mm_nasserri@yahoo.com;
mnasserri@ut.ac.ir

B. Zahraie
Center of Excellence for Engineering and
Management of Civil Infrastructures,
School of Civil Engineering,
University of Tehran, Tehran,
Iran

A. Ansari
International Institute of Earthquake Engineering
and Seismology (IIEES), Tehran,
Iran

D. P. Solomatine
UNESCO-IHE Institute for Water Education,
Delft,
The Netherlands
and
Water Resources Section,
Delft University of Technology,
The Netherlands

INTRODUCTION

Uncertainty assessment in environmental sciences has recently received significant attention. Various methods have been used to assess the uncertainty of simulations. There is a wide range of uncertainty evaluation methods which can be classified in the three important categories: probabilistic, possibilistic, and hybrid methods, as shown in Figure 1. Probabilistic methods typically refer to *cdf* (cumulative distribution function) or *pdf* (probable distribution function) of parameters, input, or output variables. This class employs different approaches, such as analytical (Tung 1996), statistical (Melching 1992; Langley 2000) and all sampling-simulation and iterative approaches, such as Monte Carlo, Markov Chain Monte Carlo, mostly under a

Bayesian framework (Beven & Binley 1992; Kuczera & Parent 1998; Thiemann *et al.* 2001; Krzysztofowicz 2002; Montanari & Brath 2004; Ruessink 2008; Mishra 2009). Broadness of uncertainty projection methods and their inevitable statistical assumptions are the most important weakness of probabilistic uncertainty simulation methods (Montanari 2007). The four major difficulties of using these methods can be listed as follows:

- Difficulty or impossibility of analytical derivation of statistical functions in case of complex models.
- Imprecision in identifying suitable statistical distribution of model parameters and variables (Montanari 2007).

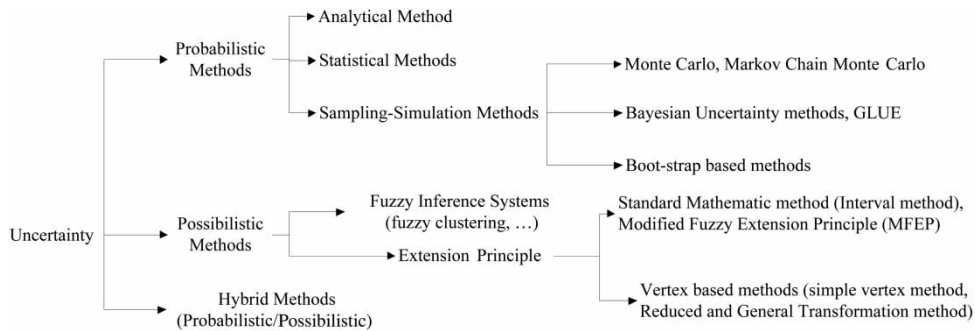


Figure 1 | Schematic overview of different uncertainty simulation methods.

- Dependency of uncertainty simulation on the statistical likelihood which is used in uncertainty simulation (Montanari 2005; Mantovan & Todini 2006).
- Theoretical and computation difficulties of using these methods in validation, i.e., for the unseen model inputs (Mantovan & Todini 2006; Stedinger *et al.* 2008).

The second type of uncertainty assessment methods lies in the possibilistic category. The foundation of many possibilistic uncertainty assessment methods is fuzzy logic. The reasons for using fuzzy approach are its fundamental underlying philosophy, small number of assumptions, and its better compatibility with the environmental systems if compared to the more traditional statistical approach (Montanari 2007). Fuzzy tools are suitable for evaluation of conceptual/structural vagueness while statistical approaches are developed to assess probability of event occurrence (Kosko 1990).

Fuzzy extension principle and pattern recognition are two of the well-known fuzzy methods which are used in uncertainty evaluation of conceptual and physical models. Zadeh's fuzzy extension principle is one of the most important theorems in the discourse of fuzzy logic which facilitates fuzzy mathematics and uncertainty simulation via fuzzy variables. The main idea of the method is based on projecting fuzzy membership value from model inputs to the outputs. Standard fuzzy mathematics (very similar to interval mathematics) and Vertex methods are the most important practical and computational solutions or approximations to implement fuzzy extension principle (Dong & Wong 1987; Juang *et al.* 1991; Zhang *et al.* 1996; Davis & Keller 1997; Hanss 2002, 2004; Alvisi & Franchini 2013; Alvisi *et al.* 2013). These methods have been utilized by

different researchers for evaluation of uncertainty in rainfall-runoff simulation (Maskey *et al.* 2004; Eder *et al.* 2005; Faybishenko 2010; Huang *et al.* 2010). Nearly all the methods belonging to this class are independent from likelihood criterion and prior probability distribution functions.

The third class of methods for uncertainty evaluation considered in this paper can be categorized as hybrid since it employs ideas both of possibilistic and probabilistic methods. Fuzzy pattern identification is another uncertainty assessment tool which belongs to this class of uncertainty assessment methods based on coupling of possibilistic and probabilistic methods. Shrestha and Solomatine (Shrestha & Solomatine 2006; Solomatine & Shrestha 2009) presented a novel hybrid method, namely UNcertainty Estimation based on local Errors and Clustering (UNECC). In this approach, the residual uncertainty (describing errors of a calibrated model) is treated probabilistically, but the method of grouping the past data patterns about model behavior employs fuzzy logic, so in this sense this technique can be attributed to hybrid methods.

Recently, Nasser *et al.* (2012) proposed a new approach to evaluate conceptual monthly water balances based on coupling fuzzy reasoning and fuzzy extension principle which is named Modified Fuzzy Extension Principle (MFEP). They have compared the MFEP method with the Generalized Uncertainty Likelihood Estimation (GLUE) and the two other approximation methods of fuzzy extension principle and illustrated the acceptable efficiency of the proposed method versus the other ones.

In the current paper, a new extension of MFEP, named Incremental Modified Fuzzy Extension Principle (IMFEP), has been proposed and examined in estimating uncertainty of hydrological systems. To show the efficiency of the

IMFEP method, two nonlinear monthly water balance models have been calibrated for the two basins in Iran and France and their results have also been compared with the outputs obtained using standard fuzzy mathematics, Vertex and UNEEC methods. In the next sections, the basics of fuzzy concepts and fuzzy extension principle for uncertainty simulation are explained. Then, concepts of Standard fuzzy mathematics, Vertex, MFEP, IMFEP, and UNEEC methods and also structures of both the monthly water balance methods and the study areas are described. Finally, the results of uncertainty estimation and their comparisons are presented.

FUZZY LOGIC

Traditional logic assumes two-valued logic and consequently (0, 1)-membership of an element in a set. Fuzzy logic was proposed by Zadeh in 1965 (Zadeh 1975) to deal with multi-valued logic when an element would belong to a set with some (fractional) degree of membership. This approach deals with inexact values with its reasoning approximation ability rather than fixed or exact (crisp) solutions. It is worth mentioning that fuzziness and vagueness are due to lack of information, imprecise determination of characteristic parameters, and the complexity of real systems.

A fuzzy set A defined in a discourse universe of X is expressed by its Membership Function (MF) value, $\mu_A(x)$. $\mu_A(x)$ express the extent to which x fulfills the category described by A . The fuzzy set A is shown with a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_A(x)) | x \in X\} \quad (1)$$

The crisp set of elements that belong to fuzzy set A at least to the degree α is called α -level set or α -cut as presented in Equation (2):

$$\tilde{A}_\alpha = \{x \in X | \mu_A(x) \geq \alpha\} \quad (2)$$

A fuzzy subset A of the set of real numbers is called a *fuzzy number* if (a) there is at least one z such that $\mu_A(z) = 1$

(normality assumption), and (b) for every real numbers a , b , c with $a < c < b$ the following is true: $\mu_A(c) \geq \min(\mu_A(a), \mu_A(b))$ (convexity assumption, meaning that the MF of a fuzzy number consists of an increasing and decreasing part, and possibly flat parts).

Figure 2 represents a general form of fuzzy and crisp set of real numbers belonging to an interval. Given a fuzzy set \tilde{A} of real numbers (or a fuzzy number) with a convex MF, the α -cut is an interval $[\tilde{A}_\alpha^-, \tilde{A}_\alpha^+]$ with the boundaries corresponding to the two MF values of α . This notation can be extended to $[\tilde{A}_\alpha^-, \tilde{A}^c, \tilde{A}_\alpha^+]$ to also include \tilde{A}^c – the real value with highest MF value (typically 1).

In practice, MF is usually presented by a continuous function. Some of the popular functions which are used as MF are triangular, piecewise linear, exponential, and Gaussian. In the next section, different popular approaches in applying fuzzy extension principle which are also used in this study are described.

Fuzzy extension principle

The uncertainty associated with the model parameters is due to the description of a real phenomenon by idealized conceptualization. For evaluation of model uncertainty with deterministic structure in fuzzy domain, fuzzy extension principle and its revisions have been advisable. In the current section, the roots of this principle and some of its important revisions are described. According to the Zadeh's fuzzy extension principle theorem (Zadeh 1975), algebraic operations on real numbers can be used for fuzzy numbers with insignificant modifications. This method propagates vagueness of model inputs to model outputs by transformations of the fuzzy membership values of the former. However, the implementation of such

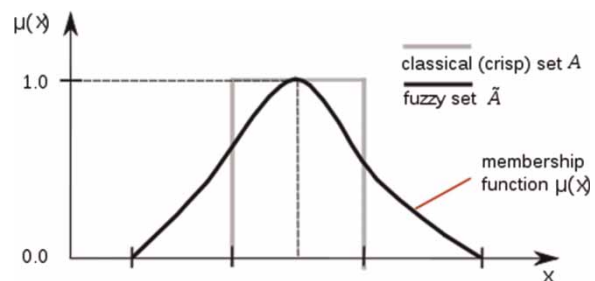


Figure 2 | General representation of fuzzy versus crisp set.

transformation is not trivial. The fuzzy extension principle is a basic concept in the fuzzy set theory that extends crisp domains of mathematical expressions to fuzzy domains.

