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An Application of Kriging to Rainfall Network Design

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Universal kriging techniques based on the generalized covariances corresponding to IRF-k theory, are applied to analyze the design of raingauging networks in regions where the spatial mean is not constant. The objective is to obtain an optimal estimate of watershed precipitation. For the purpose of analysis, symmetric and asymmetric hypothetical rainfall fields are considered. The hypothesized storms provide the bases for comparing the results of the analysis. The results are also compared with traditional approaches in current use. The investigation depicts the superiority of kriging techniques over the other methods. The effect of storm spatial variability on the network design is also examined.

Introduction

As the need for accurate estimates of water availability increases, better tools for the analysis of how a watershed responds to precipitation input have been proposed. Improved methods of calculating abstractions from rainfall such as evapotranspiration and infiltration and of routing direct runoff, all have helped increase the accuracy of hydrograph prediction. In order to fully realize the benefits from these refinements, they must be based on precipitation estimates of commensurate accuracy. The traditional approaches to estimating average basin precipitation have been to use the arithmetic mean of the gauge measurements or by weighting the measurements using the Thiessen or isohyetal methods (Linsley, Kohler and Paulhus 1975). Our current understanding of the precipitation process, however, suggests that we should employ estimation techniques which explicitly consider the spatial variability of the precipitation.

Furthermore, rainfall estimation is seldom an end by itself. Management decisions in water resources development are often based on water quantities calculated from basin precipitation or, on runoff from estimated rainfall. In these situations, the precipitation estimate must be supplied to the decision-maker together with a measure of the uncertainty, so that he will have the option of accepting the estimate or deciding to spend additional resources to reduce uncertainty.

The current tendency toward quantifying the uncertainty of hydrologic estimates rather than merely using point values further emphasizes the need toward the use of techniques of improved modern estimation methods. The use of universal kriging for the analysis of precipitation is a step in this direction. Kriging, which has found many applications in geostatistics and related fields, appears to be particularly suitable for the analysis of rainfall. This paper applies the theory of universal kriging to answer some questions on the reliability of precipitation estimates and how a gauging network design may be revised to increase the reliability of the estimates.

Theoretical Considerations

The spatial variation of the precipitation may be mathematically modeled as a stochastic process. It may be represented by a random function $Z(u)$, denoting the rainfall depth over all vectors, u, in R^2 space within a given catchment area A. A *regionalized variable* is a function that can be considered as a particular realization of this random function.

Matheron (1971) developed the theory of regionalized variables to deal with spatial data. He proposed a method of estimation which he called *kriging*, in honor of Krige (1951), who first introduced the use of moving averages to avoid systematic overestimation of reserves in the field of mining. During the last two decades increasing attention has been given to the application of this methodology to hydrologic data. Particularly, the works of Delfiner (1973) and Delhomme (1978) are noteworthy.

For the purpose of computing the mean areal precipitation (MAP), the mean of the stochastic process $Z(u)$ at the point u. Mathematically, this is written as $m(u) = E[Z(u)]$; where $E[\quad]$ denotes the expectation operator. Like $Z(u)$, $m(u)$ is defined over all points (u) in the area A. The spatial mean $m(u)$ is sometimes also called the *drift*. In flat regions, the drift may be assumed to be constant. However, in mountainous areas the spatial characteristics of precipitation are influenced by orography. Since large scale vertical movements of weather systems produce a complexity of spatial precipitation patterns, the assumption of a constant spatial mean is not reasonable and the theory which is based on this assumption becomes invalid. The need for MAP estimating procedures that do not assume a constant drift gave rise to *universal kriging*. Universal kriging proposes a particular functional form for modeling the mean. It treats the estimation as a least squares procedure by invoking the theory of intrinsic random functions of order k. Shamsi and Yoganarasimhan (1986) showed the application of universal kriging for optimal interpolation of rainfall data.

Since precipitation is usually measured at a finite number of gauges within any given catchment, the transition from point values to an areal value requires an estimation procedure and involves an estimation error. According to the theory of kriging this error does not depend upon the observed values at the gauges, but is simply a function of gauge locations over the catchment (Huijbregts 1973). If the number of gauges is large and the gauges are uniformly located, the estimation error will be comparatively smaller than if they were less in number and clustered. Kriging estimates this error in the form of the *kriging estimation variance* which can be used to measure the reliability of the estimated average value.

The following analysis essentially follows the development by Bras and Rodriguez-Iturbe (1985).

