Higher-degree moment tensor inversion using far-field broad-band recordings: theory and evaluation of the method with application to the 1994 Bolivia deep earthquake

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Accepted 1998 October 5. Received 1998 October 2; in original form 1997 December 2

SUMMARY

We present a method to estimate parameters of the extended earthquake source using higher-degree moment tensors at 27 centroid locations. We show that a Taylor series expansion of Green's functions around a single centroid is not accurate enough when working with seismic wave periods and wavelengths in the range of the rupture duration and spatial extent of the fault, respectively. Introducing a grid of 27 centroid locations on the fault and using higher-degree moment tensors we are able to model adequately body and surface waves with periods and wavelengths smaller than the rupture duration and fault dimensions. Under simplifying assumptions an iterative inversion scheme is coded to estimate parameters of planar, Haskell-type faults. Realistic inversion examples for deep and shallow earthquakes show that uni- and bidirectional rupture models, rupture direction, fault and auxiliary plane and kinematic source dimensions and times can be constrained with teleseismic body and/or surface waves. The application to the deep Bolivia event indicates a subhorizontal fault plane. Unidirectional rupture to the north is slightly preferred. The rupture duration of 25 s and fault dimensions of $47 \times 25$ km agree well with the estimates for the main pulse moment release given in other studies.

Key words: earthquake source mechanism, extended seismic source, Haskell model, inversion, moment tensor, rupture propagation.

1 INTRODUCTION

The zero-degree moment tensor (second-order tensor) representation of seismic sources is nowadays commonly used, mainly because it provides a theoretical tool to estimate efficiently general properties of point sources. Within this framework the inversion for five independent moment tensor components, moment centroid location and time is standard and has led to a wide availability of moment tensor bulletins (e.g. Dziewonski et al. 1995).

However, the more difficult task of routinely estimating kinematic parameters of the extended source and separating fault and auxiliary planes is not yet standard. This task appears to be solvable since the quality and coverage of seismic broad-band stations has improved enormously over the last few years. Information on the rupture duration, the average velocity and sense of rupture, and the geometrical dimensions of seismic sources in different provinces and depths is still incomplete. This information is highly important in understanding the physics of earthquakes. The separation of fault and auxiliary planes is important to estimate the stress field and for further tectonic studies. The discovery of active faults contributes in particular to hazard assessment.

This study is a methodical development in this direction. Several different approaches are commonly used to recover information about the finite fault.

1. Modelling of peak delay times in teleseismic body waves with respect to a reference time and location (e.g. Fukao 1972; Nabelek 1984; Estabrook & Bock 1995). The analysis is problematic when waveform peaks are not coherent over the network and cannot be uniquely defined. This inverse problem is known to be non-unique (e.g. Nabelek 1984). Ihmle (1998) showed that the coherent arrivals and peaks can be interpreted as the instantaneous centroid locations of a propagating rupture front.

2. Source tomographic studies in which a fault with given location and orientation is divided into adjacent independent subfaults that all contribute to the waveforms observed (e.g. Olson & Apsel 1982; Das & Kostrov 1990; Cotton & Campillo 1995; Ihmle & Ruegg 1997). Near-field strong
motions, regional and teleseismic body waves, and mixed seismic and geodetic data are used in the inversions to estimate the spatio-temporal slip distribution on the fault. Usually a large number of observations are necessary since the inverse problem is ill-posed and/or strongly underdetermined. The solution is often stabilized by using constraints (e.g. Hartzell & Heaton 1983; Das & Kostrov 1990). Das & Suhadolc (1996) investigated the reliability of source tomograms with synthetic noise-free data for simple Haskell-type ruptures on planar faults, and showed that heterogeneities in the rupture models obtained are often difficult to distinguish from artefacts due to the limitations of the inversion.

(3) The higher-degree moment tensor representation (e.g. Backus 1977a,b) at the centroid location is used to set up the inverse source problem (Stump & Johnson 1982; Doornbos 1982; Silver 1983; Silver & Masuda 1985; Kagan 1988; Gusev & Pavlov 1988; Jordan 1991; Bukchin 1995; Gomez et al. 1997; Das & Kostrov 1997). Body waves, surface waves and normal modes are used in different methods. Usually moments up to degree two are estimated, implying that the methods are restricted to the analysis of body or surface waves with periods and wavelengths longer than the source and rupture dimensions. In this long-period range the effects of source finiteness and rupture propagation on seismograms are relatively small. The general inverse problem is non-unique (e.g. Pavlov 1994) and usually ill-posed, so that some form of regularization has to be applied. Often positivity constraints are introduced to avoid backslip (e.g. Jordan 1991; Das & Kostrov 1997). Das & Kostrov (1997) tested Haskell-type models, amongst others, using positivity constraints. In this study we restricted the inverse problem to simple rupture models, e.g. uni- or bidirectional rupture on rectangular planar faults with a constant slip vector on the fault (see also Doornbos 1982; Gusev & Pavlov 1988). Backslip is not possible; that is, the magnitude and rake of the slip vector are time-independent. With this strong model restriction our synthetic data tests revealed that the inversion led to stable results.

We use a higher-degree moment tensor representation, based on a Taylor series expansion, and a flexible grid of moment centroids giving the possibility of working with wavelengths and periods shorter than the source dimension and rupture duration, respectively. Our method is therefore a compromise between a low-frequency moment tensor representation of higher degree and a source tomographic technique. Das & Kostrov (1997) presented a similar formulation of the inversion problem by dividing the region in which Green’s functions are approximated into smaller pieces and approximating Green’s functions separately in each piece using a root-mean-square expansion.

Some important and new aspects of our method are listed below.

(1) We use complete reflectivity seismograms for the IASP91 earth model (Kennett 1991; Kennett & Engdahl 1991), while many other polynomial moment studies use WKBJ seismograms (e.g. Doornbos 1982) or full- or half-space seismograms (Gusev & Pavlov 1988; Das & Kostrov 1997).

(2) We analyse mixed data sets, i.e. surface and body waves. We have the option of fitting amplitude spectra or full waveforms.

(3) The fault and auxiliary planes are estimated.

(4) We use a flexible discretization grid; that is, the locations of subevents depend also on the finite source parameters and are estimated during inversion. In most finite source studies a priori estimated ranges of the model space are used.

(5) The inversion is carried out for the fault-plane orientation and the parameters of the extended source simultaneously.

In the first part of the paper we derive the theory of the method. Examples with synthetic data in a homogeneous full space and for deep and shallow earthquakes in a realistic earth are presented in the second part. In the last part we apply the method to the 1994 Bolivia deep event. Applications to shallow earthquakes will be published in a second paper.

2 THEORY

2.1 Higher-degree moment tensor representation using Taylor series expansion

The Cartesian displacement vector \( \mathbf{d}(\mathbf{x}, t) \) at point \( \mathbf{x} \) and time \( t \) due to a moment tensor density distribution \( \mathbf{m}(\mathbf{r}, \tau) \) (non-zero within volume \( V \)) is given by (e.g. eq. 3.19 of Aki & Richards 1980)

\[
d_{u} = \int G_{p,q}^{m}(\mathbf{x}, \xi; t, \tau) \ast m_{pq} \, dV,
\]

where \( \ast \) denotes the time-convolution integral, \( G \) is the Green’s tensor defined for single force excitations at point \( \xi \) and time \( \tau \) and \( G_{p,q}^{m} \) denotes \( \partial G_{p,q}^{m}/\partial r_{p} \). A Taylor series expansion of Green’s functions (components of the Green’s tensor) around a centroid point \( \xi_{0} \) and a centroid time \( \tau_{0} \) leads to a representation already derived by Doornbos (1982); it is given briefly in Appendix A. Necessary notations are also given in Appendix A.