Consider a function $f()$ from space X to Y , and A is a fuzzy set on X which is typically defined in the following way:

$$A = \{\mu_1(x_1) + \mu_2(x_2) + \mu_3(x_3) + \dots + \mu_n(x_n)\} \quad (3)$$

where x_i and μ_i are the i th element of the set X and its membership value, respectively; $+$ denotes fuzzy OR operator. The extension principle projects the vagueness of the fuzzy set A under the transformation $f()$ to the fuzzy set B as follows:

$$B = \{\mu_1(f(x_1)) + \mu_2(f(x_2)) + \mu_3(f(x_3)) + \dots + \mu_n(f(x_n))\} \quad (4)$$

Various methods have been proposed in the literature of fuzzy set theory to computationally implement fuzzy extension principle. Standard fuzzy mathematics (or fuzzy interval mathematics) and Vertex method (Dong & Shah 1987) are the most well-known approaches to estimate functions of fuzzy sets.

Standard fuzzy mathematics is based on a simple structure and it is very similar to interval mathematics. For more on Standard fuzzy mathematics, readers are referred to Dong & Wong (1987), Dong & Shah (1987), and Hanss (2004).

The Vertex method (Dong & Shah 1987) is a straightforward way of computing functions with fuzzy variables and can be used in the case of monotonic functions (function without extreme points for all variables). Consider a continuous function $f(X_1, X_2, X_3, \dots, X_n)$ with n -dimensional input space which is also monotonic in its region (including the boundaries). Select a particular value of α , and make the α -cuts for each input fuzzy variable. Only two boundary values of each α -cut are considered, and $m = 2^n$ vertices (points) are formed in n -dimensional space based on all possible combinations of the α -cuts' boundaries. Function f is calculated for each vertex. For example, if a function has three variables, $2^3 = 8$ calculations of f should be done for the eight vertices; geometrical representation is given in Figure 3. The output interval Y_α (in terms of function f values) corresponding to the selected value of α can be

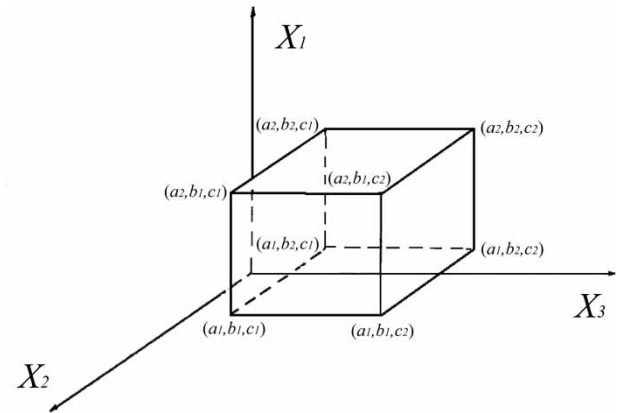


Figure 3 | Geometrical presentation of vertex points for three-dimensional representation.

identified by finding the minimum and maximum values of function f across all vertices:

$$Y_\alpha = \left[\min_i f(c_i), \max_i f(c_i) \right], i = 1, 2, 3, \dots, m \quad (5)$$

where c_i is the ordinate (function value) of the i th vertex for a particular α , and m is the number of vertices. Running this procedure for several values of α , a set of intervals for f can be found by Equation (5), and used to approximate the output MF.

Several extensions to the original Vertex method have been proposed (Hanss 2002, 2004; Klimke et al. 2004). Hanss (2002, 2004) suggested an important update of the Vertex method using a more efficient way to compute the resulting output MF, and called it the 'Fuzzy Transformation Method'. The fuzzy response is reconstructed from a set of deterministic responses, combining various values in each interval forming a sort of staggered, more sparse grid than the Vertex method, thus leading to more economical computations. One of the features of both Vertex and Fuzzy Transformation methods is that the resulting output MF based on calculations of Equation (5) is quite wide since the \min and \max operators may lead to combining and 'amplification' of fuzziness of individual variables.

One can notice that most methods aimed at identification of the output MF, including the ones presented above, use an idea of sampling (trying to do this economically, similar to how it is done in the methods associated with 'design of experiments'), using crisp arithmetic to

calculate the output, and then combining these crisp outputs to estimate the output MF. This approach is often termed in the literature as the Fuzzy Alfa-Cut (FAC) method which general form can be outlined as follows:

- The MF is cut horizontally at a finite number of α -cuts between 0 and 1.
- For each α -cut of the variable (typically using the same α for all variables), the output function is calculated (model is run) to determine the minimum and maximum possible values of the output.
- This information is then directly used to construct the corresponding MF. If the output is monotonic with respect to the dependent fuzzy variables, the process is rather simple since only two simulations will be enough for each α -cut (one for each boundary). Otherwise, optimization routines have to be carried out to determine the minimum and maximum values of the output for each α -cut.

This approach has been used for different branches of water sciences by researchers (see, e.g., Schulz & Huwe 1997, 1999; Abebe *et al.* 2000; Maskey *et al.* 2004), and it was also compared to the Monte Carlo approach. All of the above methods are computational implementations of the fuzzy extension principle to deal with real-life linear and nonlinear systems efficiently, and they concentrate on the boundary (extreme) values of MFs. In the next section, the MFEP, IMFEP, and UNEEC are explained.

MFEP and its incremental form

In the above-described methods (Standard fuzzy mathematics, Vertex method and its revisions, FAC methods), mathematical operators of the model of interest are standard algebraic operators, and it seems that fuzzy concepts may improve their mathematical operations too. In this regard, MFEP was proposed and implemented computationally by Nasser *et al.* (2012). The procedure of MFEP is founded on coupling the results of fuzzy reasoning and crisp mathematics as two parallel sub-procedures forming the unified approach. This procedure is in fact a multi-rule fuzzy inference system, in which the results of different rules are combined using ‘OR’ operator. This approach is known in fuzzy logic and control engineering applications as

Mamdani inference. Compared to Vertex method, it leads to narrower output MFs which in many practical applications would be considered to be more realistic and an advantage.

Suppose that symbolic operator ‘ Θ ’ represents any of four arithmetic operations, namely, addition, subtraction, multiplication, or division. Let **A**, **B**, and **C** be three fuzzy numbers with some crisp representative values of *a*, *b*, and *c*, respectively; these values could be obtained through any defuzzification method (Jang *et al.* 1996) applied to **A**, **B**, and **C**. It is possible to define $C = A\Theta B$ as follows:

$$\mu_C(x) = [a\Theta\mu_B(x)] \vee [b\Theta\mu_A(x)] \tag{6}$$

In Equation (6), the expression $[a\Theta\mu_B(x)]$ represents the influence of operation of crisp value ‘a’ on a fuzzy number **B** and ‘ \vee ’ represents ‘OR’ fuzzy operator. Using Equation (6), the additive interaction of fuzzy numbers **A** and **B** is avoided. In Equation (6), it is assumed that ‘ Θ ’ operation does not have any influence on increasing the fuzziness of the system and the fuzziness of the fuzzy number **C** reflects the maximum fuzziness of input numbers **A** and **B**, but not their combination. This assumption is in contrast to the extension principle where operator ‘ Θ ’ has great influence on the fuzziness of **C**. The schematic representation of operations in Equation (6) is shown in Figure 4.

Computations following the proposed MFEP are as follows. When **A** (the first fuzzy number) is divided by **B** (the second fuzzy number), the result **C** (the third fuzzy number) for a specific α -cut can be computed using the following steps.

In the first step, crisp representatives of fuzzy numbers should be calculated. In this paper, the average of the minimum and maximum values of the fuzzy range is set as this deterministic value. The crisp representative for **A** and **B**

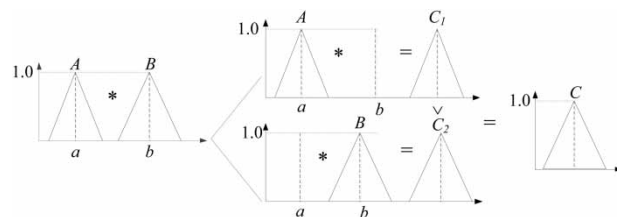


Figure 4 | Schematic flowchart of MFEP method of uncertainty simulation.

fuzzy numbers can be estimated using Equations (7) and (8):

$$\text{Crisp}(A)_\alpha = \frac{1}{2}(A_\alpha^- + A_\alpha^+) \quad (7)$$

$$\text{Crisp}(B)_\alpha = \frac{1}{2}(B_\alpha^- + B_\alpha^+) \quad (8)$$

where ‘-’ and ‘+’ superscripts represent lower and upper values of α -cut set; $\text{Crisp}(A)_\alpha$ and $\text{Crisp}(B)_\alpha$ are the representative crisp values of fuzzy sets A and B .