The spatial dependence of random processes $Z(u_1)$ and $Z(u_2)$ at any two point u_1 and u_2 can be modeled by the *semivariogram*, denoted $\gamma(u_1,u_2)$ and defined as

$$\gamma(u_1,u_2) = \frac{1}{2}\,\mathrm{Var}\,[Z(u_1)-Z(u_2)] \tag{1}$$

The random process $Z(u)$ is said to satisfy the *intrinsic hypothesis* if its first order differences $Z(u_2) - Z(u_2)$ are stationary in the mean and variance *i.e.*,

$$E[Z(u_1)-Z(u_2)] = m(h) \tag{2}$$

$$\mathrm{Var}[Z(u_1)-Z(u_2)] = 2\gamma(h) \tag{3}$$

Thus the mean and the variance of the first order difference are independent of the actual locations u_1 and u_2 and depend only on their vector difference, h.

Assume that there are n observations of the process $Z(u)$ (rainfall depths for a given storm) at locations u_i, $i=1, \ldots, n$, denoted by $Z(u_i)$. The true unknown MAP over the area A, can be expressed as

$$P = \frac{1}{A}\int_A Z(u)\,du \tag{4}$$

Our objective is to find te *best linear unbiased estimate* denoted by P^*, of the true value P. Such an estimator must possess the following three qualities.

1. *Linearity*: The estimator P^* must be a linear combination of observed values, $Z(u_i)$

$$P^* = \sum_{i=1}^{n} \lambda_i\, Z(u_i) \tag{5}$$

where λ_i, $i=1, \ldots, n$ are a set of weights to be optimized according to the criterion for the *best* estimator.

2. *Unbiasedness*: The expected value (taken over the ensemble of all realizations; an infinite number in theory) of the estimator must be equal to the expected value of the true MAP

$$E[P^*] \equiv E[P] \tag{6}$$

3. *Minimum Variance*: The best estimator must have minimum estimation variance (mean square error) defined by

$$\sigma_P^2 \equiv \mathrm{Var}[(P-P^*)] \equiv E[(P-P^*)^2] \tag{7}$$

Hence, the best linear unbiased estimate will be one which satisfies condition Eq. (6) and selects the minimum value for Eq. (7) among all possible linear estimates of the form Eq. (5). Now, substituting Eqs. (4) and (5) in Eq. (6) we have

$$E[\sum_{i=1}^{n} \lambda_i Z(u_i)] = E[\frac{1}{A}\int_A Z(u)du] \tag{8}$$

Linearity on both sides of this equation allows the following simplification

$$\sum_{i=1}^{n} \lambda_i E[Z(u_i)] \equiv \frac{1}{A}\int_A E[Z(u)du] \tag{9}$$

Universal kriging models the drift as a linear combination of ν basic functions $f^p(u)$

$$m(u) \equiv \sum_{p=0}^{\nu} a_p f^p(u) \tag{10}$$

where in order to model a polynomial drift, the $f^p(u)$ are generally assumed to be monomials.

For any arbitrary region of integration D, the continuous weighting function is defined as

$$\int_D \lambda(du) = \sum_{i=1}^{n} \lambda_i \delta_{u_i} - \frac{1}{D}\int_D du \tag{11}$$

where $\delta_{u_i} = \begin{cases} 1 & u=u_i \\ 0 & \text{otherwise} \end{cases}$

If the condition

$$\int_D f^p(u)\lambda(du) = 0 \tag{12}$$

holds for all monomials of order $p \leq k$, then

$$\int_D Z(u)\lambda(du)$$

is a *generalized increment of order k* and the random function *Z(u)* is said to be an *intrinsic random function of order k* (IRF-*k* function). Substituting in Eq. (9) the definition of mean given by Eq. (10), we obtain the following ν+1 equations which express the unbiasedness condition

$$\sum_{i=1}^{n} \lambda_i f^p(u_i) - \frac{1}{A}\int_A f^p(u)\,du = 0 \qquad p=0,1,2,\ \ldots,\nu \tag{13}$$

which by using Eq. (11), can also be written as

$$\int_A f^p(u)\lambda(du) = 0$$

implying that

$$\int_A Z(u)\lambda(du)$$

is a generalized increment of order *k*.

In practice, the most important result of IRF-*k* theory is that there exists a function *K(h)* called the *generalized covariance model* (G.C. Model) of order *k*, such that, the variance of any generalized increment of order *k* appears in the form

$$\mathrm{Var}\left[\int_A Z(u)\lambda du\right] \equiv \int_A\int_A \lambda(du_1)K(u_1-u_2)\lambda(du_2) \tag{14}$$

The G.C. Model of order 0 (for the constant drift case) is simply the semivariogram defined earlier.

