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Base Catchment Modeling in Urban Runoff Simulation

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The selection of numerical model and the transformation of catchment data into 'input data' are two fundamental problems in urban runoff simulation. They are discussed from a general point of view. A numerical solution method for the kinematic wave equations is proposed for base catchment modeling. In connection to this solution a methodology for the generation of input data representing the individual base catchment is presented.

Introduction

In the application of urban runoff models one of the most important problems is the transformation of catchment characteristics into 'input data'. A transformation which is too detailed is ineffective and very costly. On the other hand a coarse description of the catchment will often result in poor model performance. The selection of numerical model is another factor affecting the model performance. The discretization of geometrical input data should then always be discussed with the properties of available numerical models as a base.

An urban runoff model is principally built up of two main sub-models. One treats the collection of storm water on the surface including the transport to the sewer network system. The other describes the transportation of water within the network system. In available runoff models a wide range of approach is used for overland flow routing from simple time offset methods to the kinematic wave approximation (Huber 1977, National Water Council 1981, Lindberg *et al.* 1986).

The model user is in practice not able to describe the catchment geometry with every pavement and roof in detail. In generating the model input he has to simplify and this he does by defining a main sewer network. The upstream ends of the network are connection points to what will here be called base catchments. These will normally contain several different runoff surfaces, gutters and small diameter sewers. The base catchment is represented by a simplified geometry and the runoff from it by a model containing an overland flow routing element. The important geometrical discretization of the catchment in the model input is thus given by the definition of the main network. As the network is usually well specified, the main approximations and difficulties will be in the modeling of runoff from the base catchments.

In a ordinary case of design or analysis we need to know the hydraulic properties in a number of points along the main sewer network, not just the outflow from the catchment. For practical reasons, the base catchments have to be comparatively large. The time of concentration in the base catchment and in the main network system (greater velocities) is in reality often of the same magnitude. Most of the design points will then significantly be influenced by the base catchment modeling.

In conclusion, a balanced geometrical discretization and sound base catchment modeling is essential for the effective and precise use of urban runoff models. Different methods of catchment aggregation have been discussed in literature, but mostly based on specific properties of the SWMM model (Proctor and Redfern 1977, Zaghloul 1981). The object of this paper is to discuss base catchment modeling in general terms and to present a base catchment model based on the kinematic wave theory.

Basic Runoff Models

The shallow water equations are the basic equations describing free surface flow on surfaces, in gutters and in sewers (Yevjevich 1975, Sjöberg 1976). Using the Manning friction relation the equations are given by

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{1a}$$

$$Q = \frac{1}{n} A \ R^{2/3} \Big(S_b - \underbrace{\frac{\partial Y}{\partial x}}_{\substack{\text{pressure}\\ \text{force}\\ \text{term}}} \underbrace{- \frac{1}{gA} \frac{\partial}{\partial x} \Big(\frac{Q^2}{A} \Big) + \frac{1}{gA} \frac{\partial Q}{\partial t}}_{\substack{\text{acceleration}\\ \text{terms}}} \Big)^{1/2} \tag{1b}$$

where Q is flow, A is cross-sectional area of flow, q is lateral inflow, R is hydraulic radius, Y is water depth, S_b is slope and n is the Manning coefficient of roughness. x

and t are the space and time variables respectively. The division of the model into overland flow and sewer flow sub models is not intended to reflect the physical difference between water movement above and below ground. The main point is to divide the catchment into regions where different levels of numerical analysis and sophistication in input data can be applied. If the possibility of back water analysis is restricted to the main sewer line, a natural choice of base catchment model is the kinematic wave model, first presented by Lighthill and Whitham (1955). The model has a well documented performance in cases where no significant influence from the downstream boundary condition exist (Morris and Woolhiser 1980). The kinematic wave equations are obtained from the basic equations by neglecting pressure force and acceleration terms

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{2a}$$

$$Q = \frac{1}{n} A \, R \, S_b^{1/2} \tag{2b}$$

Further simplification of the above relations is done by assuming that the flow velocity invariant in time or space. Invariance in space is obtained by applying the kinematic wave equations on the base catchment in one single space step. The approach may physically be interpreted as flow over a surface with uniform water depth along the reach. In some urban runoff models this solution is presented as a kinematic wave solution. However if the cross-sectional area is transformed into a reservoir volume ($s = A \, L$ where L is the length in flow direction) the corresponding equations are given by

$$ds = (Q_{in} - Q_{out}) \, dt \tag{3a}$$

$$s = C_1 \, (Q_{out})^{C_2} \tag{3b}$$

where Q_{in} and Q_{out} is inflow and outflow respectively and C_1 and C_2 can be identified by Eq. (1b). The equations describe a nonlinear reservoir.