In the second step, the represented crisp values, the fuzzy parameters, and the proposed operation have been used to achieve two sets of results. As shown in Figure 3, for each set, a crisp value and a fuzzy parameter are combined using the proposed fuzzy operator. For example, for division operator, Equations (9) and (10) show the procedure of combining the two fuzzy sets:

$$C_\alpha^1 = \left[\frac{\text{Crisp}(A)_\alpha}{B_\alpha^+}, \frac{\text{Crisp}(A)_\alpha}{B_\alpha^-} \right] \quad (9)$$

$$C_\alpha^2 = \left[\frac{A_\alpha^-}{\text{Crisp}(B)_\alpha}, \frac{A_\alpha^+}{\text{Crisp}(B)_\alpha} \right] \quad (10)$$

In the final step, ‘OR’ operator should be used to find the final upper and lower bounds (UB and LB) of the resultant fuzzy variable. In Equation (11), the final mathematical step of achieving C (as shown in Figure 4) is presented:

$$C_\alpha = \left[\min[C_\alpha^1(1), C_\alpha^2(1)], \frac{A^c}{B^c}, \max[C_\alpha^1(2), C_\alpha^2(2)] \right] \quad (11)$$

where $C_\alpha^i(1)$ and $C_\alpha^i(2)$ are the first and second members of $C_\alpha^i()$ array in Equation (10). When the mathematical operator is changed to addition, only Equations (9) and (10) should be changed:

$$C_\alpha^1 = [\text{Crisp}(A)_\alpha + B_\alpha^-, \text{Crisp}(A)_\alpha + B_\alpha^+] \quad (12)$$

$$C_\alpha^2 = [\text{Crisp}(B)_\alpha + A_\alpha^-, \text{Crisp}(B)_\alpha + A_\alpha^+] \quad (13)$$

The final result for addition operator is equal to:

$$C_\alpha = [\min[C_\alpha^1(1), C_\alpha^2(1)], A^c + B^c, \max[C_\alpha^1(2), C_\alpha^2(2)]] \quad (14)$$

These two examples illustrate the basic logic of the proposed MFEP method. In brief, in the first step of the proposed method, fuzziness of each parameter has been calculated. Then, in the second step, the fuzziness has been projected using the mathematical operator Θ (such as division, addition, etc.). In the Appendix (available online at <http://www.iwaponline.com/jh/015/159.pdf>), the major mathematical operators of MFEP are illustrated.

While the Vertex method and its revisions are sampling-based approaches in applying fuzzy extension principle, MFEP has a straightforward procedure similar to the one used in Standard fuzzy mathematics method and it is one of the important advantages of the computational scheme of the MFEP method.

To implement the MFEP method in uncertainty assessment practice, three steps should be taken. The first step is identification of fuzzy MFs for each behavioral model parameter. The second step is to apply the proposed fuzzy arithmetic operators in the main models, and the last one is to use suitable criteria for evaluating the results. These steps are described in more detail below:

- **Identification of MFs.** MFs of all fuzzy parameters should be identified as pre-processing information to model uncertainty by the fuzzy extension principle methods (using Standard fuzzy mathematics, Vertex method, MFEP, or IMFEP). In this paper, MFs have been defined using acceptable minimum, maximum, and most frequent values extracted from the results of sampling-simulation procedure as most of FAC methods. The sampling-simulation procedure used in this step is similar to sampling in Monte Carlo simulation. Nash–Sutcliffe (NS) efficiency criterion has been utilized to evaluate the results.
- **Replacing the mathematical operators used in the model.** Proposed arithmetic operations are different from their deterministic forms. So in this step, mathematical operators of models should be rewritten to be suitable for the proposed uncertainty simulation. This means that four basic mathematical operators (+, −, ×, and ÷) and also other mathematical functions, such as $\exp()$, $\sin()$, etc. should be replaced with the new forms of the operators.
- **Evaluation of the uncertainty simulations.** Various metrics can be used; in this paper three indices are used for assessing the simulated uncertainty bounds.

Although fuzziness by itself is an objective fact, its representation through MFs is subjective, so this means that there is no fixed rule to determine fuzzy MFs. Fortunately, in fuzzy mathematics the results of mathematical fuzzy operations often are not too sensitive to the details of different MFs, however it is important to ensure that the MFs are compatible with the nature of uncertain parameters. For identifying the fuzzy MFs, the two approaches of using expert experience and statistical information (if it is available) are suggested.

In the first approach, an expert adjusts the types and domain of each MF based on some prior practices or logical assumptions (Juang et al. 1999; Huang et al. 2010). The second approach is based on inferring shapes and parameters of MFs from the statistical information, such as analytical or empirical cumulative or partial statistical distributions (*cdf* or *pdf*). Civanlar & Trussell (1986) described how the acceptable fuzzy MF can be inferred for each variable from the available raw statistical data. This approach is feasible in accordance with the possibility–probability consistency principle (Civanlar & Trussell 1986). This type of fuzzification of model parameters makes the procedure more physically based. It also benefits from the advantage of dependability of the possibility–probability concept (Civanlar & Trussell 1986; Juang et al. 1991; Davis & Keller 1997; Abebe et al. 2000; Nasserri & Zahraie 2011). In this paper, characteristics of the suitable fuzzy MFs for the model parameters have been determined as the result of a two-step procedure. In the first step, based on a selected threshold NS as the performance criterion, sampling-simulations are identified. After that, maximum, minimum, most frequent values, and the shapes of the uncertain parameters are extracted from the pool of accepted samples and are used for defining the MFs. It is worth mentioning that the resulting MFs of inputs have also been used for evaluating output uncertainty using Standard fuzzy mathematics and Vertex method as well.

A new general form of MFEP, namely IMFEP is proposed in the current paper. The basic idea of IMFEP is based on coupling the MFEP method and the implemented methodology of FAC for different α -cut uncertainty assessment. This method is able to project different convex types of input fuzzy MFs to the model output. In the initial form

of MFEP, MFs of outputs are triangular and different α -cuts of the model outputs are calculated linearly using the center points ($\alpha = 1$) and the UB and LB of the model results ($\alpha = 0$). These are the two major simplifications used in MFEP. The major difference between MFEP and IMFEP is that in IMFEP, the UB and LB of MF can be estimated for any α -cut level and not only for 0 and 1. The differences in identifying UB and LB (\hat{Q}_n) of the model uncertainty around the center (Q) of the model response with MFEP and IMFEP methods have been depicted in Figure 5 for various α -cuts ($h = 0, 0.25, 0.5, 0.75, \text{ and } 1$).

The first step in using IMFEP is ‘decomposition of the input fuzzy numbers’. At first, the MF ranges for each parameter are subdivided into m equally spaced α -cuts with

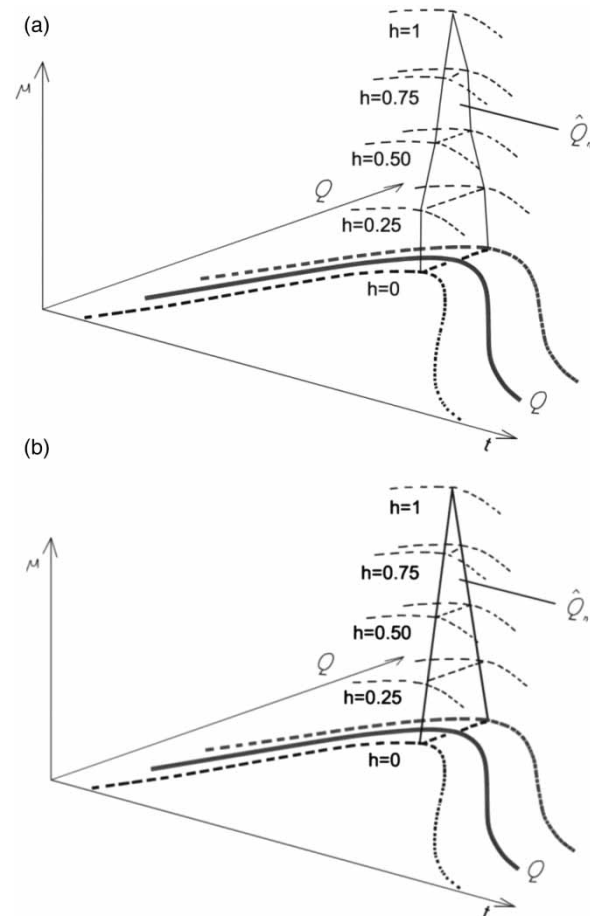


Figure 5 | Uncertainty estimation of model outputs based on (a) IMFEP and (b) MFEP methods for different α -cuts ($h = 0, 0.25, 0.5, 0.75, \text{ and } 1$). (Q , simulated runoff; \hat{Q}_n , simulated fuzzy uncertainty response.)

the distance along the ordinate of $1/m$. The number of discretization steps (m) is constant for all model parameters and it can be selected based on engineering judgment. Figure 6 represents a sample discretization of a hypothetical fuzzy MF. As a result of this discretization, m sets of UB and LB of the m equally spaced α -cuts are available for fuzzy uncertainty simulation. The second step of using IMFEP is ‘uncertainty assessment of each α -cut’. In this step, the model under study should be executed for each of the m uncertainty levels (α -cuts). The most important inherent assumption of IMFEP is the correspondence of uncertainty level of model outputs to the model inputs in each fuzzy level (α -cut). In this respect, IMFEP and other FAC methods are similar, and the difference between them is in the mathematical operators used.

Note that in practice, due to the use of fuzzy arithmetic used by MFEP and IMFEP, these methods can be used only for relatively simple models where the number of mathematical operators to be replaced by their fuzzy equivalents is not too high (however, many lumped conceptual hydrological models fall into this category).

