to either of Professor Trefethen's questions. Although we are fairly confident of what they would reveal, such experiments could well produce surprises and probably should be made.

On the Shock Wave Velocity and Impact Pressure in High-Speed Liquid-Solid Impact[1]

B. C. SYAMALA RAO[2] and N. S. LAKSHMANA RAO[2]

We wish to congratulate the author for his useful contribution on a subject of interest to several investigators. The paper very elegantly shows that for high-speed liquid-solid impact, the propagation velocity (C) of the shock wave is different from the acoustic velocity (C_0) in the undisturbed liquid.

While estimating the impact pressure on an elastic target, the particle velocity change in the liquid (V_1) was related to the impact velocity (V_0) and the particle velocity (V_2) in the target by the author through the equation

$$V_1 = (V_0 - V_2) \tag{18}$$

This equation is true in the case of high-speed solid-solid impact only as shown by Engel.[3] In the case of liquid-solid impact, there will be a radial outflow of the liquid at the impact interface. Hence, equation (18) becomes

$$V_1 = \alpha(V_0 - V_2) \tag{19}$$

where α is a nondimensional coefficient less than unity. Engel[3] also gives an equation to estimate as

$$\alpha = \frac{0.41}{1 + (0.59Z_0/Z_2)} \tag{20}$$

Using equation (19), the impact pressure expressed by the author in equation (7) becomes

$$P = \frac{\alpha\rho_0 C V_0}{1 + (\alpha\rho_0 C/\rho_2 C_2)} \tag{21}$$

Since the values of α as obtained from equation (20) are always less than 0.41, the impact pressures estimated taking $\alpha = 1.0$ will be considerably large. Would the author kindly explain why he chose a constant value of $\alpha = 1.0$ in his analysis?

If α is included in the analysis, the several equations derived by the author get modified as discussed below:

The main quadratic equation (9), used for deriving the value of (V_1/V_0) becomes

$$\frac{Z_0 k M_0}{Z_2}\left(\frac{V_1}{V_0}\right)^2 + \left(\frac{Z_2 + \alpha Z_0}{\alpha Z_2}\right)\left(\frac{V_1}{V_0}\right) - 1 = 0 \tag{22}$$

The first approximate solution for (V_1/V_0) obtained from equation (22) is

$$\frac{V_1}{V_0} \cong \frac{\alpha}{1 + (\alpha Z_0/Z_2)} \tag{23}$$

which is valid when

$$\frac{\alpha k M_0}{2 + (\alpha Z_0/Z_2) + (Z_2/\alpha Z_0)} \ll 1 \tag{24}$$

The second approximate solution of equation (22) applicable for higher values of M_0, is

$$\frac{V_1}{V_0} \cong \frac{(Z_2/\alpha Z_0) + \alpha k M_0}{1 + (Z_2/\alpha Z_0) + 2\alpha k M_0} \tag{25}$$

which is valid when

$$\frac{\alpha k M_0(1 + \alpha k M_0)}{[1 + (Z_2/\alpha Z_0) + 2\alpha k M_0]^2} \ll 1 \tag{26}$$

[1] By F. J. Heymann, published in the September, 1968, issue of the JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 90, pp. 400–402.

[2] Lecturer and Associate Professor, respectively, Dept. of Civil and Hydraulic Engineering, Indian Institute of Science, Bangalore, India.

[3] Engel, O. G., "Note on Particle Velocity in Collisions Between Liquid Drops and Solids," *Journal of Research*, National Bureau of Standards, Vol. 62A, 1960, pp. 497–498.

Author's Closure

The author wishes to thank the discussers for their interest. He disagrees, however, with discussers' contention that a coefficient α should be incorporated in the expressions for liquid/solid impact. The reason for this is two-fold:

Firstly, the analysis presented in the paper is explicitly restricted to the one-dimensional case, i.e., of two infinite plane surfaces coming into contact. This was clearly stated in both the Introduction and the Summary of the paper, because the author well realizes that two or three-dimensional impact, between nonplane surfaces, is a much more complicated affair. Lateral flow or particle motion is therefore ruled out here, and Engel's equations (19) and (20), which relate to the impact of a liquid sphere onto a solid surface, do not apply to the present case.

Secondly, the author doubts that Engel's analyses (see footnote in discussion) [12][4] are applicable, even to the impact of a liquid drop onto a solid surface, during that brief but crucial initial stage when compressibility phenomena govern the liquid response to the impact and maximum impact pressures are developed.