By assuming the flow velocity constant in time but not in space the following relation is obtained

$$Q(t) = \int_{t-t_c}^{t} \frac{dA_c(t-\tau)}{d\tau} \, i(t) \, d\tau \tag{4}$$

where A_c is contributing area, t_c is the time of concentration (here taken as L/wave velocity) and i is the rain intensity. The equation describes the Time-Area Method in a continuous form.

Both the nonlinear reservoir model and the Time-Area Method give rise to difficulties when applied in practice. In Eq. (3b) C_1 has, despite the relationship with the Manning formula, to be evaluated from rainfall-runoff measurements.

Usually such data are not available for the individual base catchment. The Time-Area Method parameter, t_c is principally related to the rain intensity. When a time varying flow is to be simulated, it is not possible to select a value of t_c that gives a resulting simulated flow which fits at all times during the rainfall event. The performance of the Time-Area Method is improved if the time of concentration is allowed to vary with the rain intensity, Johansen (1985). This shows clearly that this method represents a too simplified version of the basic equation.

The kinematic wave model does not suffer from any of above mentioned problems. As will be shown below the model complexity will not result in difficulties in the numerical solution. The kinematic wave model is therefore regarded as the most suitable alternative for base catchment modeling.

The Kinematic Wave – Numerical Solution and Attenuation

The kinematic wave equations describe a non attenuating wave propagation. This seems to be a drawback as it is in contrast to a real wave emanating from a rainfall event. However, in application the numerical solution introduces an artificial attenuation which is normally greater than the real one. The problem is then rather to keep the artificial attenuation in the numerical solution as small as possible. There are several alternative numerical solutions presented in literature. The most general approach is the weighted box scheme given in Fig. 1.

In the continuity equation the differentiation of the derivatives by the weighted box scheme takes the form

$$\frac{\partial Q}{\partial x} \equiv \frac{\beta\,(Q_{j+1}^{m+1} = Q_{j}^{m+1}) + (1-\beta)\,(Q_{j+1}^{m} - Q_{j}^{m})}{\Delta x} \qquad (5a)$$

$$\frac{\partial A}{\partial t} \equiv \frac{\alpha\,(A_{j}^{m+1} - A_{j}^{m}) + (1-\alpha)\,(A_{j+1}^{m+1} - A_{j+1}^{m})}{\Delta t} \qquad (5b)$$

It can be shown by Taylor series expansion that the weighted scheme is a better approximation of the equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + D_n \frac{\partial^2 Q}{\partial x^2} = q \qquad (6)$$

where

$$D_n = (2\alpha - 1)\,\frac{\Delta x}{2} + (1 - 2\beta)\,c_k\,\frac{\Delta t}{2} \qquad (7)$$

and c_k is the wave velocity.

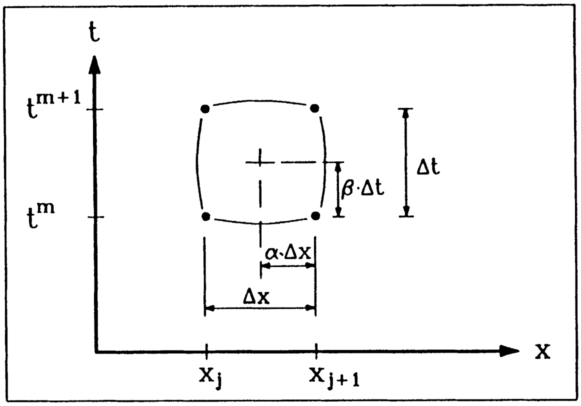

Fig. 1. The relation between the center point and weighting factors.

A common selection of weights is $\alpha = 0$, $\beta = 0.5$, which are used in the ILLUDAS and the SSARR models (Price 1980). With this set of weights the attenuation will only be influenced by Δx (Eq. (7)). Applied to an ordinary base catchment this model will need a $\Delta x/L$ of approximately 1/20 to provide an attenuation similar to the natural situation, Lyngfelt (1985). It is advantageous to reduce the influence on the attenuation from Δx. According to Eq. (5) this is done by using $\alpha = 0.5$. In a series of tests it was found that $\Delta x/L$ could be increased to 1/4 using the values of β shown in Table 1.