UNEEC

UNEEC is a hybrid uncertainty assessment method presented by Shrestha & Solomatine (2006) and Solomatine & Shrestha (2009). It is aimed at estimation of the total residual uncertainty (expressed in the form of two or more percentiles) of the model results. In the original version of this method it is assumed that the model uncertainty is manifested by the model errors (so the

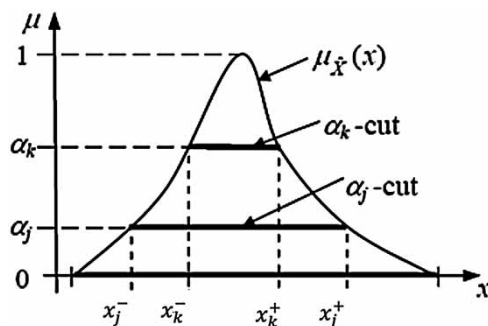


Figure 6 | A sample discretization of a hypothetical fuzzy membership function in two different α -cuts.

parametric uncertainty is not considered), and a machine learning model able to predict uncertainty of the model output for future situations is built. (In the method's latest update, termed UNEEC-P (Pianosi *et al.* 2010), parametric uncertainty is considered as well.) UNEEC includes the following steps:

- In the data set representing the past model performance (errors), find fuzzy clusters of data (fuzzy c -clustering method is used). Clustering is done in the space of specially selected descriptive, relevant variables (RV) (for hydrological conceptual models these can be differently lagged and pre-processed rainfall data, soil moisture, flow, etc.). Average mutual information (AMI) and correlation analysis can be used to select these predictors (Bowden *et al.* 2005a, 2005b). An advantage of using AMI is its ability to detect nonlinear relationships between two parameters (Battiti 1994). In the case of using clustering as the pattern recognition approach, optimality of cluster numbers must be confirmed. Various indicators have been proposed for offline identification of the best number of clusters in the literature (Xie & Beni 1991; Halkidi *et al.* 2001; Nasser & Zahraie 2011). This step is the possibilistic part of UNEEC.
- For each cluster c calculate M percentiles Q_c^m ($m = 1, \dots, M$) of the empirical distribution of the model error for each cluster c , taking however into account the membership degree of each data point (weighted counting is conducted).
- Using the calculated percentiles Q_c^m for each cluster c , calculate the ‘global’ estimate of the percentile Q^m for each data vector RV, by weighting the cluster percentile by the corresponding degree of membership of the given data vector to this cluster. Form the input–output data tables for each percentile m where the ‘global’ percentile values Q^m are used as outputs.
- For each percentile m , train a machine learning (statistical) model (e.g., an artificial neural network) or induced instance-based learning approach able to predict the percentile value Q^m given the input vector of RV. If $M = 2$, the percentiles will form the confidence interval (CI) with the confidence level determined by the percentiles used.

The most important advantage of UNEEC is that it makes no assumptions about the error distribution, and

that the use of clustering allows for taking into account the local character of dependency of the error percentiles on the descriptive (relevant) variables, and the use of fuzzy logic reflects the smooth nature of variability in environmental variables and makes the transition between these 'local' models gradual. In the current application, the instance-based learning and fuzzy *c*-means clustering method, as described by Shrestha & Solomatine (2006), have been used to calculate the uncertainty bounds for validation periods and unseen data sets.

CASE STUDIES

Two basins have been selected to test the proposed fuzzy uncertainty simulation in monthly water balance modeling. The first basin is Adour-Garrone (at Pont de Maussac) Basin in France. This basin is located in the south-west of France and in this paper is referred to as AG in this paper. The data used in this study are the same used by Perrin *et al.* (2001). Snowmelt's contribution is very low in the total runoff of this basin. The available data set from AG Basin is from January 1954 to January 1996.

The second study area is Karoon III Basin (referred to as K in this paper). It is a sub-basin of the Great Karoon River Basin in the south-west of Iran. This basin is one of the largest river basins in Iran in which several dams have been constructed. Information about this basin has been extracted from the Iranian hydrological data bank provided by the Iranian Ministry of Energy. The available data set for the K Basin is recorded from October 1974 to April 2003. Brief statistical information about the hydrology of these basins is presented in Table 1. The location maps of the basins are also shown in Figure 7. For both basins, 60% of the available data sets has been used for calibration and the remainder has been used for validation of the water

balance models. It should be mentioned here that different physiographic and climatological characteristics of these basins make them good cases for evaluation of the proposed method versus other fuzzy-based uncertainty simulation tools. In the next section, selected monthly water balance models are described in brief.

MONTHLY WATER BALANCE MODELS

Monthly water balance models have been utilized by different researchers to investigate monthly hydrological behavior of basins with different climatic conditions. Mass balance of soil water storage is the building block of nearly all conceptual monthly water balance models, and they differ in the meteorological variables they use (e.g., evaporation, temperature) and also in the mathematical relationships between precipitation, temperature, evaporation, and runoff (Xu & Singh 1998).

In the current paper, two monthly water balance models, *abc* (Alley 1984; Selle & Hannah 2010) and the two-parameter water balance model presented by Guo *et al.* (2002), have been used. In Figures 8 and 9, schematic conceptual structures of these models and their mathematical formulations are shown. The *abc* model is known as a semi-nonlinear and the second model is fully nonlinear. These models have been suggested for basins without considerable snowmelt, and are based on water mass balance. Surface runoff, subsurface flow, and soil water content are the main hydrologic concepts included in these models. In spite of the fact that these models have relatively simple mathematical structures, they are complex enough to make them quite adequate for the considered cases studies, and they were selected as good candidates for testing the efficiency of the proposed uncertainty analysis methods.

Table 1 | Characteristics of the selected basins in Iran and France

No.	Location	Basin	Area (km ²)	Rainfall (mm)		Evaporation (mm)		Runoff (mm)	
				Max	Min	Max	Min	Max	Min
1	Iran	Karoon (K)	24,200	399.51	0	399.31	8.03	175.7	5.27
2	France	Adour-Garrone, Luzège (at Pont de Maussac) (AG)	82.3	439.5	2.8	131.2	12.9	257.0	0.3

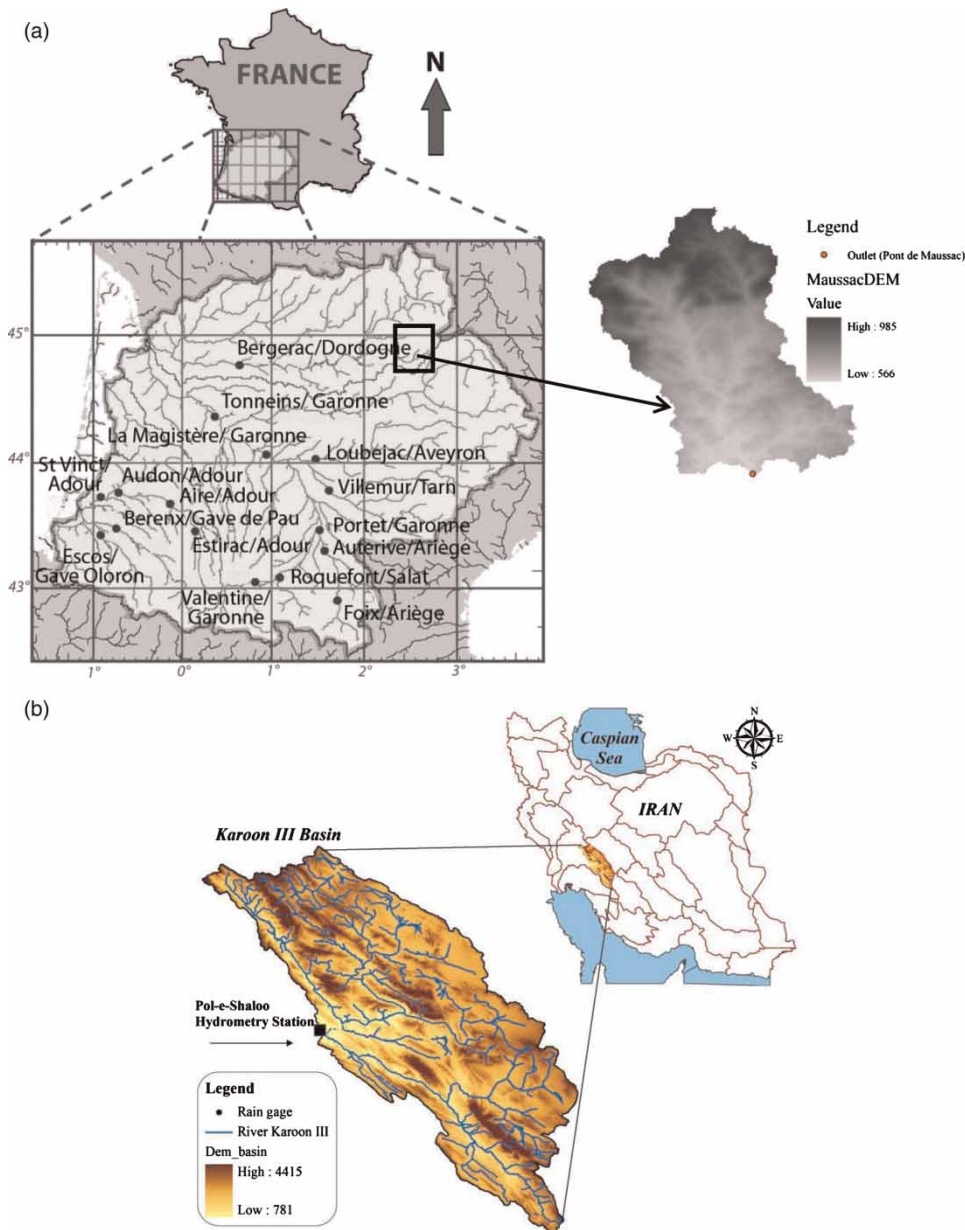


Figure 7 | Two basins of interest in France and Iran. (a) Locations of Adour-Garonne at Pont de Maussac in France; (b) Karoon III Basin in Iran.