No rigorous analysis of this type of impact, taking compressibility into account, has yet come to the author's attention. The author, however, favors an argument expounded in detail by Bowden and Field [13]. According to this, an essential feature of the impact process, between a curved liquid surface and a plane solid surface, is an initial stage during which the response of the liquid is entirely compressible, and *no* lateral out-flow (such as would introduce the α of equation (19)) can occur. That is so because the perimeter of the impact interface moves tangentially outward at a speed which initially *exceeds* the velocity of the shock waves generated by the impact, see Fig. 3. The resulting shock front is, therefore, attached to the solid surface; and the compressed liquid, being bounded entirely by the solid surface on one side and by the shock front separating it from undisturbed liquid on the other, cannot flow. It is only when shock waves can overtake the interface perimeter, and reach a "free" surface, i.e., when the shock front becomse detached, that lateral flow is able to begin.

Bowden and Field [13] also concluded that, during this initial "compressible" stage, the impact pressure is uniform and equal to the "one-dimensional" pressure as given by equation (3). The latter conclusion appears to be somewhat oversimplified, and a more accurate picture can perhaps be deduced from Skalak and Feit's [3] results, which apply to the closely related case of a rounded or wedge-shaped solid body impacting onto a plane liquid surface. These results confirm that there is an initial stage without flow, and that the *average* impact pressure during this stage equals the one-dimensional pressure $\rho_0 C V$; but suggest that the pressure

[4] Numbers [12–17] in brackets designate References at end of closure.

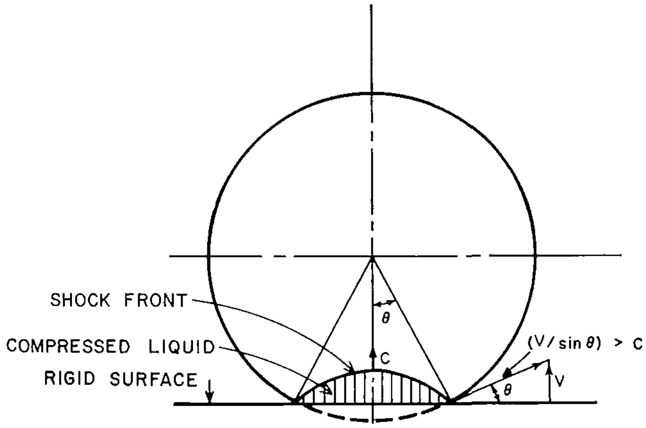

Fig. 1 Initial (compressible) stage of impact between liquid sphere and rigid surface, showing attached shock front

distribution becomes nonuniform, with the pressure at the perimeter *increasing,* and that under the center *decreasing,* as the interface angle θ grows. The significant point here is that the maximum stress experienced by the solid surface will, therefore, be *higher* than that predicted by the one-dimensional model, rather than lower as discussers have suggested. This may be one reason why liquid impingement damage and erosion has often been found to occur at unexpectedly low impact speeds.

Once lateral flow does begin, the picture changes drastically, of course, and the impact pressures do decrease. It is noteworthy that the lateral outflow velocities can be several times greater than the impact velocity, as has been observed by numerous workers [1, 2, 12]. While a rigorous treatment of this is lacking once again, Brunton [1, 14] has pointed out that the process involved is similar to that occurring in shaped charge detonations and also in explosive welding or cladding, where a high-speed jet is formed by a kind of fluid wedging action.

The latter has been analytically treated by Walsh, et al. [15], and by Harlow and Pracht [16]. They confirmed that below a critical "collapse angle" no jetting occurs. This is consistent with the previously stated assumption of no lateral outflow during the early stage of droplet impact.

It is to be hoped that in the near future someone will treat the liquid drop impact process in a rigorous manner, perhaps by means of a time-incremented numerical approach similar to the "Particle-in-Cell" method used by Harlow and Pracht [16]. The results should be most interesting from an academic viewpoint as well as useful in the context of liquid impact damage.

In a final comment (not related to the question raised by discussers), the author would like to refer to a recent contribution by Ruoff [17], which develops a more rigorous justification for a linear shock wave velocity relationship such as here proposed in equation (5). According to Ruoff, the shock velocity can be expressed as a Maclaurin expansion in the form

$$u_s = c + su_p + s'u_p^2 + \ldots \qquad (27)$$

where u_s is the shock velocity, u_p the particle velocity, and c, s, and s' are quantities which can be calculated from densities and ultrasonically measured properties of the material. Ruoff shows that for several materials s' is very nearly zero, because of the cancellation of terms which contribute to it. Equation (27) then reduces to the form of equation (5). The experimental data shown in Fig. 1 suggest, however, that s' for water is negative and not negligibly small.

The author is indebted to Dr. J. E. Field for bringing references [15] and [16] to his attention.

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