Table 1 – Optimal β values ($\Delta x/L=1/4$)

Δt(s)	L(m)	α	β
30	>15	0.5	0.61
30	10	0.5	0.66
30	5	0.5	0.71
60	>15	0.5	0.72
60	10	0.5	0.77
60	5	0.5	0.82

Geometric Representation of the Base Catchment

To use a very detailed description of the base catchment violates the basic idea of the discretization of the runoff system into a main sewer line and base catchments. The geometric representation has to be more generalized in order to reduce the effort of generating input data. During normal application the base catchment includes several separate surfaces and a connecting sewer system. There are several

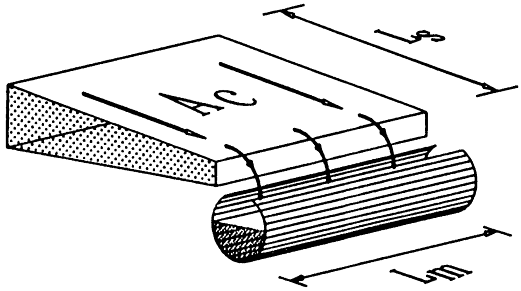

Fig. 2. Physical representation of the base catchment by the KW6S-model.

alternative approaches to obtain a simplified geometrical representation of such a catchment. A very simple representation, shown in Fig. 2, is built up by a rectangular surface feeding a single sewer line. The single sewer represents the main sewer line within the base catchment and is characterized by its length L_m, slope $S_m = \triangle H/L_m$ and diameter D_m. The rectangular surface (parameters; contributing area A_c, length and slope in flow direction L_s and S_s respectively) represents the surface-gutter and sewer branch flows in the real base catchment.

The model is based on the use of the kinematic wave theory, it has 6 parameters and includes a sewer and it is therefore named the KW6S-model. Comparisons with alternative schematic representations show that the model has a well balanced complexity (Lyngfelt 1985).

The six parameters in the KW6S-model may be evaluated in three steps:

- Choose a main line sewer in the sewer system of the base catchment and evaluate length L_m and mean slope S_m from the available data.
- Choose a flow path which characterizes the lateral flow into the above defined main sewer line of the base catchment. The path usually includes the elements of runoff surface, gutter and sewer which are evaluated by length, slope etc.
- Estimate the equivalent KW6S-surface length, $(L_s)_{\mathrm{KW6S}}$, from evaluated base catchment parameters by the relation

$$(L_S)_{\mathrm{KW6S}} = (L_s^{3/5} + C_1 L_g^{3/4} + C_2 L_p^{3/4})^{5/3} \tag{8}$$

where

$$C_1 \equiv C_3\, i^{3/20}\, S_s^{3/10}\, S_g^{-3/8}\, L_s^{-1/4}$$

$$C_2 = C_4\, i^{3/20}\, S_s^{3/10}\, S_m^{-3/8}\, A_c^{-1/4}$$

and

$$C_3 = n_g^{3/4} (4(\frac{1}{z} + z))^{1/4}$$

$$C_4 \equiv n_p^{3/4} (4(\frac{1}{z} + z))^{1/4}$$

In the above equations i is a rain intensity representative for the storms which are to be simulated. z is the side wall slope and s, g, p are indices for surface, gutter and pipe parameters. C_3 and C_4 represent the shape and roughness of gutter and pipe which are normally not varied between different base catchments. In ordinary Swedish catchments relevant values for C_3 and C_4 are 0.058 and 0.036, respectively. Eq. (8) is derived from the kinematic wave equations which have an analytical solution in the case of constant rain intensity. It is based on equality between the travel times for waves over the KW6S surface and along the defined characteristic flow path. A similar approach has been discussed for the SWMM-model by Marsalek (1983).

Performance of the KW6S-Model

The performance of the KW6S-model was investigated by comparative studies of a model using the kinematic wave theory with a very detailed description of the catchment. In this model practically every surface and gutter in the catchmens are simulated. Three residential areas (Fig. 3) were used for the investigation. The relevance of the detailed model was shown by comparisons between simulated and measured runoff for 21 different storm events (Lyngfelt 1985). In the comparison between the KW6S and detailed model corresponding real storms have been used.

The Bergsjön and Linköping catchments were subdivided into base catchments according to the above proposed methodology. Three levels of subdivision were investigated:

Level 1; nine base catchments, each $\approx 0.5 \times 10^{-4}$ m^2 contributing area
Level 2; three base catchments, each $\approx 2 \times 10^{-4}$ m^2 contributing area
Level 3; one base catchment, $\approx 5 \times 10^{-4}$ m^2 contributing area

Suitable parameters for comparing simulated runoff are levels and corresponding times for peak flow values and the shape of the hydrographs. The investigation generally showed a good agreement between these parameters – when there was similarity in flow peaks there was also similarity in time and hydrograph shapes. In this paper the emphasis will therefore be on flow peaks. A more general discussion is given in Lyngfelt (1985). In Fig. 4 runoff simulations using the different models are shown for one of the storm events in Bergsjön.