Representing Green’s functions in the frequency domain \( (\omega) \) and using, for layered media, the Sommerfeld integral leads to a representation of Green’s functions as a sum of kernels depending on horizontal slowness \( u \) (reflectivity method, e.g. Müller 1985):

\[
G_{p}^{m}(\omega) = \int_{0}^{\infty} \mathfrak{g}_{p}^{m}(\omega, u) \, du.
\]

The kernel \( \mathfrak{g} \) is synthesized from up- and downgoing \( P \) and \( S \) waves. In the far field, neglecting terms with \( O(1/R^{2}) \), the partial derivatives of Green’s functions are

\[
\frac{\partial G_{p}^{m}}{\partial \xi_{q}} = \int_{0}^{\infty} j \omega \mathfrak{g}_{p}^{m,\xi_{q}} \, du, \quad j^{2} = -1,
\]

where \( \mathfrak{s}(u) \) is the slowness vector at the source. Using this method and keeping the letter \( G \) to denote Green’s kernels (instead of using \( \mathfrak{g} \)) the moment tensor representation in Appendix A can be rewritten as

\[
d_{u}(\mathbf{x}, \omega) = M_{pq} \int_{0}^{\infty} \left\{ G_{p,q}^{m} \ast -j \omega \frac{\partial G_{p,q}^{m}}{\partial \xi_{q}} \Delta(\tau) \right. \\
+ j \omega G_{p,q}^{m,s,q} \Delta(\xi_{s}) + \frac{\omega^{2}}{2} G_{p,q}^{m,s,q} \Delta(\xi_{s})^{2} \\
\left. - \frac{\omega^{2}}{2} G_{p,q}^{m,s,q} \Delta(\xi_{s})^{2} \right\} \, du.
\]

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In Appendix B we derive formulae appropriate for calculating higher-order Green’s functions of layered media with the reflectivity method.

2.2 Application to Haskell’s fault model

2.2.1 Unidirectional rupture

The quantities \( \Delta(\tau) \) and \( \Delta(\xi) \) in (2) define the location of the moment centroid with respect to the location \( \xi^0 \) and \( \phi^0 \) used to calculate Green’s functions. We assume that an optimal location can be found such that \( \Delta(\tau) = \Delta(\xi) = 0 \). The remaining higher-degree tensors in (2) can then be interpreted in terms of kinematic source parameters, for example given by the rectangular Haskell model (length \( L \), width \( W \)) with unidirectional rupture propagation with velocity \( v \), rupture duration \( T = L/v \) and constant final slip \( [u] \) (Fig. 1). We assume that \( T \) is much longer than the rise time \( \Theta = [u]/c \) (\( c \) is the slip velocity). For instance, Bernard (1997) estimated with near-field accelerograms \( \Theta \approx 0.35 \) s and \( T \approx 6 \) s for the 1992 Erzincan, Turkey, \( M_S \) 6.9 earthquake. Synthetic data have shown that \( \Theta \) can barely be resolved with far-field data. \( \Theta \) is therefore not considered here. In this model the healing of the fault surfaces and the ceasing of motion on the fault begins at the initiation point of rupture, prior to the rupture front reaching the end of the fault. The entire fault surface is never simultaneously moving. Greatly simplified models with only a few model parameters have been chosen to regularize the ill-posed and non-unique general inversion problem, and to find a stable method applicable to teleseismic far-field recordings. Other models may alternatively be tested.

Using first a rupture fault within the \( x-y \) plane, the stress glat rate is defined by

\[
\dot{\sigma}_{xz} = \mu [u] \delta(z) \delta \left( \frac{x - L}{2} \right) \left[ H(x) - H(x - L) \right] 
\]

\[
\times \left[ H \left( y + \frac{W}{2} \right) - H \left( y - \frac{W}{2} \right) \right],
\]

where \( H \) is the Heaviside step function, \( \delta \) is Dirac’s delta function and \( \mu \) is the shear modulus. The stress glat rate is non-zero for \(-W/2 \leq y \leq W/2, 0 \leq x \leq L \) and \( 0 \leq \tau \leq L/v \).

With (A1) in Appendix A one finds

\[
\dot{M}_{xz} = \mu [u]WL = A, \quad \Delta(\xi^2) = \frac{T^2}{12} = B, \quad \Delta(\xi) = 0.
\]

(3)

\[
\Delta(\tau) = \frac{LT}{12} = D, \quad \Delta(\xi^2) = \frac{L^2}{12} = E, \quad \Delta(\xi) = \frac{W^2}{12} = F.
\]

We used \( \phi^0 = T/2 \) and \( \xi^0 = (L/2, 0, 0)^T \). All other source components are zero. A rupture in the opposite direction, starting at \( x = L \) and stopping at \( x = 0 \), is described by changing the sign of \( D \) in (3).

In Fig. 2 we give an example of the angular radiation patterns corresponding to higher-order terms \( M_{xz}, \Delta(\tau^2), M_{zz}, \Delta(\tau^2) \), and \( M_{zz}, \Delta(\xi^2) \). (A1) and \( \Delta(\xi) \) in (A1).

Directivity effects break the symmetry between rupture plane and auxiliary plane, especially for \( S \) waves (or surface waves), mainly due to their smaller propagation velocity.

Eq. (3) is generalized for faults with arbitrary strike \( \Phi \), dip \( \delta \), rake \( \lambda \) (defining the slip direction) and a second rake angle \( \lambda_L \) (defining the rupture direction) by means of tensor rotation, which leads to

\[
\dot{M}_{ij} = A_{ij}^{(0)}, \quad \Delta(\xi) = C_{ij}^{(1)}, \quad \Delta(\tau) = D_{ij}^{(1)},
\]

\[
\Delta(\xi^2) = E_{ij}^{(2)}(\lambda_L) + F_{ij}^{(2)}(\lambda_L - 90^\circ),
\]

with

\[
\begin{align*}
\dot{f}_{11}^{(0)} &= - \sin \delta \cos \lambda \sin (2\Phi) - \sin (2\delta) \sin \lambda \sin^2 \Phi, \\
\dot{f}_{12}^{(0)} &= \sin \delta \cos \lambda \sin (2\Phi) + \frac{1}{2} \sin (2\delta) \sin (2\Phi) = f_{12}^{(0)}, \\
\dot{f}_{13}^{(0)} &= - \cos \delta \cos \lambda \cos \Phi - \cos (2\delta) \sin \lambda \sin \Phi = f_{13}^{(0)}, \\
\dot{f}_{22}^{(0)} &= \sin \delta \cos \lambda \sin (2\Phi) - \sin (2\delta) \sin \lambda \sin^2 \Phi, \\
\dot{f}_{23}^{(0)} &= - \cos \delta \cos \lambda \sin \Phi + \cos (2\delta) \sin \lambda \sin \Phi = f_{23}^{(0)}, \\
\dot{f}_{33}^{(0)} &= \sin (2\delta) \sin \lambda.
\end{align*}
\]

