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discussion

Conditions for a Pipe to Run Full When Discharging Liquid Into a Space Filled With Gas[1]

D. S. MILLER.[2] The data on vertical downward flow have a much wider interest than the authors concern with nuclear reactors. For instance, heat exchanger and condenser cooling water outlet pipes on large petro-chemical plants often discharge vertically. If, as has happened, the outlet pipe discharges into a free flowing culvert the pipe may fail to prime, resulting in loss of syphon head and vibration of the pipe and heat exchanger.

In civil engineering the problems of priming large syphons are well known. In cases where there is no water seal at outlet devices are used to deflect the flow to form a seal, causing the syphon to prime without the need for an excessively large supply head.

In horizontal flow a pipe runs full if the critical depth exceeds the pipe diameter and, if the normal depth of flow is less than the pipe diameter, a critical section forms at the pipe outlet. Between these two flow conditions is a region requiring experimental data, such as the authors have provided, in order to specify the water profile. Similarly, two flow conditions can be specified for vertical downward flow. If the velocity is sufficiently high, for friction losses to equal or exceed the hydrostatic head change, the pipe will run full and if a critical section is present at inlet free fall flow, modified by contact with the wall, occurs. Between friction control and inlet control is a region for which experimental data are required.

In many engineering situations flow enters a vertical pipe through a bend or other arrangement which is conducive to the formation of an inlet critical section. The authors inlet flow arrangement appears to have been particularly effective in priming the vertical pipe. It would, therefore, be unwise to apply the results to other inlet configurations. It would be useful to know if for the larger pipes it was necessary to manually prime the syphon before obtaining the results in Fig. 13.

The phenomenon of large bubbles "riding the wall" is particularly evident in large pipes. A bubble at the wall is subject to a lower drag than a bubble in the bulk of the fluid. If, due to a variation in velocity at the wall, the bubble's nose is displaced off the wall, the whole bubble moves toward the centre of the pipe to be swept downwards until it can move back onto the wall. In large systems the cyclic movement of the bubble up the wall and then down in the centre of a pipe can cause considerable pressure fluctuations. It would be interesting to know what the sub-atmospheric pressures are at outlet and how they vary with size, velocity and surface tension. Perhaps these could be measured during the further work proposed in the paper.

[1]By G. B. Wallis, C. J. Crowley, and Y. Hagi, published in the June, 1977, issue of the JOURNAL OF FLUIDS ENGINEERING, TRANS. ASME, Series I, Vol. 99, No. 2, p. 405.

[2]Principal Research Engineer, BHRA Fluid Engineering, Cranfield, Bedford, England.

Authors' Closure

Dr. Miller's comments are a useful addition to our paper.

The larger pipes were primed by using a water flow rate sufficiently large to wash out all the air bubbles. Once the pipe runs full the conditions for "unpriming" at the bottom and for washing out large bubbles are independent of how the pipe was originally primed; our results should therefore be valid for many inlet configurations as long as the liquid velocity distribution is not significantly altered.

We agree that upstream effects, particularly bends, steps or corners that cause the liquid flow to separate from the wall, can influence behavior in the unprimed condition.

Lift on a Rotating Porous Cylinder[1]

C. DALTON.[2] The author of the paper has presented a very effective analysis of the problem of a rotating cylinder in a uniform flow field with surface suction. The treatment considers the boundary layer to be laminar, separation of the boundary layer is omitted and no effect of the surface curvature on the boundary layer is considered. I am not sure that omission of all of these effects is consistent and would like the author to comment on the following points: First, it might well be true that surface curvature effects are negligible due to the effects of uniform suction but displacement effects are not necessarily negligible over the entire surface. I think that further discussion on this point is necessary. It is not sufficient, even in the interest of conciseness, to omit effects just because a prior investigator says they are negligible?

Secondly, the assumption of a laminar boundary layer and a rotational velocity great enough to cause the stagnation point to be removed from the cylinder surface seem inconsistent. A rotational velocity of this magnitude could conceivably cause the boundary layer to become turbulent. Has there been any experimental work that could identify the values of the parameters involved to determine the transition criterion?

If the author can provide satisfactory answers to these questions, then I believe that his analysis can lead to a better understanding of the fluid mechanics of wind power generation.

A. B. WARDLAW, JR.,[3] **AND R. L. ADAMS.**[3] In the above paper the author obtains a solution for viscous flow about a rotating

[1]By R. M. C. So, published in the December, 1977, issue of the JOURNAL OF FLUIDS ENGINEERING, TRANS. ASME, Series I, Vol. 99, No. 4, p. 753.

[2]Associate Professor of Mechanical Engineering, University of Houston, Houston, Texas.