ASSESSMENT OF UNCERTAINTY SIMULATION

In this section, three statistical indicators are used to assess results of described uncertainty analysis. Evaluating the quality of the uncertainty analysis methods that do not involve fully fledged Monte Carlo simulation or analytical representation of uncertainty is difficult for a simple reason: there is no 'real' MF or *pdf* of the output variable

to compare with. However, it is possible to compare different uncertainty analysis methods by using statistical criteria that provide some indirect characterization of the methods' adequacy and quality.

Jin *et al.* (2010) developed two indicators to compare different uncertainty analysis methods. They used these indicators for 95% CI. These indicators are crisp and computed by using UB and LB of the CIs in probabilistically

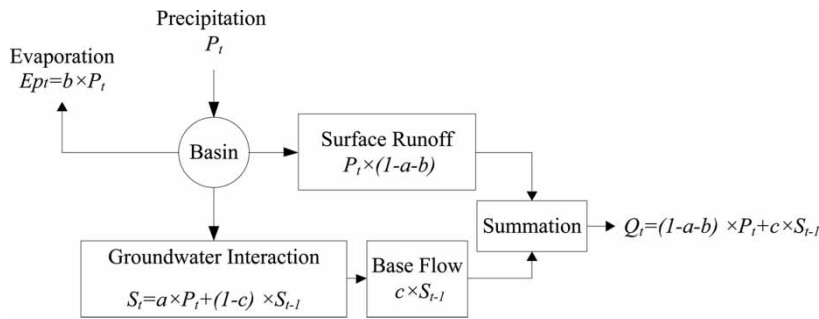


Figure 8 | Structure of *abc* model (*a*, *b*, and *c* are the model parameters, and P_t , S_t , and Q_t are monthly precipitation, soil water content, and runoff at month t).

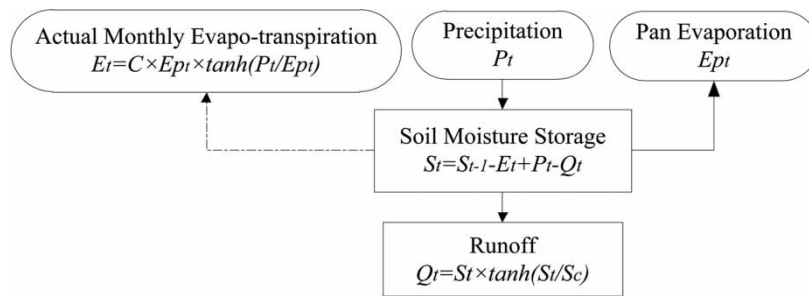


Figure 9 | Structure of two-parameter monthly water balance (C and S_c are model parameters, and P_t , S_t , E_t , and Q_t are monthly precipitation, soil water content, evapotranspiration, and runoff at month t).

uncertainty modeling. Average relative interval length (ARIL) is one of these indicators which describe the wide-ness of uncertainty bounds versus the observed values. This index is presented in Equation (15):

$$ARIL = \frac{1}{n} \times \left(\sum_{t=1}^n \frac{UpLi_t - LoLi_t}{Obs_t} \right) \tag{15}$$

where UpLi and LoLi are UB and LB of uncertainty (simu-lated runoff in the considered case studies) of the selected α -cut or statistical CI, and Obs is the observed runoff; t is the time step (month); n is the total number of time steps. In principle, the lower the ARIL, the lower the uncertainty. In a case of no uncertainty, ARIL is equal to zero. Note that ARIL may reflect the uncertainty of the investigated main model and not the quality of the uncertainty analysis method per se; however, it can be used in comparative studies (when different uncertainty analysis methods are applied to propagate the same uncertain inputs through the model) as it is done in this paper.

The second indicator describes the coverage of obser-vations by the uncertainty bounds (often also termed prediction interval coverage probability (PICP), see Soloma-tine & Shrestha (2009)). It is formulated as follows:

$$P_{Level} = \frac{NQ_{in}}{n} \times 100 \tag{16}$$

where NQ_{in} is the number of observations which are bracketed by the uncertainty bounds in the simulated uncer-tainty bound. A good uncertainty method should result with P_{level} being equal to the selected α -cut or the confidence level used (e.g., 0.9) (Lu *et al.* 2010). These two behavioral indi-cators have been used in this study for evaluation of different uncertainty analysis methods. It is clear that proximity of P_{level} to the confidence level (often set to 0.9 or 0.95) is an indicator of the quality of the uncertainty analysis method.

Obviously, ARIL and P_{level} are in a way conflicting indi-cators, and to take into account the interaction between these two criteria, a new aggregate index, namely the nor-malized uncertainty efficiency (NUE) for each fuzzy level

has been proposed by Nasser & Zahraie (2011), and it was also used in this study. NUE can be estimated using the following equation:

$$\text{NUE} = \frac{P_{\text{Level}}}{w \times \text{ARIL}} \quad (17)$$

where w is the scale factor of P_{level} and ARIL. The modeled uncertainty with higher NUE is preferred over the lower ones. Higher NUE means a higher ration of the number of observations within the upper and lower uncertainty bounds to the area between them. In this paper, it is assumed that the weighting factor between P_{level} and ARIL is equal to one ($w = 1$).

In general, the only criterion that we can safely use is the following one: 'how close is P_{level} to the confidence level'. The criterion based on minimizing ARIL can be used as well as an auxiliary one, so it can be said that when comparing different uncertainty assessment methods that the methods with lower ARIL and higher P_{level} are preferred. If a single method with maximum P_{level} and minimum ARIL cannot be identified, NUE can be used as an auxiliary statistical indicator to compare different methods (higher NUE values would correspond to a better method). In the next section, the results of uncertainty simulation with the IMFEP and other specified methods are described.

RESULTS AND DISCUSSION

Both model-basins are calibrated with 60% of the available data set to achieve the optimum parameter values. The single objective optimization was conducted by the real-coded genetic algorithm (GA). NS efficiency criteria have been used as the fitness function in optimizing the conceptual model responses. The results of the optimization model including the best achieved NS and their optimized parameters are presented in Table 2. The performance of the *abc* model in the K Basin has been acceptable in simulating base and mid-flows while it has shown weaker performance in simulating some of the peak flows. The two-parameter water balance model has shown good performance in simulating low, mid, and peak flows of the AG Basin.

Table 2 | Calibrated parameters and statistics for both model-basins (their optimum NS efficiency criteria and their abbreviations)

Monthly water balance models			
<i>abc</i> model		Two-parameter model	
<i>Abv.</i>	K (NS = 0.71)	<i>Abv.</i>	AG (NS = 0.85)
<i>a</i>	0.619	<i>c</i>	1.238
<i>b</i>	0.242	<i>Sc</i>	1,313.1
<i>c</i>	0.283	–	–
$S_{(1)}$	3.742	$S_{(1)}$	179.9

In the next step, all fuzzy MFs for the model parameters have been identified. As mentioned before, sampling-simulation is the selected approach to define MFs. This procedure is similar to Monte Carlo simulation, or, more accurately, a version of it – the GLUE method. Beven & Binley (1992) suggested the threshold of NS = 0.6 to filter out the least acceptable models in sampling-simulation procedure. In this study, least acceptable NS values have been set to 0.6 and 0.75 for K and AG Basins, respectively. Histograms of the acceptable parameters for both basins are shown in Figures 10 and 11. These histograms are bell-shaped and convex.

According to the shapes of the resulting histograms, triangular MFs have been used to represent uncertainty of the model parameters. Thus, LB and UB of the acceptable parameters have been set as LU and UB of the triangular MFs and the optimized parameters have been used as the center value for the triangular form of MFs as well. These LBs and UBs and their NS criteria are shown in Table 3. In the final step, uncertain responses of both model-basins have been assessed using the Standard fuzzy mathematic, Vertex, MFEP, and IMFEP methods. Efficiency indicators of the four methods for different fuzzy α -cuts and for both basins are reported in Table 4.