Using \( f_1^{(1)} = \cos \lambda_L \cos \Phi + \cos \delta \sin \lambda_L \sin \Phi \),

\[
\begin{align*}
\dot{f}_{12}^{(1)} &= \cos \lambda_L \sin \Phi - \cos \delta \sin \lambda_L \cos \Phi, \\
\dot{f}_{13}^{(1)} &= - \sin \delta \sin \lambda_L, \\
\dot{f}_{11}^{(2)} &= \cos \lambda_L \cos \Phi + \cos \delta \sin \lambda_L \sin \Phi, \\
\dot{f}_{12}^{(2)} &= \cos \lambda_L \sin \Phi - \cos \delta \sin \lambda_L \cos \Phi, \\
\dot{f}_{13}^{(2)} &= - \cos \delta \cos \lambda_L \cos \Phi - \cos (2\delta) \sin \lambda \sin \Phi = f_{13}^{(2)}, \\
\dot{f}_{22}^{(2)} &= \cos \lambda_L \sin \Phi - \cos \delta \sin \lambda_L \cos \Phi, \\
\dot{f}_{23}^{(2)} &= - \cos \delta \cos \lambda_L \cos \Phi - \cos (2\delta) \sin \lambda \sin \Phi = f_{23}^{(2)}, \\
\dot{f}_{33}^{(2)} &= \sin^2 \delta \sin^2 \lambda_L.
\end{align*}
\]

Source components of the other possible fault plane follows from (4) and (5) by using \( \Phi, \delta \) and \( \lambda \) of the auxiliary plane.

2.2.2 Bidirectional rupture

Bidirectional rupture is considered in an analogous way, leading to slightly different pre-factors \( A, B, C, \) etc. in (3), which are specified in Table 1. We tested uni- and bidirectional models with synthetic data and give examples below.

2.3 Improved source approximation

We find that the one-centroid approximation used to derive (2) is good when wavelengths \( \Lambda \) are longer than \( 2L \) or \( 3L \). At higher frequencies (shorter wavelengths) the approximation

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breaks down. For example, synthetic S-wave pulses from a pure strike-slip source within a homogeneous full space are badly fitted at stations in the opposite rupture direction when using $\Lambda_{\min} \approx 1.2L$ (Fig. 3). Low-pass filtering the theoretical and modelled data in Fig. 3 with a cut-off frequency at 0.07 Hz (corresponding to $\Lambda_{\min} \approx 2.5L$) results in a visually good fit. However, at such low frequencies (large wavelengths) the directivity effects of the extended source and rupture are very small and are negligible when noise is present.

The theoretical data used in Fig. 3 and below (forward problem) have been calculated by superposing seismograms from several hundred densely spaced point sources. This commonly used approach leads to realistic seismograms of finite sources and rupture.

To improve the method for shorter wavelengths and periods, for which directivity effects are larger, we split the integrals in (1) into three parts and expanded the kernels afterwards. Therefore, we used an approach in between the Taylor series expansion around a single moment centroid and a superposition of many point sources, taking advantage of both. Using three time centroids at $q^{(1)} \sim T/6$, $q^{(2)} \sim T/2$ and $q^{(3)} \sim 5T/6$ to improve the approximation of the time integral,

$$\int_0^T \ldots dt = \int_{T/3}^{2T/3} \ldots dt = \int_{2T/3}^T \ldots dt,$$

leads to three additional sets of Green’s functions (Fig. 4, left). The Green’s functions at centroids before and after $q_0$ are exactly and quickly calculated from those at $q_0$ by time-shifting the traces. Green’s functions corresponding to spatial centroids $j_0 (at L/6, L/2, 5L/6, and at -W/3, 0, W/3)$ are approximated by time-shifted Green’s functions also. The

Table 1. Weights $A \rightarrow A^{(ok)}$, $B \rightarrow B^{(ok)}$, etc. that enter (4) in the improved approximation.

<table>
<thead>
<tr>
<th></th>
<th>$A^{(ok)}$</th>
<th>$B^{(ok)}$</th>
<th>$C^{(ok)}$</th>
<th>$D^{(ok)}$</th>
<th>$E^{(ok)}$</th>
<th>$F^{(ok)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unidir.</td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
<td>$D$</td>
<td>$(1+12(j-2)^2)/9$</td>
<td>$E$</td>
</tr>
<tr>
<td></td>
<td>$27/81$</td>
<td>$81$</td>
<td>$(j-2)L/9$</td>
<td>$81$</td>
<td>$(1+12(j-2)^2)/9$</td>
<td>$81$</td>
</tr>
<tr>
<td>Bidir.</td>
<td>$A$</td>
<td>$(1+3(j-2)^2)/81$</td>
<td>$(j-2)(j-2)L/9$</td>
<td>$(j-2)/9$</td>
<td>$(7-6(j-2)^2)/81$</td>
<td>$E$</td>
</tr>
<tr>
<td></td>
<td>$27/81$</td>
<td>$81$</td>
<td>$81$</td>
<td>$81$</td>
<td>$(7-6(j-2)^2)/81$ for $j = 2$</td>
<td>$81$</td>
</tr>
</tbody>
</table>
time-shift is a function of the slowness vector at the source and the rupture direction (Fig. 4, right). This approximation is valid for separated body-wave phases observed at stations far from the source. An alternative and exact approach would be to calculate new Green’s functions for every chosen trial orientation and trial dimension of the source during the iterations. We disregarded this here since the computing time would be too great to be applicable in a routine analysis, and since our results with synthetic data are promising.

Altogether we have for the improved method 27 sets of Green’s functions substituting one original set, namely $G_{pqs}^{ijk} \rightarrow (ijk) G_{pqs}^{n}$, $G_{pqs}^{ijk} \rightarrow (ijk) G_{pqs}^{sr}$, etc., where $i, j, k \sim 1, 2, 3$ correspond to the shifts in the $T$, $L$ and $W$ directions, respectively. For unidirectional rupture the time centroids are at $\tau^{(ij)} = (j-2)T/3 + T/2 + (t-2)T/9$ and the spatial centroids are at $\varsigma_{T}^{(ij)} = L/2 + (j-2)L/3$ and $\varsigma_{W}^{(ij)} = (k-2)W/3$. For bidirectional rupture we choose $\tau^{(ij)} = T/6 + (t-2)T/9$ for $j=2$ and $\tau^{(ij)} = 4T/6 + (t-2)T/9$ for $j=1, 3$, and the same spatial centroids. The related weights $A, B, \ldots$ are given in Table 1.

Using the improved method to explain the test data of Fig. 3 leads to a much better fit than before (Fig. 5). The cut-off frequency used in Fig. 5 is higher ($f_c \sim 0.25$ Hz) than that used in Fig. 3, allowing shorter wavelengths to be present ($\lambda_{\text{min}} \approx 13.8 \text{ km} \approx 2L/3$). More complex and more realistic seismograms can also be fitted at high frequencies with the improved method, as is shown below.

At high frequencies, directivity effects in the seismograms are larger, and therefore the residuals between theoretical and modelled waveforms using a wrong model are larger; that is, the misfit minimum in the parameter space has the potential to be much more pronounced. A deep minimum is important to estimate reliable parameters with real or noisy data. With the improved method we are able to make use of the deep minima at high frequencies, where the simple approximation totally breaks down (Fig. 6).