[3]Staff Members, MIT Lincoln Laboratory, Lexington, Mass.

cylinder in uniform flow with suction. The general approach is to transform the problem so that the boundary conditions are identical to those on a rotating cylinder without suction. Following the approach developed by Glauret[4] for the nonsuction case, an asymptotic solution is obtained in the parameter $\alpha = 2U_0/Q$. Although this solution clearly satisfies the formulated problem, it does not appear to be the appropriate one when strong suction exists.

The boundary layer profiles calculated in the above paper presume that the velocity at the edge of the boundary layer, U, is equal to the sum of two components. The first is the potential solution for a cylinder in uniform crossflow, $2U_0 \cos(x/d)$, while the second is the tangential velocity component, Q, induced on the inviscid outer flow region by the cylinder rotation. As will be shown in the following paragraph, presumption of a Q component is not necessary when a strong suction exists. Under these conditions an alternative solution to the problem exists which features an edge velocity solely equal to the cylinder in crossflow solution.

To illustrate these two possible solutions, consider the problem of a rotating cylinder in a stationary fluid with strong suction. Due to symmetry, $dU/dx = 0$, and the boundary layer equations resemble those on a flat plate. Using the symbols defined in the above paper, a solution to the governing equations (i.e., equations (1) and (2)) is:

$$u = q + (U - q)\exp(-\mathrm{Re}\, y) \qquad \text{where} \qquad \mathrm{Re} = v_0/\nu \quad (1)$$

When there is neither suction or blowing, $v_0 = 0$ and $q = Q = u$. For blowing $v_0 < 0$ and U must also be set equal to q in order to obtain a finite velocity at the edge of the boundary layer. Hence in both of these cases, equation (1) provides a solution in agreement with that obtained in the above paper for a rotating cylinder in a stationary fluid (i.e., limit $\alpha \to 0$). When suction is present, $v_0 > 0$, and several solutions to equation (1) are possible. The edge velocity, U, can again be set to q which is consistent with the solution in the above paper. Setting $U = 0$ produces:

$$u = q\exp(-\mathrm{Re}\, y) \simeq q(1 - \mathrm{Re}\, y + 0(\mathrm{Re}^2 y^2)) \qquad (2)$$

A solution in this form is cited in footnote 5 as being applicable to a flat plate with uniform suction far from the leading edge. Its validity is limited to small values of v_0 in order to assure preservation of the inviscid boundary conditions. Such a restriction is not necessary for the rotating cylinder problem since the inviscid solution can be constructed by the superposition of a sink of arbitrary strength on the cylinder in crossflow solution.

To determine the correct solution to the problem posed in the preceding paragraph, the Navier Stokes equations are applied to a rotating, circular cylinder (radius equal to unity) in a stationary fluid. Upon setting $\partial/\partial\theta = 0$, the mass conservation equation yields $\tilde{v} = -v_0/r$. Here v is the radial velocity component and r is the radial distance measured from the center of the cylinder. Using this result in the tangential momentum equation produces the following equation for the tangential velocity component, $\tilde{u}$:

$$d^2\tilde{u}/dr^2 + (1/r)(d\tilde{u}/dr)(1 + \mathrm{Re}) + (\tilde{u}/r^2)(\mathrm{Re} - 1) = 0 \qquad (3)$$

This equation has the solution:

$$\tilde{u} = C_1/r + C_2/r^{(\mathrm{Re}-1)} \qquad (4)$$

Applying the boundary condition $u(1) = q$ and $u(a) = 0$ yields:

$$C_1 = -q/(a^{\mathrm{Re}-2} - 1) \qquad C_2 = q - C_1 \qquad (5)$$

[4]Glauert, M. B., "The Flow Past a Rapidly Rotating Circular Cylinder," *Proc. Roy. Soc.*, *A242*, 1957, pp. 108–115.

[5]Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, 6th Edition, New York/London, 1968, p. 369.

Here a is some arbitrary radial location. In the limit $a \to \infty$, C_1 and C_2 become:

$$C_1 \to 0 \quad \text{and} \quad C_2 \to q \qquad \mathrm{Re} > 2$$

$$C_1 \to q \quad \text{and} \quad C_2 \to 0 \qquad \mathrm{Re} < 2$$

For sufficiently large values of suction $\mathrm{Re} > 2$ and

$$u = q/r^{(\mathrm{Re}-1)} \sim q(1 - \mathrm{Re}\, y + 0(\mathrm{Re}^2 y^2)) \qquad \text{where} \qquad r = 1 + y \qquad (6)$$

The expansions of equations (6) and (2) are in agreement indicating that the appropriate edge boundary condition is $U = 0$ for the a rotating, stationary cylinder with suction.