As illustrated in Table 4, relatively high ARIL values show that the modeled uncertainty by Standard fuzzy mathematics is quite wide in all presented α -cuts in the AG Basin. This means that all observed points are scattered between LB and UB of the uncertainty interval. Consistency in increasing the rate of ARIL with increasing uncertainty level (decreasing values of α) is noticeable. Table 4 also shows that with decreasing membership level, NUE values are decreased as well. This means that the rate of increase

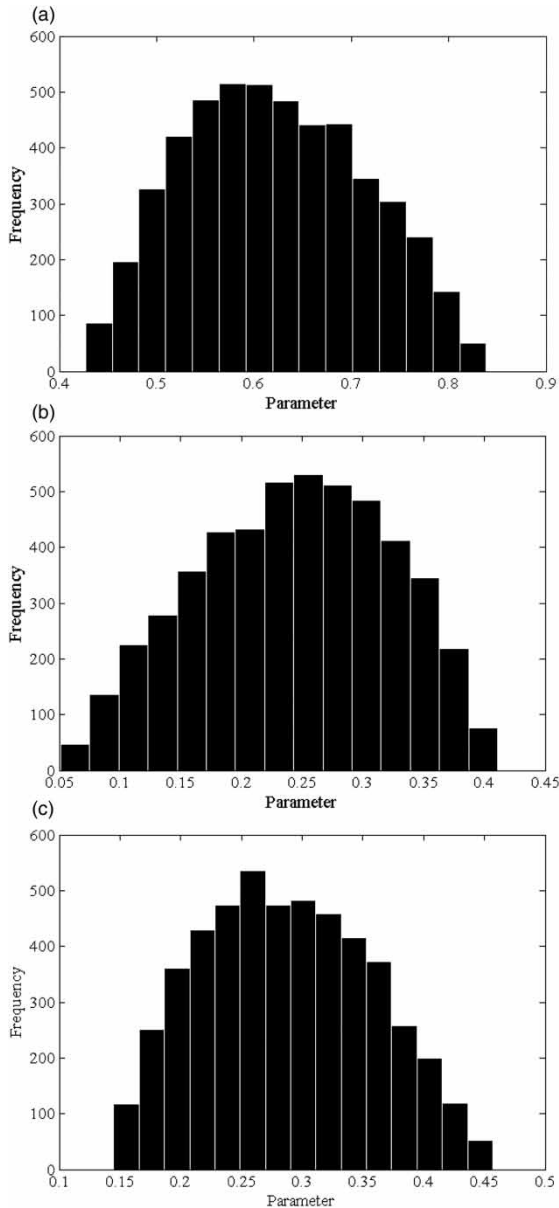


Figure 10 | Histograms of the three acceptable parameters for *abc* monthly water balance model in K Basin ($NS \geq 60$). (a) Histogram of *a* samples. (b) Histogram of *b* samples. (c) Histogram of *c* samples.

in P_{level} and decrease in ARIL values are not consistent. In the AG Basin, the results with lower uncertainty (higher α -cuts) depict higher NUE values. It is obvious that very wide uncertainty bounds covered nearly all of the observed runoff values, but a very large value of ARIL shows that it is not an eligible uncertainty response.

As is shown in Table 4, the Vertex method demonstrated better behavior regarding wideness and coverage of

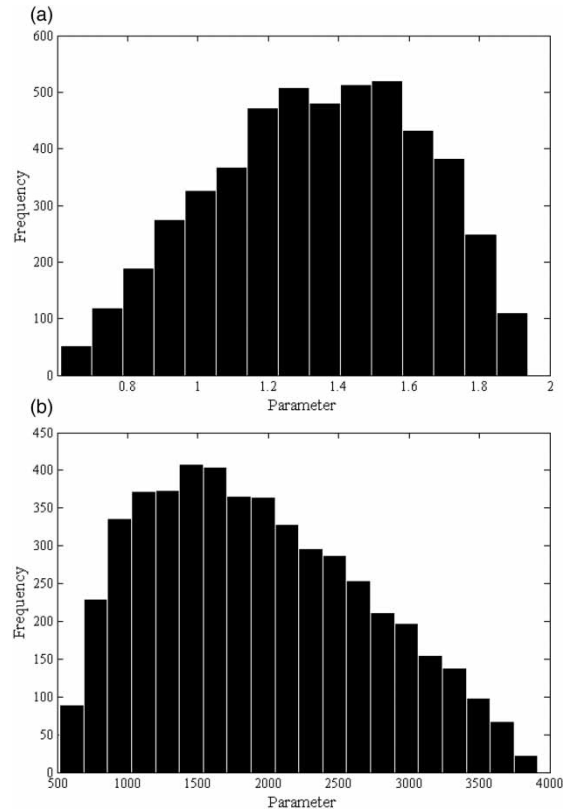


Figure 11 | Histograms of the two acceptable parameters for two-parameter monthly water balance in AG Basin ($NS \geq 75$). (a) Histogram of *C* samples. (b) Histogram of *SC* samples.

Table 3 | Effective parameter bounds for both models and basins based on sampling-simulation results

No. of parameters	Model (K) ($NS \geq 0.6$)			Model (AG) ($NS \geq 0.7$)		
	Abv.	LB	UB	Abv.	LB	UB
1	<i>a</i>	0.426	0.83	<i>c</i>	0.61	1.936
2	<i>b</i>	0.05	0.41	<i>Sc</i>	518.37	3,916
3	<i>c</i>	0.15	0.456	–	–	–

uncertainty bounds (based on P_{level} , ARIL, and NUE values) compared with Standard fuzzy mathematics. P_{level} , ARIL, and NUE indicators in both basins show that the results of higher α -cuts (or lower fuzzy uncertainty levels) represent more acceptable behavior than the results of lower α -cuts. Figures 12 and 13 represent uncertainty bounds for AG and K Basins with Vertex method and Standard fuzzy mathematics with the same α -cut equal to 0.9. It

Table 4 | Statistical evaluations of the Standard fuzzy mathematic, Vertex, MFEP, and IMFEP methods for different α -cuts

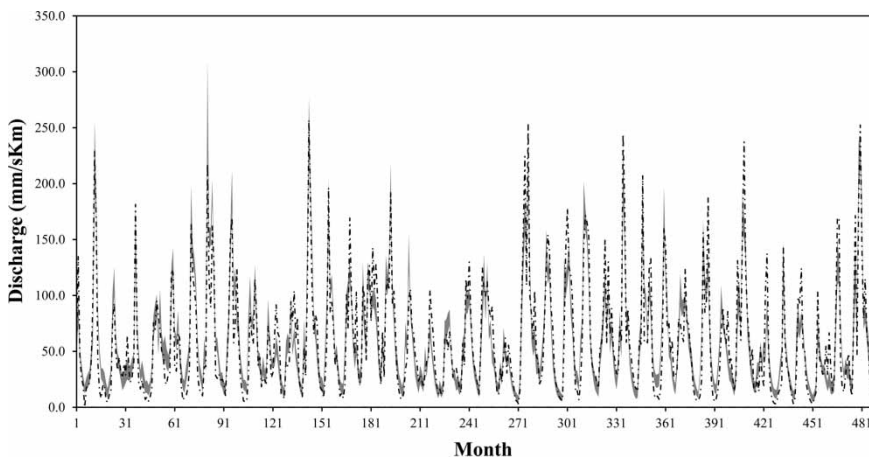
No. of parameters	α -cut	Model (K)			Model (AG)		
		ARIL	P_{level}	NUE	ARIL	P_{level}	NUE
Standard fuzzy mathematics	0.9	0.538	0.365	0.678	>> 10	1	<< 0.1
	0.7	1.612	0.825	0.512	>> 10	1	<< 0.1
	0.5	2.687	0.972	0.362	>> 10	1	<< 0.1
	0.2	4.3	1	0.233	>> 10	1	<< 0.1
Vertex method	0.9	0.185	0.209	1.130	0.462	0.321	0.695
	0.7	0.553	0.509	0.920	1.353	0.771	0.570
	0.5	0.919	0.705	0.767	2.191	0.930	0.424
	0.2	1.455	0.852	0.586	>> 10	0.994	<< 0.1
MFEP	0.9	0.106	0.102	0.962	0.096	0.087	0.906
	0.7	0.287	0.307	1.070	0.262	0.277	1.057
	0.5	0.529	0.511	0.966	0.437	0.392	0.897
	0.2	0.817	0.763	0.934	0.699	0.518	0.741
IMFEP	0.9	0.122	0.136	1.12	0.226	0.189	0.84
	0.7	0.342	0.353	1.03	0.51	0.415	0.884
	0.5	0.548	0.569	1.04	0.652	0.487	0.747
	0.2	0.837	0.781	0.932	0.877	0.510	0.582

should be noted that ARIL values achieved by Vertex method are much lower than those obtained from Standard fuzzy mathematics. The last important point inferred from Table 4 is the consistency between the variations of fuzzy α -cuts and NUE values. Higher α -cut values provided lower NUE values, which shows that no optimum point for uncertainty behavior of the model results can be found based on this indicator.

The P_{level} values in Table 4 show that the observation points within the uncertainty bounds obtained from the

MFEP method are lower than both Standard fuzzy mathematics and Vertex methods. Airl values estimated for MFEP results also show fewer areas between the uncertainty bounds compared with the results of the other two methods. For both basin-models, α -cut equal to 0.7 is selected as the best uncertainty level based on the NUE values.

The results of uncertainty simulation in K and AG Basins are shown in Figure 14. This figure illustrates uncertainty bounds for α -cut equal to 0.7. The results of the Vertex method in the K Basin at fuzzy level equal to 0.9 are better

**Figure 12** | Uncertainty simulation of AG Basin with Vertex method (dashed line, observations, grey bound: uncertainty bound) (α -cut = 0.9).

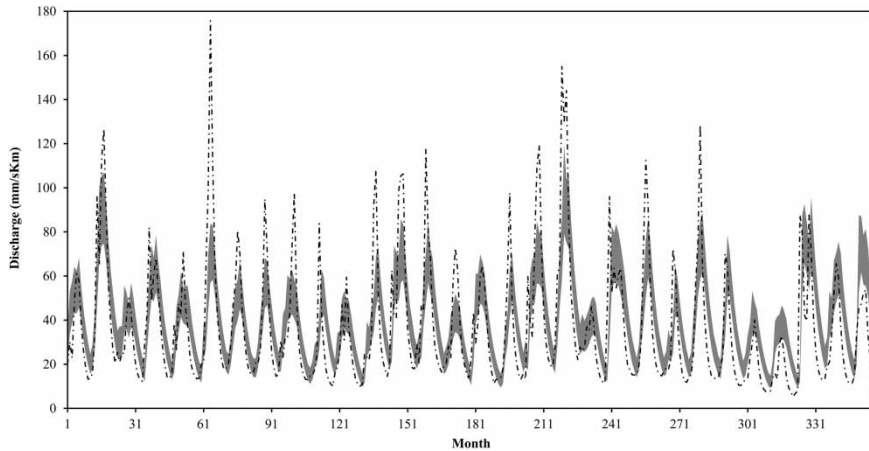


Figure 13 | Uncertainty simulation of K Basin with standard method (dashed line, observations; grey bound, uncertainty bound; α -cut = 0.9).

than MFEP, but in all other fuzzy levels and the AG Basin, the results of MFEP are significantly better than the Vertex method.