**Figure 3.** (Left) Source (centroid at origin, $\Phi=0^\circ$, $\delta=90^\circ$, $\lambda=0^\circ$, $L = 20 \text{ km}$ in the $-x$-direction, $W = 10 \text{ km}$ in the $y$-direction, $e = 3 \text{ km s}^{-1}$) and station configuration in homogeneous full space ($s = 6 \text{ km s}^{-1}$, $\beta = 3.46 \text{ km s}^{-1}$). (Right) Theoretical (continuous line) and modelled (dashed line) $S$-wave pulses (transverse component) using the one-centroid approximation. A low-pass filter (non-causal) with cut-off frequency at 0.14 Hz has been applied, so that the shortest wavelengths are $\pm 24 \text{ km}$.

**Figure 4.** (Left) Sketch showing how a smooth Green’s function (versus rupture time $\tau$) is approximated using three centroids at $T/6$, $T/2$ and $5T/6$. Dashed lines indicate the first derivative term of the Taylor series expansion using three centroids (thick) and one centroid at $T/2$ (thin). (Right) The nine spatial centroid locations are indicated by stars on the fault plane with length $L$ and width $W$. The time shift $\delta t$ for a given phase propagating with slowness $s$ from two neighbouring centroids is a function of the angle between rupture and slowness direction.

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2.4 Inversion approach

The inversion is in two steps. The first step is a linear inversion for a standard moment tensor (point source) and optimal source coordinates $q^0$ and $j^0$ using low-frequency data only. We use only acausal filters, which we found is an important practical point of the approach for retrieving unbiased parameters. [Then $m_l \sim q_0$.] One may apply station travel-time corrections at this point to obtain an optimal fit between synthetic and real data within the point source approximation.

Delay times up to 5 s relative to expected $P$ and $S$ arrival times from 1-D earth models are commonly observed for well-located events. They are due to unconsidered 3-D structure. Numerical simulations with periods larger than 30–50 s in a 3-D earth showed that traveltime corrections remove most of the effects from unconsidered structure (Igel & Gudmundsson 1997).

The second step is to include higher-frequency data and to solve (2) using a linearized, iterative approach (gradient method, e.g. eq. 9.13 of Menke 1984). We therefore estimate the fault parameters $M_0$, $\Phi$, $\delta$, $\lambda$, $\lambda_0$, $T$, $L$, and $W$. The optimal source coordinates are not improved in the second step. The necessary derivatives of (2) with respect to source parameters of the Haskell model have been analytically calculated and are not listed here. We stop calculation when the residual sum does not decrease with iteration.

We have the option of working with waveforms or amplitude spectra of body waves (and/or surface waves). For body-wave spectra convergence is slower, but results are insensitive to station delay times.

3 INVERSION EXAMPLES

3.1 Homogeneous medium

First we used body waves in a homogeneous full space to investigate the convergence and resolution of the method. Strike, dip, rake and moment of the source are easily constrained when using phases at 10 or fewer stations within an azimuthal range of 90° (Table 2). As long as the deviation

Table 2. Source and ray parameters. Distance $R$ was 2000 km.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth (°)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>Take-off (°)</td>
<td>70</td>
<td>50</td>
<td>80</td>
<td>90</td>
<td>70</td>
<td>70</td>
<td>90</td>
<td>120</td>
<td>100</td>
<td>70</td>
</tr>
</tbody>
</table>

Figure 5. Theoretical (continuous line) and modelled (dashed line) $P$ and $S$ waves using the improved approximation. The fit is nearly perfect. A low-pass filter with cut-off frequency at 0.25 Hz has been applied so that the shortest wavelengths are $\approx 13.8$ km. The geometry of the problem is given in Fig. 3.

Figure 6. Sum over absolute residuals between theoretical and modelled waveforms versus rupture duration $T$ and fault length $L$ estimated from the single centroid (dotted line) and improved method (continuous line), at low (0.04 Hz) and high frequencies (0.25 Hz). Note the different scales in the plots on the right. Curves have been calculated by varying $T$ or $L$ only and keeping all other parameters at their correct values. The theoretical expected locations of the minima are indicated by arrows. The geometry of the problem is given in Fig. 3.
between initial and correct angles is less than \( \approx 30^0 \), these parameters converge within three or four iterations to the correct values (Fig. 7). Exceptions are fault planes with a dip of 0\(^0\). Then, equations to constrain strike and rake are linearly dependent; that is, strike and rake cannot be estimated independently.

The examples that we studied showed that the convergence radius and resolution of \( T, L, W \) and \( \lambda_L \) depend on the problem geometry. For instance, noisy data (20 per cent noise, station and source geometry in Table 2) gave satisfactory results for \( T, L \) and \( W \), since for the majority of start values the theoretical values are found (Fig. 8). The noise factor is given with respect to the maximum peak amplitude of the full seismogram (usually the \( S \) pulse). In most cases the rupture duration \( T \), the rupture direction \( \lambda_L \) and the length parallel to the rupture direction \( (W) \) are well constrained. The length orthogonal to the rupture direction \( (W) \) is less constrained with sparse data. In Fig. 8 the rupture direction \( \lambda_L \) has a global residual minimum at \( 75^0 \) and not at the theoretical value (here \( 90^0 \)). This bias is possibly introduced by the noise as well as the station geometry. We therefore suggest investigating the resolvability and convergence of source parameters for a given source–station geometry with synthetic data.

Applying the single centroid approximation for the source we noticed a strong systematic bias in the source parameters obtained (values are too small or too large) which is small or not present with the improved approximation. The bias increases with increasing cut-off frequency, and can be explained by the breakdown of the single centroid approximation at high frequencies. Therefore, there is a danger that source estimates using methods based on Taylor series expansions around a single centroid point (e.g. Doornbos 1982; Bukchin 1995) are systematically biased.

The distinction between fault and auxiliary planes is an important task in source seismology. Within a wide range of frequencies, the residuals (here and below the sum over squared residuals) at the orientation of the fault plane are much smaller than those at the orientation of the auxiliary plane (Fig. 9, left), so that both planes can be separated, even under 20 per cent noise. For example, at 0.2 Hz the sum over squared residuals is \( 6.2 \text{ m}^2 \) and \( 8.2 \text{ m}^2 \) for the fault and auxiliary planes, respectively. For the 20 traces with 60 samples and eight

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**Figure 7.** Iteration paths of point-source parameters \( M_0, \Phi, \delta \) and \( \lambda \) from different initial values (small circles) to the end values (open stars). The correct value is given by a large filled circle. In this test every seismogram at the stations has been superposed with 20 per cent Gaussian noise with respect to its maximum amplitude.

**Figure 8.** (Left) Iteration paths of extended-source parameters \( T, L, W \) and \( \lambda_L \) from different initial values (small circles) to end values (open stars). For \( \lambda_L \), iteration paths are not plotted. The correct values at the global minimum are indicated by large filled circles. 20 per cent noise has been added to the data. (Right) Example of best fit (dashed lines) to data (solid lines) with cut-off frequency at 0.44 Hz. Although noise is larger than the \( P \) phases, \( T, L \) and \( W \) (and \( \lambda_L \)) have been resolved in this example.