Equation (1) suggests an asymptotic solution to the general problem of a rotating cylinder in uniform cross flow in the limit $\mathrm{Re} \to \infty$. The boundary layer thickness is $0(\mathrm{Re}^{-1})$. Assuming:

$$y = \tilde{y}/\mathrm{Re}$$

$$\tilde{u} = q(u_1(x, y) + u_2(x, y)/\mathrm{Re} + u_3(x, y)/\mathrm{Re}^2 + 0(\mathrm{Re}^{-3}))$$

$$\tilde{v} = v_0(v_1(x, y) + v_2(x, y)/\mathrm{Re} + v_3(x, y)/\mathrm{Re}^2 + 0(\mathrm{Re}^{-3})) \quad (8)$$

and substituting into equations (1) and (2) of the preceding paper produces:

$$\left.\begin{aligned} &\partial v_1/\partial\tilde{y} = 0 \\ &v_1\partial u_1/\partial\tilde{y} = \partial^2 u_1/\partial\tilde{y}^2 \end{aligned}\right\} 0(\mathrm{Re})$$

$$\left.\begin{aligned} &\partial u_1/\partial x + (v_0/q)\partial v_2/\partial\tilde{y} = 0 \\ &u_1\partial u_1/\partial x + (v_0/q)(v_1\partial u_2/\partial\tilde{y} + v_2\partial u_1/\partial\tilde{y}) \\ &\qquad = GG' + (v_0/q)\partial^2 u_2/\partial\tilde{y}^2 \end{aligned}\right\} 0(1) \qquad (9)$$

Here $G = (2U_0/q)\cos(x/d)$ and $G' = dG/dx$. The accompanying boundary conditions are:

$$y = 0 \quad v_1 = -1, \quad v_2 = 0 \quad u_1 = 1 \quad u_2 = 0$$

$$y = \infty \quad u_1 = G \quad u_2 = 0$$

Here q, U_0, and v_0 are assumed to be of the same order. The first and second order solutions to u and v are:

$$u/q = [G + (1 - G)e^{-\tilde{y}}] + (1/\mathrm{Re})(q/v_0)(G')(\tilde{y}e^{-\tilde{y}})\left[(2G - 1) - \frac{(1 - G)}{2}\tilde{y}\right] + 0(1/\mathrm{Re}^2) \qquad (10)$$

$$v/v_0 = -1 + (1/\mathrm{Re})(q/v_0)G'(1 - e^{-\tilde{y}} - \tilde{y}) + 0(1/\mathrm{Re}^2) \qquad (11)$$

Substitution of the above solution into the y momentum equation indicates that to the dominant order the pressure is constant across the boundary layer and is established by the cross flow. Hence the presence of suction destroys the lift on a rotating cylinder. This is in contrast with the solution presented in the above paper.

Author's Closure

This author has to disagree with Professor Dalton concerning the effects of surface curvature and displacement. If the Prandtl boundary-layer approximations are invoked (as indeed in the present analysis) and the flow is assumed laminar, then it can be easily shown that both surface curvature and displacement effects are second-order and are proportional to $\mathrm{Re}^{-1/2}$ (for a detailed discussion on higher-order effects in boundary-layer theory, see van Dyke [8]).[6] Since the expansion for ψ is inde-

[6]Numbers 8–11 in brackets designate Additional References at end of closure.

pendent of Re, there is no need to correct for these effects in the present analysis. They are not ignored simply because Glauert [1] had neglected these effects.

It is a known fact that suction helps to stabilize the boundary layer and thereby delays transition to turbulent flow [7]. In view of this, the assumption of a laminar flow around the cylinder is more likely to hold true than the case of zero suction considered by Glauert [1]. On the other hand, rotation is found to have a stabilizing or destabilizing effect on boundary layer development depending on the direction of action of the resultant Coriolis force [9]. For the present case, the direction of rotation is such that a stabilizing Coriolis force would result. Consequently, the assumptions of laminar flow and a high rotational velocity are not inconsistent as Professor Dalton implied.

Finally, the author wishes to thank Professor Dalton for his thoughtful comments.

The author simply cannot agree with the conclusion drawn by Drs. Wardlaw and Adams. As a matter of fact, the author does not think that any conclusion about the lift on a rotating cylinder can be drawn from their analysis as presented in the discussion. This observation is based on the following reasons.

It is correct that to the dominant order the pressure is constant across the boundary layer. However, this is only a consequence of the Prandtl boundary-layer approximations made and the discussers merely demonstrate that it is indeed correct even in the limit of $(v_0/\nu)\to\infty$. The lift, on the other hand, does not depend on the normal pressure gradient across the boundary layer. It only depends on the circulation around the cylinder and the velocity of the uniform cross flow. Therefore the lift can be calculated if the circulation around the cylinder is known. Alternatively, it can also be calcu'ated if the surface pressure distribution around the cylinder is available. The author finds it very difficult to extract these informations from the analysis given by the discussers. Without these informations, one cannot draw any conclusion concerning the lift acting on the cylinder.