Reported statistical indicators in Table 4 describe the different behavior of the IMFEP method in modeling uncertainty of the models and basins when compared with MFEP results. Both the primary indicators (ARIL and P_{level}) are higher than the values estimated for MFEP results, but rates of these changes are different and inconsistent for the two basins. In the K Basin, the improvement in the

results, even with uncertainty levels as wide as 0.5 or 0.2, is noticeable. Based on the NUE indicator, the best achieved uncertainty bounds for this basin are better than those obtained from MFEP. Also, the general results of the IMFEP method is the best among the other fuzzy extension-based methods. In the AG Basin, IMFEP results are not as good as MFEP results, but its uncertainty responses are better than Vertex and Standard mathematical fuzzy methods. Figure 15 depicts uncertainty bounds using the IMFEP method with α -cut equal to 0.7.

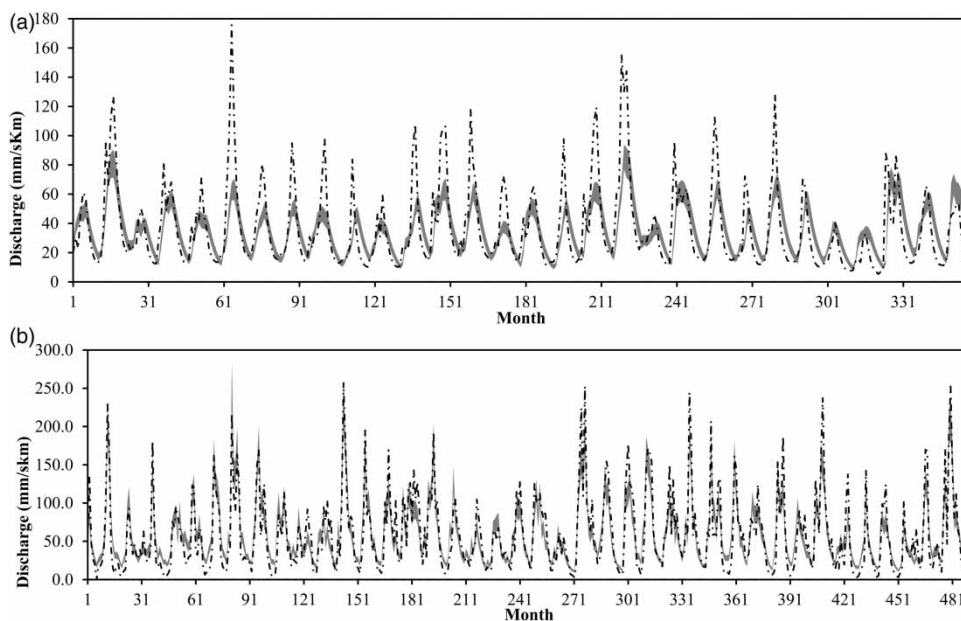


Figure 14 | Uncertainty simulation using MFEP for monthly water balance models of (a) K and (b) AG Basins (dashed line, observations; grey bound, uncertainty bound; α -cut = 0.7).

For uncertainty estimation using the UNEEC method, first the relevant parameters must be selected. As described in the description of the UNEEC method, AMI has been used to do this task. In Table 5, AMI for different pairs of model parameters (model inputs and state variables) and residuals for both basins are shown. It should be noted that the two values of lags of each parameter have been included in the evaluation procedure as well. According to the AMI results, seven (AMI >0.07) and six (AMI >0.06) parameters for K and AG Basins, respectively, have been selected for the UNEEC methods to predict uncertainty. These criteria have been selected based on expert judgment. The selected parameters are differentiated by grey shading in Table 5.

After that, UNEEC employs fuzzy *c*-means clustering for identifying groups of data for which percentiles will be estimated. Selection of an appropriate number of clusters was performed using the Xie and Beni's index and Dunn's index, following Halkidi *et al.* (2001) and Solomatine & Shrestha (2009). However, based on these results, cluster numbers 3 and 9 are appropriate choices for K and AG Basins, using Xie and Beni's indicator, but 4 and 10 clusters should be selected for K and AG Basins in practice, respectively. For the validation period, as stated in the description of the UNEEC method, instance-based learning method has been used for estimating uncertainty bounds as was also done by Solomatine & Shrestha (2009). The smoothing exponential coefficient of fuzzy *c*-mean has been found by

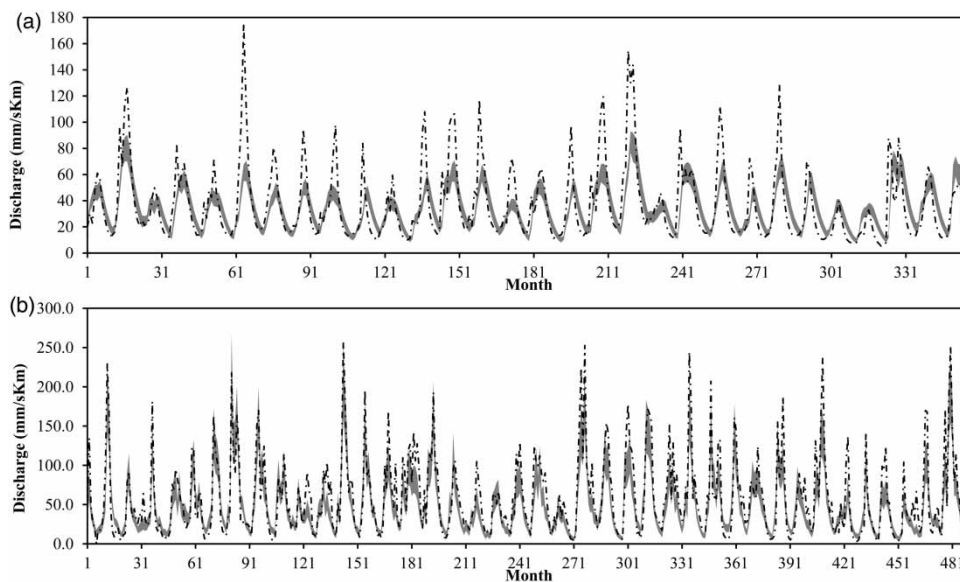


Figure 15 | Uncertainty simulation using IMFEP for monthly water balance models of (a) K and (b) AG Basins (dashed line, observations; grey bound, uncertainty bound; α -cut = 0.7).

Table 5 | AMI between different model inputs and state variables of both models and basins and their residuals

Probable predictors for residual estimation	Basins and lags					
	0		1		2	
	K	AG	K	AG	K	AG
Rainfall	0.0564	0.0506	0.0239	0.0274	0.0286	0.0300
Pan evaporation (PE)	0.0919	0.0601	0.0526	0.0695	0.0761	0.0549
Evapotranspiration	–	0.0583	–	0.0529	–	0.0501
Soil moisture (S)	0.1237	0.0972	0.0853	0.0665	0.0677	0.0572
Computed runoff	0.1311	0.0936	0.1033	0.0612	0.0896	0.4640

calibration to be 2 and 3.5 for K and AG Basins, respectively.

Tables 6 and 7 present the three statistical indicators estimated for UNEEC results for the two basins with different cluster numbers and percentile levels. The maximum detected P_{level} in these tables are equal to 0.63 and 0.91 for K and AG Basins, respectively, but their braced areas are not the lowest in these cases. If P_{level} is selected as the only statistical indicator, CI equal to 90% for both basins should be selected as the best uncertainty level ignoring their coverage areas. But, considering the secondary indicator, NUE values, the best CIs for K and AG Basins are equal to 30 and 10%, correspondingly. Uncertainty simulation in AG Basin with seven clusters and CI equal to 10 has been selected as the best result of UNEEC for this basin. The results for AG and K Basins for the selected cluster numbers and CIs are presented in Figure 16. Judging by the NUE indicator, for both basins, UNEEC performances for lower CIs are often higher than for the higher CIs. The highest NUE estimated for Standard fuzzy mathematics results for K Basin is higher than that obtained from UNEEC results. Also, NUE values for the UNEEC method represent the acceptable performance compared with Standard fuzzy mathematics and Vertex method. Generally, considering P_{level} values, the results of MFEP and UNEEC have roughly similar performance.

The results of UNEEC obviously depend on the choice of appropriate number of clusters, parameters of fuzzy clustering, and the choice of the variables used in predicting percentiles. It is worthwhile to note that the results of the UNEEC method are based on uncertainty modeling of model residuals, so from this point of view UNEEC has a different structure versus the other methods used in this study. One of the advantages of IMFEP is that its results are only affected by MFs, so less effort by the modeler is needed for selecting the model parameters.