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Moment tensor inversion using far-field broad-band recordings.
parameters to be fitted, the average data variance is 6.2/1192, so that $\sigma^2$ s are 1192 and 1576 for fault and auxiliary planes, respectively. Using the method described in Press et al. (1992) (p. 654) we derive a ‘goodness-of-fit’ probability of 0.5 for the fault plane and 0.4 $\times 10^{-12}$ for the auxiliary plane, indicating that the auxiliary plane is an incorrect model.

A comparison of residuals obtained for different models (uni- or bidirectional rupture) gives the possibility of estimating the best model (Fig. 9, right).

3.2 Deep event

We also studied more realistic examples using reflectivity seismograms calculated for the IASP91 earth model (Kennett 1991; Kennett & Engdahl 1991) and a source at 641 km depth. Source parameters are $\Phi=0^\circ$, $\delta=0^\circ$, $\lambda=0^\circ$, and $W=30$ km. $L=92$ km and $T=23$ s, rupturing eastwards orthogonal to the slip direction ($\lambda=90^\circ$). Reflectivity seismograms at stations between 20$^\circ$ and 90$^\circ$ epicentral distance show a large number of body-wave phases that can be used for the inversion (Fig. 10). The polarity of $P$ phases in Fig. 10 regularly jumps between upwards and downwards with increasing distance because we systematically increased the station azimuth in steps of 35$^\circ$, beginning at 70$^\circ$. While the polarity of phases is controlled by the orientations of the fault and the slip, the width and shape of pulses are influenced by the finiteness of the fault and rupture. We find that our method is able to fit width and shape of separated body-wave phases, namely $S$, $ScS$, $sS$ (transversal component) and $P$, $Pp$, $sP$ (vertical component, Fig. 11). Using $S$ waves in addition to $P$ waves improves the resolution of the method, mainly because their propagation velocity is smaller, leading to larger directivity effects in $S$-wave pulses. The best fit corresponds to the correct rupture model and fault plane. The rupture duration $T$, direction $\lambda_L$ and length $L$ are well constrained; the width $W$ cannot be estimated with the data used (Fig. 12). Using only $P$ phases still resolves $T$ and $\lambda_L$, while for $L$ the depth of the normalized residual minimum is much smaller (Fig. 12, dashed lines), so that $L$ cannot be resolved confidently. More phases may be added to constrain the source parameters better. However, a mixture of different phases at the same arrival time, waves near cusp points, and waves that change pulse forms when reflected at the surface (e.g. shallow-incident $SV$ waves) should be avoided.

Figure 9. Sum of squared residuals versus cut-off frequency of the low-pass filter. (Left) Convergence to the correct fault-plane orientation (Table 1, continuous line) results in smaller residuals than convergence to the auxiliary plane (dashed line) using initial values close to the auxiliary-plane parameters. (Right) Residuals of the correct rupture model (here unidirectional rupture parallel to slip, continuous line) are smaller than those of other models (bidirectional rupture parallel to slip, uni- and bidirectional rupture orthogonal to slip, dashed lines). 20 per cent Gaussian noise has been added to the data used for inversion in both cases, and 60 samples have been used for the time-series of the 10 $P$ and 10 $S$ traces.

3.2 Deep event

We also studied more realistic examples using reflectivity seismograms calculated for the IASP91 earth model (Kennett 1991; Kennett & Engdahl 1991) and a source at 641 km depth. Source parameters are $\Phi=0^\circ$, $\delta=0^\circ$, $\lambda=0^\circ$, and $W=30$ km. $L=92$ km and $T=23$ s, rupturing eastwards orthogonal to the slip direction ($\lambda=90^\circ$). Reflectivity seismograms at stations between 20$^\circ$ and 90$^\circ$ epicentral distance show a large number of body-wave phases that can be used for the inversion (Fig. 10). The polarity of $P$ phases in Fig. 10 regularly jumps between upwards and downwards with increasing distance because we systematically increased the station azimuth in steps of 35$^\circ$, beginning at 70$^\circ$. While the polarity of phases is controlled by the orientations of the fault and the slip, the width and shape of pulses are influenced by the finiteness of the fault and rupture. We find that our method is able to fit width and shape of separated body-wave phases, namely $S$, $ScS$, $sS$ (transversal component) and $P$, $Pp$, $sP$ (vertical component, Fig. 11). Using $S$ waves in addition to $P$ waves improves the resolution of the method, mainly because their propagation velocity is smaller, leading to larger directivity effects in $S$-wave pulses. The best fit corresponds to the correct rupture model and fault plane. The rupture duration $T$, direction $\lambda_L$ and length $L$ are well constrained; the width $W$ cannot be estimated with the data used (Fig. 12). Using only $P$ phases still resolves $T$ and $\lambda_L$, while for $L$ the depth of the normalized residual minimum is much smaller (Fig. 12, dashed lines), so that $L$ cannot be resolved confidently. More phases may be added to constrain the source parameters better. However, a mixture of different phases at the same arrival time, waves near cusp points, and waves that change pulse forms when reflected at the surface (e.g. shallow-incident $SV$ waves) should be avoided.

Figure 10. Test data (vertical component) for a horizontal slip event ($z=641$ km, $\Phi=0^\circ$, $\delta=0^\circ$, $\lambda=0^\circ$) and rupture in an easterly direction ($L=92$ km, $W=30$ km) lasting 23 s. Maximum frequencies are 0.40 Hz, and traces have been multiplied by epicentral distance.

Figure 11. Test data (solid lines) are well fitted by modelled data (dashed lines). The amplitudes have been multiplied by epicentral distances. The time length of waveforms fitted can be individually chosen to find an optimal separation of phases. The cut-off frequency was 0.3 Hz.
Using the correct rupture plane and model, the waveforms fit the theoretical pulses well, whilst at some azimuths waveforms from different rupture models or from a rupture on the auxiliary plane are distinct (Fig. 13). In this example, the $pP$ phases help considerably to separate the fault and auxiliary planes, since they take off from the source in an upward direction, whilst rupture propagation is in a horizontal direction (see Fig. 13). The waveform comparison at many stations leads, during inversion, to the best model which has the least residuals.

### 3.3 Shallow event

We tested a strike-slip source ($\Phi = \delta = 90^\circ$, $\lambda = 180^\circ$) extending from 0 to 12 km depth with a length of 38 km. During 15.2 s the rupture propagates unilaterally from west to east. The seismograms are plotted in Fig. 14. Rupture on a shallow fault generates more complex waveforms than on a deep one. For instance, the depth phases ($pP$, $sP$ and $sS$) arriving shortly after direct $P$ and $S$ waves may lead to complex pulses.
Superposition and mixing of phases is not considered in the method and, if disregarded, may bias the estimates of source dimensions. However, as an empirical approach, we used \( P \) and \( S \) phases as if they were isolated, and further included \( ScS \) phases (transverse component, ranging from \( 18^\circ \) to \( 22^\circ \) and \( 45^\circ \) to \( 58^\circ \)) and \( SS \) phases (transverse component) up to 0.25 Hz in the analysis.