In the boundary-layer flow around the cylinder, the correct characteristic velocity is given by the circulatory velocity, Q, and not by the peripheral velocity, q. This was discussed in detail by Glauert [1]. Consequently, the stream function, ψ, in the present analysis and that of Glauert [1] is normalized with respect to Q rather than q. From the analysis, an expression for the lift can be derived in terms of q, U_0 and the suction parameter, a.

Since the resultant flow field around the cylinder is strongly dependent on the cylinder rotational speed, one is not at liberty to specify both the resultant flow field and the rotational speed. In the present analysis and that of Glauert [1], the choice is made to specify the resultant flow field and then proceed to calculate the rotational speed that would be responsible for such a flow field and hence the lift. The solution is given in equation (25) of the present analysis and if a is set to zero, Glauert's [1] result is obtained. From the analysis proposed by the discussers, it is not clear, to this author at least, whether the rotational speed is assumed known or whether the resultant flow field is assumed known. As a result, this author finds it difficult to draw any conclusion concerning the lift from their analysis.

Physically, it is difficult to visualize how very large suction can destroy the lift on a circular cylinder rotating in a uniform cross flow. The limit $(v_0/\nu)\to\infty$ can be approached either by letting $v_0\to\infty$ in a fluid of constant ν, or by letting $\nu\to 0$ while maintaining v_0 constant. Since the lift on a circular cylinder rotating in a uniform stream is independent of viscosity [4], one would certainly expect the lift to remain finite on a circular cylinder with uniform suction when $(v_0/\nu)\to\infty$ as $\nu\to 0$. Therefore, it is not reasonable to expect the limiting behavior of the lift to change abruptly when the limit $(v_0/\nu)\to\infty$ is approached by letting $v_0\to\infty$.

Finally, the velocity profile obtained by setting $U = 0$ in equation (1) of the discussion represents the asymptotic suction profile on a flat plate [7] as well as on a circular cylinder in an axial stream [10] and in a uniform cross flow [11]. It is not the same as the velocity profile, u, given by equation (6) of the discussion. Consequently, it is difficult to see that equation (1) of the discussion indeed suggests an asymptotic solution to the general problem of a rotating cylinder in a uniform cross flow in the limit of $(v_0/\nu)\to\infty$.

Additional References

8 van Dyke, M., "Higher-Order Boundary-Layer Theory," *Annual Review of Fluid Mech.*, Vol. 1, 1969, pp. 265–292.

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10 Wuest, W., "Asymptotische Absaugegrenzschichten an Längsangeströmten zylindrischen Körpern," *Ing.-Arch.*, Vol. 23, 1955, pp. 198–208.

11 Rosenhead, L., ed., *Laminar Boundary Layers*, Clarendon Press, Oxford, 1963, pp. 345–348.

Numerical Modelling of Unsteady, Separated Viscous Flow[1]

JOEL H. FERZIGER.[2] This paper contains a number of interesting results, especially those concerning the nature of separation and reattachment. Both of these are, of course, research topics of high interest. My comments relate to two areas: the first is the nature of the methods needed to predict separated flow and the second is in the nature of general cautions about computational work.

In the introduction, the author states that since Prandtl's boundary layer equations do not hold in the separated region, the full Navier-Stokes equations must be used. Actually, there are several approximations and assumptions in the Prandtl theory and not all of them break down in the presence of separation. Since there seems to be some confusion on this point in the literature, it is probably worth discussion. Let us look at the assumption one at a time.

1 Prandtl assumes that the streamwise diffusion of momentum by the action of viscosity may be neglected; this assumption renders the momentum equation parabolic. In flows such as the one the author is dealing with, this assumption is probably valid in nearly all of the flow and invoking it might not affect the results significantly. It would have been helpful if the author had assessed the importance of streamwise momentum diffusion. Separated flows have been reported in which Prandtl's equations remain valid even in the separation zone.

2 Prandtl assumes that the boundary layer is thin. This leads to the conclusions that the normal velocity is small and that the pressure does not vary vertically in the boundary layer. These assumptions are obviously violated to a significant degree in the flow treated in this paper.

3 It is usually assumed that the flow in the streamwise direction does not change sign. This assumption is necessary if the equations are to be solved by a marching method, but serves no other function. It is quite possible to have situations in which the flow is reversed in some region, but the boundary layer equations are valid everywhere.

[1]By A. R. Giaquinta, published in the December, 1977, issue of the Journal of Fluids Engineering, Trans. ASME, Series I, Vol. 99, No. 4, pp. 659-665.

[2]Thermosciences Division, Department of Mechanical Engineering, Stanford University, Stanford, Calif.