In Fig. 5 peak flows obtained in the simulations by the detailed and KW6S models have been plotted. It shows that the performance at all levels of discretization is good. This is also evident when the commonly used statistical parameters for the peaks (mean ratio $\lambda_p = 0.96$, mean standard deviation $\sigma_p = 0.13$ and absolute error $\varepsilon_p = 11\,\%$) are compared with values obtained for corresponding parameters in model performances discussed in the literature (Colyer 1977, Price 1980).

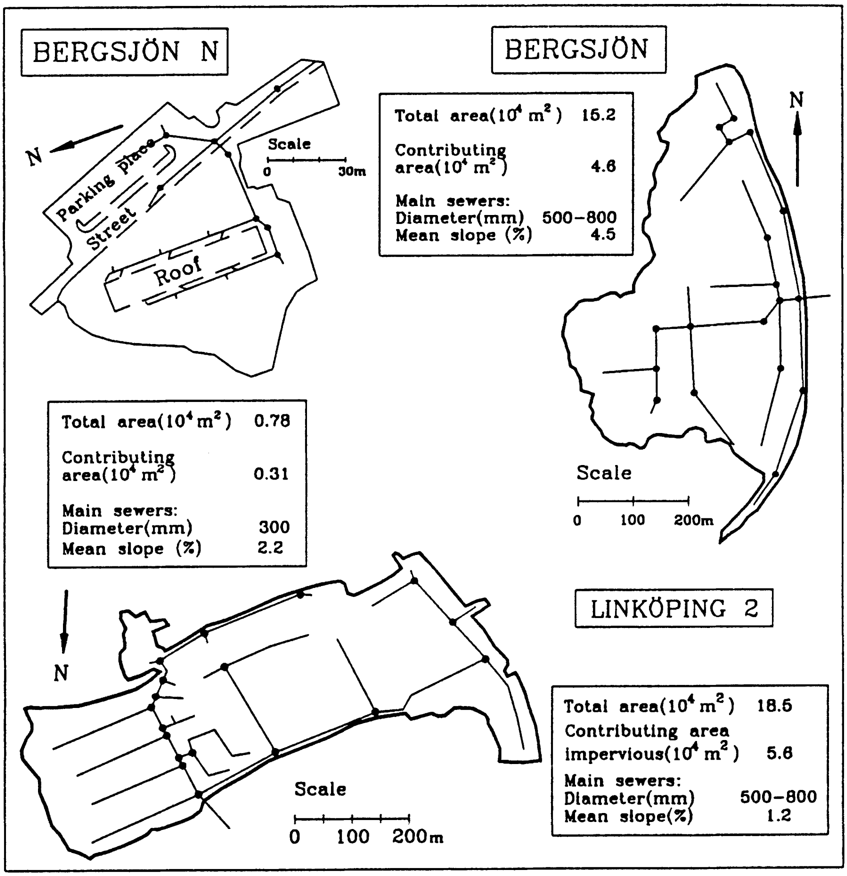

Fig. 3. The urban catchments Bergsjön N, Bergsjön and Linköping.

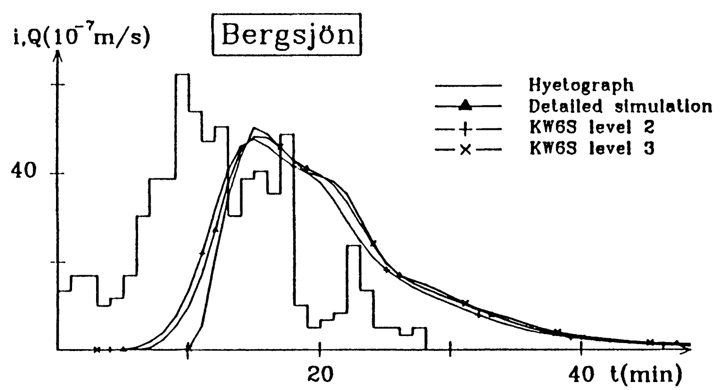

Fig.. 4. Runoff simulations from a storm event in Bergsjön using different levels of discretization.