Usually amplitudes of surface waves are largest in seismograms of shallow events (see Fig. 14). Directivity effects are also quite large for surface waves. To test the usefulness of surface waves in our method we additionally extracted time-windowed surface wave trains from the test data, and filtered them with a cut-off frequency of 0.08 Hz. It was necessary to restrict the inversion to amplitude spectra of the surface waves, since the complicated dispersive character of the wave trains cannot be modelled with our approach. The time-shifts of surface waves in Green’s functions are calculated assuming horizontal wave propagation with average group velocities of 3.65 km s\(^{-1}\) for Love and 3.24 km s\(^{-1}\) for Rayleigh waves (see Fig. 14).

Using body and surface waves we were able to constrain the rupture duration \( T \), the rupture direction \( \lambda_L \) and the length \( L \) of the fault, although the estimates are slightly biased (Fig. 15, e.g. the estimate of \( L \) is about 5 km too small). This bias may reflect the approximate validity of our approach for shallow events as described above. For instance, changing the average group velocity of the surface waves changes the estimate of \( L \). The fault width \( W \) is again not resolved. For surface waves this is expected, since they cannot contribute to resolve the depth extent of a strike-slip fault. It is interesting that \( T \) is constrained by body and surface waves, \( \lambda_L \) is best constrained by body waves and \( L \) by surface waves (Fig. 15, dashed and dotted lines).

Although the improved method studied here was not developed for dispersive surface waves, we find surface waves of shallow earthquakes advantageous in discriminating between fault and auxiliary planes and between different rupture models. The normalized sum of squared residuals increases from 0.03 at the global minimum of the fault-plane orientation to 0.09 at the local minimum of the auxiliary plane orientation. Fig. 16 shows the good fit between theoretical and modelled surface-wave amplitude spectra (top) and the large effect of an incorrect model on the amplitude spectra (bottom). Therefore, the fault plane, rupture model and rupture sense are well constrained.

### 4 APPLICATION TO THE 1994 BOLIVIA DEEP EVENT

The 1994 June 9 Bolivia \( M_{w} \) 8.2 deep event (636 km) occurred at a major bend in the deep seismic zone marking the subduction of the Nazca plate. The largest deep event so
far recorded has been studied extensively (e.g. Wallace 1995; Antolik et al. 1996; Ihmle 1998), and is therefore well suited to a comparison of our results with those of other studies. It had a large double-couple component and only small changes in mechanism during rupture (e.g. Goes & Ritsema 1995; Wu et al. 1995), so that rupture on a planar fault seems to be a sensible approximation of the rupture process. Several authors have retrieved a complex rupture history on a subhorizontal ‘planar fault’ with subevents and/or changing rupture velocities (e.g. Goes & Ritsema 1995; Estabrook & Bock 1995; Beck et al. 1995; Ihmle 1998). Our approximation uses a constant average rupture velocity. This has to be considered when comparing best models.

We utilize good-quality broad-band body waves from 35 stations in the epicentral distance $19^\circ$–$105^\circ$ (Fig. 17), namely $P$, $pP$, $sP$, $S$ and $sS$ phases. The instrument transfer functions have been deconvolved, and Green’s functions are calculated for the IASP91 velocity model (Kennett 1991; Kennett & Engdahl 1991) and PREM attenuation model (Dziewonski & Anderson 1981). We worked with amplitude spectra derived from time derivatives of the traces to amplify higher frequencies. However, displacement-proportional spectra lead to very similar results.

Using the Harvard CMT location (latitude = $-13.82^\circ$ N, longitude = $-67.25^\circ$ E, $z = 647$ km, Dziewonski et al. 1995) we analysed the data in the frequency range 0.001–0.01 Hz to retrieve a best point-source model, $M_0 = 2.3 \times 10^{21}$ Nm, $\Phi_{1/2} = 92^\circ/311^\circ$, $\delta_{1/2} = 78^\circ/15^\circ$, $\lambda_{1/2} = -99^\circ/ -53^\circ$. These values have been used as a starting model for the following inversion for kinematic source parameters as constrained by amplitude spectra in the frequency range 0.001–0.15 Hz. For $T$, $L$, $W$ and $\lambda_L$ a variety of starting values have been taken during many inversions to exclude convergence to a local residual minimum. The first step was to find a best rupture model and the correct fault plane. We tested both nodal planes as fault planes and found the horizontal plane to be superior (Fig. 18). For a low-pass cut-off frequency at $f_c = 0.15$ Hz and a unidirectional rupture the variance reduction is 72.6 and 69.4

![Figure 17. Best fault-plane solution (lower-hemisphere projection), epicentre location (star) and distribution of broad-band stations (triangles) used in the analysis.](https://academic.oup.com/gji/article-abstract/137/1/35/700855)

![Figure 18. Normalized sum over squared residuals versus the cut-off frequency of the low-pass filter. Thick and thin continuous lines correspond to uni- and bidirectional rupture on a horizontal plane, respectively. The dotted lines show ruptures on vertical planes.](https://academic.oup.com/gji/article-abstract/137/1/35/700855)
Subhorizontal rupture was also indicated by other studies (e.g. Kikuchi & Kanamori 1994; Beck et al. 1995; Estabrook & Bock 1995). In Fig. 18 a cut-off frequency between 0.08 and 0.24 Hz has been used. For $f_c > 0.14$ Hz the unidirectional rupture model is slightly preferred. This is not surprising since subevent analysis (e.g. Kikuchi & Kanamori 1994; Beck et al. 1995) and source tomography studies (e.g. Ihmle 1998) indicate that the main pulse of the moment release corresponds to a bidirectional or fan-shaped rupture for which a unidirectional branch propagating to the north is dominant.

The best-fit unidirectional rupture model indicated a rupture duration $T = 25.0 \pm 0.8$ s, a fault length $L = 47 \pm 5$ km, a width $W = 25 \pm 20$ km and an average rupture direction $\lambda_L = 350.4^\circ \pm 8^\circ$ (Fig. 19). For bidirectional rupture we estimated $T = 24.6$ s, $L = 68$ km, $W = 40$ km and $\lambda_L = 347^\circ$ and similar confidence limits. The moment, strike, dip and rake of uni/bidirectional ruptures are similar:

Figure 19. Shape and dimensions of the best-fit uni- and bidirectional rupture models are plotted as enlarged boxes. The rupture directions are given with arrows, the slip directions with two parallel arrows. The length and width (km), the average rupture velocity (km s$^{-1}$) and the average slip (m) are given. The star indicates the epicentre, triangles the deep seismicity (300–700 km), light and dark circles the intermediate (70–300 km) and shallow (0–70 km) seismicity (PDE locations). The strike of the deep seismicity is highlighted by two line segments.

Figure 20. Misfit function projected on the $T$, $\lambda_L$, $L$ and $W$ axes. Continuous and dashed lines correspond to uni- and bidirectional rupture, respectively.
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$M_0 = 2.19/2.33 \times 10^{23}$ Nm, $\Phi = 344.2^\circ / 344.3^\circ$, $\delta = 13.6^\circ / 11.7^\circ$ and $\lambda = -18.1^\circ / -15.9^\circ$. The shape of the misfit function (Fig. 20) shows that the rupture duration and the average rupture direction are best constrained, while the fault width is not well constrained from the data used. A constant variance increase of 0.125 per cent has arbitrarily been defined to estimate confidence limits. Best-model synthetics explain the major features of observed amplitude spectra (Fig. 21). The shape and magnitude of the peak at 0.02 Hz in Fig. 21 are fitted at nearly all 154 phases. At higher frequencies spectra for some phases are well fitted (e.g. BFO $S$, BAR $S$, LBTB $SS$, DPC $P$, ANMO $SP$); however, some features remain unexplained for other phases. This is not unexpected since the inversion problem is highly overdetermined; 154 phase spectra have to be explained with eight model parameters that approximate the real rupture process. Due to the larger amplitudes of $S$ compared to $P$ phases, the $S$ spectra are in general better explained by the least-squares model.