To assess the general efficiency of the fuzzy uncertainty simulation methods, weighted summation of NUE with confidence level for both basins are presented in Table 8. To calculate these values, NUEs for higher/lower CIs (higher α -cut) receive higher/lower weights. In general, Standard fuzzy mathematics shows the lowest performance in uncertainty modeling in both basins. In the K Basin, IMFEP, MFEP, Vertex method, and UNEEC (with four

Table 6 | Statistical evaluations using fuzzy UNEEC for different CIs and clusters in K Basin

Number of clusters Confidence intervals and bounds	2			3			4			5			6		
	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE
90 [5, 95]	1.711	0.631	0.369	1.628	0.628	0.386	1.569	0.578	0.369	1.479	0.584	0.395	1.466	0.57	0.389
80 [10, 90]	1.270	0.514	0.405	1.160	0.511	0.441	1.112	0.478	0.429	1.066	0.483	0.454	1.068	0.469	0.440
50 [25, 75]	0.583	0.285	0.488	0.571	0.279	0.489	0.547	0.268	0.490	0.518	0.215	0.415	0.518	0.215	0.416
30 [35, 65]	0.307	0.17	0.555	0.306	0.154	0.502	0.296	0.176	0.594	0.305	0.128	0.421	0.294	0.126	0.427
10 [45, 55]	0.087	0.042	0.483	0.096	0.05	0.522	0.097	0.053	0.545	0.108	0.059	0.543	0.101	0.053	0.526

Table 7 | Statistical evaluations using fuzzy UNEEC for different CIs and clusters in AG Basin

Number of clusters Confidence intervals and bounds	7			8			9			10			11			12		
	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE	ARIL	P_{level}	NUE
90 [5, 95]	2.112	0.912	0.432	2.006	0.906	0.452	1.988	0.91	0.458	1.976	0.91	0.461	2.05	0.91	0.444	1.993	0.912	0.46
80 [10, 90]	1.5	0.816	0.544	1.477	0.818	0.554	1.478	0.82	0.555	1.487	0.824	0.554	1.468	0.82	0.559	1.444	0.816	0.57
50 [25, 75]	0.711	0.527	0.741	0.685	0.511	0.746	0.694	0.517	0.745	0.699	0.521	0.745	0.686	0.513	0.747	0.683	0.513	0.75
30 [35, 65]	0.388	0.299	0.772	0.386	0.303	0.786	0.386	0.301	0.78	0.383	0.297	0.776	0.381	0.305	0.802	0.383	0.307	0.8
10 [45, 55]	0.12	0.11	0.915	0.113	0.108	0.853	0.125	0.11	0.875	0.12	0.116	0.862	0.123	0.116	0.839	0.129	0.116	0.898

clusters) represent the descending order of efficiencies. In the AG Basin, MFEP has shown the best performance and after that IMFEP, UNEEC, and Vertex method are the second to fourth recommended methods. Thus it can be concluded that Standard fuzzy mathematics performs least for both basins. Based on the depicted results of Table 8, UNEEC, MFEP, and IMFEP methods, against Standard mathematics and Vertex method, have the same performance in both K and AG Basins. In general, UNEEC and Vertex methods (also often Standard fuzzy mathematics) generally show similar behavior in simulating uncertainty by means of the NUE indicator for both models and basins.

CONCLUSIONS

In this paper, we use the fuzzy logic-based characterization of uncertainty (which is axiomatically different from the more widely used probabilistic characterization), and propose a generalized form of MFEP, named IMFEP, for fuzzy parametric uncertainty assessments. The presented model is tested for uncertainty simulation of the results of two conceptual monthly water balance models. The IMFEP method is based on coupling discretized convex MFs of model inputs, as other FAC methods, and on MFEP to assess the uncertainty of the model outputs with the same α -cuts. Efficiency of the proposed IMFEP has been also compared with four possibilistic and a hybrid uncertainty assessment method. Three of these fuzzy-based methods, namely Standard fuzzy mathematics, Vertex method, and MFEP method, have been presented before in the realm of possibilistic uncertainty simulation based on fuzzy extension principle (and FAC terminology), and UNEEC (which essence is probabilistic) can be also seen as a hybrid approach since it employs the ideas of fuzzy logic as well.

Based on the presented experiments, the Standard fuzzy mathematics in one of the basins has shown very wide uncertainty bounds. The Vertex method should be seen as a more effective method (based on P_{level} , ARIL, and NUE indicators) than the Standard fuzzy mathematics. Against the simplicity of implementing Standard fuzzy mathematics and its straightforward structure, its results may suffer from

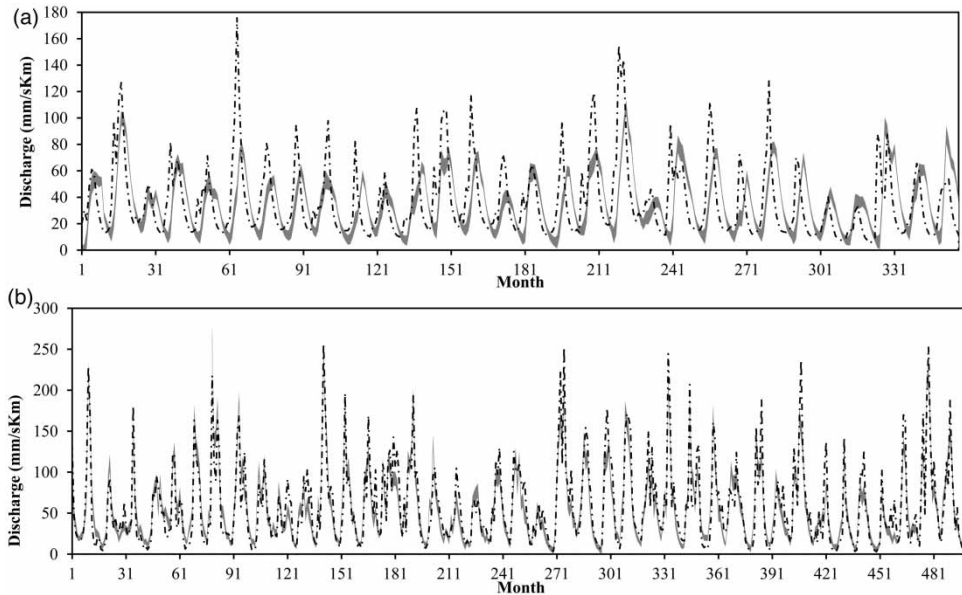


Figure 16 | Uncertainty simulation of AG and K Basins with UNEEC method (dashed line, observations; grey bound, uncertainty bound). (a) K Basin (four clusters and CI equal to [35, 65]). (b) AG Basin (seven clusters and CI equal to 10%).

Table 8 | General assessment of the interested fuzzy uncertainty method (weighted summation of NUE with confidence level or α -cuts)

Uncertainty assessment method	Model (K)	Model (AG)
Standard fuzzy mathematic	0.52	<< 1
Vertex method	0.94	0.59
MFEP method	0.99	0.94
IMFEP method	1.06	0.81
UNEEC	0.58	0.49

wideness of uncertainty coverage area. When using the Vertex method, a high number of simulations must be carried out to achieve acceptable results. As for the important differences between the structures of UNEEC and Vertex methods, UNEEC results have been roughly the same as the results of the Vertex method.

One of the most important results of implementing MFEP and IMFEP is their low variance of NUE in different α -cuts. IMFEP is characterized by the best or near the best NUE values in all fuzzy levels compared with the other three methods. As for UNEEC, it requires identification of several parameters: type and number of inputs, type and structure of the selected clustering and regression methods and may be characterized as a data-hungry method since it

requires some critical mass of data for reconstruction of the empirical distributions for each cluster, and all this may limit the universal applicability of this method.

Comparing the results of MFEP versus Vertex or Standard fuzzy mathematics, as mentioned before, it can be seen that fuzzy MFs of the resulted uncertainties can only be of triangular form. To add a new degree of freedom in fuzzy shapes and forms of the results, IMFEP has been presented in the current paper for the first time. Based on the proposed incremental simulation of different FAC methods, achieving fuzzy MFs of various shapes is possible. The proposed MFEP and its generalized form, the IMFEP method, have the following advantages compared with the other fuzzy extension-based methods tested in this study: (1) it is as straightforward as standard fuzzy mathematics; (2) the method of projecting uncertainty is based on mathematical relationships of the models and uses the 'OR' fuzzy operator following the approach of MFEP (reason of narrower uncertainty bounds); (3) it is computationally more efficient than methods based on sampling, like the Vertex method. On the other hand, due to the very fact that the Vertex method and its variants use sampling strategy (and hence less efficient since they require more model runs), they can be used for models of any complexity, while the methods directly

using fuzzy arithmetic (so also like MFEP and IMFEP methods) are applicable only for models with open sources and functions convertible to their fuzzy variants.

It should be mentioned also that one has to be careful in comparing methods without knowing the ‘truth’ and reliable performance measures. In the case of the methods considered, we do not know the ‘real’ uncertainty of a model under question, and, as explained earlier, the criteria we use for estimation of a model output uncertainty are quite approximate. An advisable way of addressing this problem is to use several methods each of which is based on slightly different assumptions and focuses on various aspects of uncertainty.

Further research can be focused on testing other forms of the fuzzy OR operator in MFEP and IMFEP methods projecting input uncertainties to the outputs. It would also be interesting to perform a more comprehensive comparison with various types of the widely used probabilistic Monte Carlo-based methods (following, e.g., Abebe *et al.* 2000), combining fuzzy and probabilistic Monte Carlo approaches (e.g., Franks *et al.* 1998), and testing the usefulness of using fuzzy-random variables (Kwakernaak 1978) in the context of analyzing uncertainty of hydrological models.

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