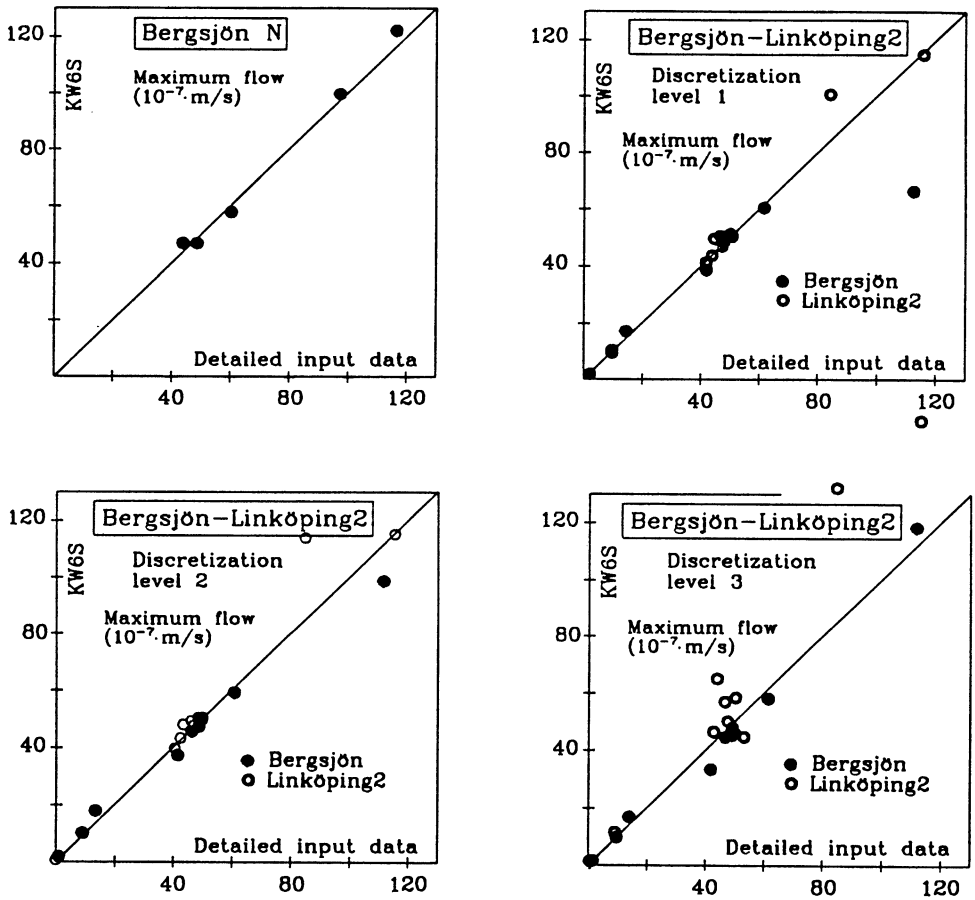

Fig. 5. Comparison between simulated peak flow values obtained by a kinematic wave model using detailed input data and the KW6S-model. Three different discretization levels are used.

Generally the investigation show a weak but notable deterioration of the performance with increasing base catchment area. However, there seems to be no definite upper limit beyond which the base catchment area should be selected using the KW6S-model. For each selection of discretization the performance of the simulation is in principle better downstream compared to upstream. The main sewer net should therefore be defined at least one or two sewer reaches upstream of the points where the hydraulic properties are analyzed.

Conclusions

In runoff simulation the kinematic wave equations are from both a theoretical and practical point of view a suitable base catchment model. There is no reason using very simplified versions of the basic equations such as the nonlinear reservoir model and the Time-Area Method.

In practical application it is usually necessary to describe the catchment in a simplified way. This is preferably done by subdivision into a suitable number of base catchments where each is represented by the KW6S-model using 6 parameters. A good performance implies serious consideration of these parameters. This may be done with maps and simple field observations as a base.

If the proposed method is applied, the study shows that reasonably large catchments may be used – an important property in practice.

It should be stressed that a proper evaluation of contributing area is very important for satisfactory model performance, independent of the model used.

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Notation

A	(m^2)	– cross-section of flow
A_c	(m^2)	– contributing catchment area
C_1- C_4		– parameters (defined by Eq. (8))
i	(m/s)	– rain intensity
j	(-)	– space step
m	(-)	– time step
n	($m^{-1/3}s$)	– Manning's coefficient of roughness
Q	(m^3/s)	– flow rate
R	(m)	– hydraulic radius
S_b	(-)	– slope in flow direction
t	(s)	– time
x	(m)	– space coordinate
Y	(m)	– cross-sectional water depth
z	(-)	– slope factor of side walls
α	(-)	– numerical parameter in the weighted box scheme
β	(-)	– numerical parameter in the weighted box scheme
ε_p	(%)	– mean absolute error in compared peak flow values
λ_p	(-)	– mean of the ratio between flow peaks
σ_p	(-)	– standard deviation of the ratio between compared flow peaks

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