A comparison of our results with other results shows that the simple rupture model is useful and leads to meaningful results. The rupture plane is resolved, and the similar fit of uni- and bidirectional rupture models indicates that, on average, a dominant unidirectional rupture to the north is accompanied by a rupture front with smaller moment or duration to the south. The true rupture may have been more complex and may consist of several subevents or changing rupture velocities, as indicated by peaks in high-frequency waveforms.

The main moment release of the earthquake was preceded by a small foreshock 10–11 s before and 30 km east of the main shock (e.g. Kikuchi & Kanamori 1994; Beck et al. 1995). Our estimate of 25 s for the main pulse moment release is similar to other estimates (e.g. Ihmle 1998). However, the rupture

**Figure 21.** Fit of theoretical (continuous lines) to observed amplitude spectra (grey shaded, linear–linear plot) of velocity-proportional seismograms. Amplitudes have been multiplied by the epicentral distance at the stations. The epicentral distances increase from 19° at the top to 105° at the bottom. Station numbers are as follows: (P) 1 = BOCO, 2 = SJG, 3 = RPN, 4 = UNM, 5 = CEH, 6 = PMSA, 7 = CCM, 8 = HRV 9 = BINY, 10 = TBT, 11 = TUC, 12 = ANMO, 13 = DUG, 14 = GLA, 15 = PFO, 16 = BAR, 17 = CMB, 18 = SPA, 19 = NEW, 20 = COR, 21 = PAB, 22 = SUR, 23 = BOSA, 24 = ESK, 25 = LBTB, 26 = BAR, 27 = BFO, 28 = DPC, 29 = KONO, 30 = KIP, 31 = ALE, 32 = COL, 33 = AFI, 34 = SNZO, 35 = ANTO; (S) 1 = RPN, 2 = UNM, 3 = PMSA, 4 = CCM, 5 = HRV 6 = TBT, 7 = TUC, 8 = ANMO, 9 = GLA, 10 = BAR, 11 = PFO, 12 = CMB, 13 = COR, 14 = PAB, 15 = SUR, 16 = BOSA, 17 = ESK, 18 = LBTB, 19 = RAR, 20 = BFO, 21 = DPC, 22 = KONO, 23 = KIP, 24 = ALE, 25 = COL, 26 = AFI, 27 = SNZO, 28 = ANTO.
directions given in the literature vary since many authors included the smaller moment releases before and after the main rupture in their estimates. Our rupture dimensions of about 47 km × 26 km (unidirectional model) and 68 × 40 km (bidirectional model) are also well within the range of estimates of fault dimensions corresponding to the main moment release.

The rupture direction we found is parallel to the strike of the deep seismicity to the north and south of the Bolivia event, and approximately 10° inclined to the slip direction (Fig. 19). When assuming a scenario where the rupture direction on the subhorizontal plane is constrained by the strike of the slab, this result indicates that the slab itself is broken off at the bend near the epicentre of the event. This may explain the lack of deep seismicity in the region of the Bolivian event.

5 CONCLUSIONS

In this paper, the seismic source is represented in terms of higher-degree moment tensors at 27 correlated moment centroids. The higher-degree Green’s functions necessary to solve the inversion problem can be calculated for one centroid prior to analysis.

To reduce the large number of terms originally present in higher-degree moment tensor representation we included simplifying assumptions of planar, Haskell-type faults. Experiments with inversion of synthetic data showed that the estimation of seismic moment, strike, dip, rake, average length and width of the fault, rupture duration and rupture duration is possible. Realistic examples using teleseismic body and surface waves of shallow and deep events indicate that the above-mentioned parameters are well constrained, with the exception of the fault dimension orthogonal to rupture direction.

We included uni- and bidirectional rupture models in the method. The study showed that the rupture model, rupture direction and fault and auxiliary planes can be resolved. For example, we think that the method is well suited to discriminating between fault and auxiliary planes of strong and weak deep events. Often for these events, a study using aftershocks or a master event technique is not possible.

The application to the Bolivia deep event confirmed that the method retrieves average parameters corresponding to the main moment release. The horizontal fault plane was resolved. Therefore, the approach can also be used to estimate a starting model for source tomographic studies.

ACKNOWLEDGMENTS

We wish to thank Gerhard Müller and Frank Scherbaum for their helpful comments on the manuscript. We also thank J. Ungerer for providing the basis version of the reflectivity code. Suggestions from two anonymous reviewers and one adjudicator referee led to an improvement of the manuscript and helped to clarify the theoretical role of backslip in the inversion problem.

REFERENCES


APPENDIX A: HIGHER-DEGREE MOMENT TENSOR REPRESENTATION

The following representation (e.g. Doornbos 1982) can be derived:
\[
d_{\mu \nu} = M_{\mu \nu} \left\{ G_{\mu \nu}^0 - G_{\mu \nu}^0 \Delta(t) + \frac{1}{2} G_{\mu \nu}^0 \Delta(t) \right\},
\]
\[
= G_{\mu \nu}^0 \Delta(t) + \frac{1}{2} G_{\mu \nu}^0 \Delta(t) + O \left( \frac{1}{R^2} \right),
\]
(A1)
with
\[
M_{\mu \nu} = \int m_{\mu \nu} dV,
\]
and
\[
M_{\mu \nu} = \left( \varepsilon_{\mu \nu} - \frac{\partial}{\partial t} \right) m_{\mu \nu} dV
\]
and
\[
M_{\mu \nu} = \left( \varepsilon_{\mu \nu} - \frac{\partial}{\partial t} \right) m_{\mu \nu} dV,
\]
and
\[
M_{\mu \nu} = \left( \varepsilon_{\mu \nu} - \frac{\partial}{\partial t} \right) M_{\mu \nu} d\tau.
\]

\[
R \text{ is the distance between source and station, and first-degree (} M_{\mu \nu} \text{) and second-degree moment tensors (} M_{\mu \nu} \text{) denote Cartesian tensors of third and fourth order, respectively. The Green’s functions are estimated at centroidal points } \varepsilon_0 \text{ and } \varepsilon^0. \]

This approximate formula is valid for sources with a simple space–time history, e.g uniform rupture on a rectangular or circular fault. The following interpretation can be made (after Doornbos 1982): the centroid coordinates are determined from \( \Delta(\tau) \) and \( \Delta(\xi) \), the rupture velocity averaged over the source region from \( \Delta(\tau_{\xi}) \) and \( \Delta(\tau) \), and the size and shape of the final static source region from \( \Delta(\tau_{\xi}) \).

APPENDIX B: HIGHER-DEGREE REFLECTIVITY GREEN’S FUNCTIONS

To calculate complete Green’s functions appropriate for layered media in the far field we used the reflectivity method, which is well established. The wavefield is decomposed into up (+) and downgoing (−) \( P \) and \( S \) waves, which are multiplied by the reflectivity and transmitivity of the media prior to integration over horizontal slownesses \( u \) (see Müller 1985). Higher-order Green’s functions in (2) are calculated by multiplying the original Green’s function kernels by \( j_0 u \) and the appropriate slowness vector \( s \) where
\[
s^{(P)} = (u, 0, a)^T, \quad s^{(P)} = (u, 0, -a)^T,
\]
\[
s^{(S)} = (u, 0, b)^T, \quad s^{(S)} = (u, 0, -b)^T.
\]
\( a \) and \( b \) are the vertical slownesses of \( P \) and \( S \) waves, respectively.

For simplicity we begin from the single force excitation and derive then expressions for higher-order Green’s functions, which correspond to moment tensors of zero, first and second degree. Due to the cylindrical symmetry of the problem we do not have nine independent Green’s functions for a single force excitation \( F = (F_1, F_2, F_3)^T \); we have only five elementary seismograms, which are related by functions of the station azimuth \( \varphi \). This simplification is also traced to higher-order excitations.

The five elementary seismograms are denoted by \( G_1, G_2, G_3, G_4 \) and \( G_5 \), and are related to the displacement by
\[
\begin{bmatrix}
    d_1 \\
    d_2 \\
    d_3 \\
    d_4 \\
    d_5
\end{bmatrix} = \int_0^\infty \left\{ \epsilon_1 \left[ G_1(u) \varepsilon_1 G_1(u) \right] + \epsilon_2 \left[ G_2(u) \varepsilon_2 G_2(u) \right] \right\} du,
\]
(B1)
with
\[
\epsilon_1 = F_1, \quad \epsilon_2 = F_1 \cos \varphi + F_2 \sin \varphi, \quad \eta = -F_1 \sin \varphi + F_2 \cos \varphi.
\]
Displacements \( d_1, d_2, \) and \( d_3 \) are measured in radial, vertical and transverse directions, respectively. A comparison with (82, 83) in Müller (1985) gives analytical expressions for \( G_1, G_3, \) etc. The weights \( \epsilon_1, \epsilon_2 \) and \( \eta \) are functions of the single force components, which can be calculated using tensor rotation (e.g. Appendix in Dahm 1996).

We follow this concept to derive weighting functions for the second-order excitations (zero-degree moment tensor) in (2),
leading to

\[
\begin{bmatrix}
  d_1 \\
  \vdots \\
  d_n
\end{bmatrix} = \int_0^\infty \left\{ k_1 \begin{bmatrix}
  G_{x_1} \\
  G_{x_2}
\end{bmatrix} + k_2 \begin{bmatrix}
  (G_{x_1} + G_{x_2}) \\
  (G_{x_1} + G_{x_2})
\end{bmatrix} + k_3 \begin{bmatrix}
  G_{x_1} \\
  G_{x_2}
\end{bmatrix} \right\} du,
\]

with

\[
\begin{align*}
  k_1 &= M_{11} \cos^2 \phi + M_{22} \sin^2 \phi + M_{12} \sin 2\phi, \\
  k_2 &= M_{13} \cos \phi + M_{23} \sin \phi, \\
  k_3 &= M_{33}, \\
  \lambda_1 &= -\frac{1}{2} \left( M_{11} - M_{22} \right) \sin 2\phi + M_{12} \cos 2\phi, \\
  \lambda_2 &= -M_{13} \sin \phi + M_{23} \cos \phi,
\end{align*}
\]

which represents the term

\[
d_0 = \int_0^\infty \left\{ \lambda_1 G_{x_1} + \lambda_2 G_{x_2} \right\} du,
\]

We cross-checked this equation with (94, 95) in Müller (1985).

The higher-order term

\[
d_i = \int_0^\infty G_{x_0}^i s_d du,
\]

not studied in Müller (1985), is given by

\[
\begin{align*}
d_0 &= \int_0^\infty \left( \gamma_1 G_{x_1} + \gamma_2 G_{x_2} + \gamma_3 G_{x_1}^2 + \gamma_4 G_{x_2}^2 + \gamma_5 G_{x_1} G_{x_2} \\
&+ \gamma_6 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

\[
\begin{align*}
d_1 &= \int_0^\infty \left( \gamma_1 G_{x_1} + \gamma_2 G_{x_2} + \gamma_3 G_{x_1}^2 + \gamma_4 G_{x_2}^2 + \gamma_5 G_{x_1} G_{x_2} \\
&+ \gamma_6 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

\[
\begin{align*}
d_2 &= \int_0^\infty \mu_1 G_{x_1} G_{x_2} + \mu_2 G_{x_1} G_{x_2} + \mu_3 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

The \( d_i \) component depends on six elementary seismograms \((G_{x_1}, G_{x_2}, G_{x_1}^2, G_{x_2}^2, G_{x_1} G_{x_2}, G_{x_1} G_{x_2})\). \( d_i \) on another six (superscript \( r \) is replaced by \( z \)) and \( d_i \) on three.

The elementary seismograms corresponding to the second-degree moment tensor in (2),

\[
d_i = \int_0^\infty G_{x_0}^i s_d du,
\]

are derived in an analogous way, which leads to the following representation:

\[
\begin{align*}
d_0 &= \int_0^\infty v_1 G_{x_1} + v_2 G_{x_2} + v_3 G_{x_1}^2 + v_4 G_{x_2}^2 \\
&+ v_5 G_{x_1} G_{x_2} + v_6 G_{x_1} G_{x_2} + v_7 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

\[
\begin{align*}
d_1 &= \int_0^\infty v_1 G_{x_1} + v_2 G_{x_2} + v_3 G_{x_1}^2 + v_4 G_{x_2}^2 \\
&+ v_5 G_{x_1} G_{x_2} + v_6 G_{x_1} G_{x_2} + v_7 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

\[
\begin{align*}
d_2 &= \int_0^\infty \mu_1 G_{x_1} G_{x_2} + \mu_2 G_{x_1} G_{x_2} + \mu_3 G_{x_1} G_{x_2} \right) du,
\end{align*}
\]

with

\[
\begin{align*}
\gamma_1 &= \kappa_1 (\Delta(\xi_1) \cos \phi + \Delta(\xi_2) \sin \phi), \\
\gamma_2 &= \kappa_1 (\Delta(\xi_1) + \Delta(\xi_2) \cos \phi + \Delta(\xi_2) \sin \phi), \\
\gamma_3 &= \kappa_2 \Delta(\xi_1) \sin \phi + \Delta(\xi_1) \cos \phi, \\
\gamma_4 &= \kappa_2 \Delta(\xi_2) \sin \phi + \Delta(\xi_2) \cos \phi, \\
\gamma_5 &= \kappa_3 \Delta(\xi_1) \cos \phi + \Delta(\xi_1) \sin \phi, \\
\gamma_6 &= \kappa_3 \Delta(\xi_1) \cos \phi + \Delta(\xi_1) \sin \phi, \\
\mu_1 &= \lambda_1 (\Delta(\xi_1) \cos \phi + \Delta(\xi_1) \sin \phi), \\
\mu_2 &= \lambda_1 (\Delta(\xi_2) \cos \phi + \Delta(\xi_2) \sin \phi), \\
\mu_3 &= \lambda_1 (\Delta(\xi_3) \sin \phi).
\end{align*}
\]

We modified an existing reflectivity code to include the calculation of (B2) and (B3).

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