Eq. (14) can be minimized under ν constraints given by Eq, (13), resulting in the following linear system of equations called the *kriging system*

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_i K(\underset{\sim}{u_i}-u_j) - \mu_0 - \sum_{p=1}^{\nu} \mu_p f^p(\underset{\sim}{u_i}) = \frac{1}{A}\int_A K(\underset{\sim}{u_i}-u)\,d(u) \qquad i=1,2,\ldots,n\\
&\sum_{i=1}^{n} \lambda_i = 1 \\
&\sum_{i=1}^{n} \lambda_i f^p(u_i) = \frac{1}{A}\int_A f^p(u)\,du \qquad p=1,2,\ \ldots\nu
\end{aligned} \tag{15}$$

The solution of these $n+\nu+1$ equations gives the n optimal weights λ_i^* and $\nu+1$ Lagrangian multipliers μ_p. The kriging estimation variance is expressed as:

$$\sigma_k^2 \equiv \frac{1}{A}\int_A\int_A K(u_1-u_2)\,du_1\,du_2 - \frac{1}{A}\sum_{i=1}^{n}\lambda_i\int_A K(u-u_1)\,du + \frac{1}{A}\sum_{p=0}^{\nu}\mu_p\int_A f^p(u)\,du \tag{16}$$

It is important to mention here that all functions can not represent a generalized covariance. They must satisfy certain conditions of mathematical consistence which ensure that the variances of increments are always positive. In particular, the G.C. Models developed by Delfiner (1976) can be used. The algorithm AKRIP for determining the G.C. Model including the order k of the IRF, that best describes the data, and for computing the MAP values and corresponding estimation variances has been developed by Kafritsas and Bras (1981). A modified IBM compatible version of AKRIP written in FORTRAN IV (Shamsi 1984) and a contour plotting subroutine is available from the authors.

Reinforcement of the Network

We now come to the questions raised at the beginning of the paper. The kriged estimate is based on the irregularly located samples over the area. The sampled values are only partially related to the kriged value. The degree of this relationship is expressed by the G.C. Model. We would, therefore not expect the estimated MAP to be exact. The variance σ^2_k given by Eq. (16) is a measure of this inexactness. The smaller is the variance, the greater is the reliability of the estimate. Conversely, the estimate with a large associated variance must be used with caution. Thus, the kriging variance gives a measure of confidence that can be used to: (a) decide on the advisability of reinforcing the network, and (b) determine the optimal location of a possible additional measurement point. These two cases can be analysed by employing the following method.

Fictitious Point Method

As seen from Eq. (16), the kriging variance does not depend on the functional values $Z(u)$, but only on the G.C. Model values. It is, therefore, a function of only the location of data points $u_1, \ldots, u_n$. This remarkable property can be used in the optimization of the gauging network. The procedure will involve the following steps:

1) For the existing network consisting of n gauges, located at the points u_i, $i = 1, \ldots, n$, compute the kriging variance $\sigma^2_{k,n}$.
2) Consider an additional point u_{n+1}, without making any assumption for the value at the additional fictitious point.
3) Develop the kriging system corresponding to the set of (n+1) data points u_i, $i = 1, \ldots, (n+1)$.
4) Compute the corresponding kriging variance $\sigma^2_{k,n+1}$.
5) Compute the relative variance reduction by the equation

$$R = \frac{\sigma^2_{k,n} - \sigma^2_{k,n+1}}{\sigma^2_{k,n}} \qquad (17)$$

6) Locate the fictitious point successively on all the candidate points within the catchment and compute the variance reduction each time (*i.e.* repeat steps two to five). The candidate points will be determined by geographic, economic or climatic constraints. In the absence of such constraints, the fictitious point can be arbitrarily introduced in the high variance regions of the catchment. High variance regions can be identified by computing the kriging variance on the nodes of an arbitrary grid constructed on the catchment area and then plotting a contour map from the point values. Delhomme (1979) suggests constructing a grid of suitable size on the area and then locating the fictitious point on every grid node successively.
7) The location giving the maximum reduction in variance identifies the optimal site for the additional gauge. If variance reductions are marginal, then the reinforcement of the network is not called for. Optimal site location may also be done by contouring the variance reductions over the entire catchment and selecting the locations with the highest contour values (*e.g.* a local maxima).

Case Studies

According to Matheron's theory the spatial pattern of a rainfall field can be considered as a realization of a random function $Z(u)$. To show how the theory is applied, two hypothetical storms are considered. Both are assumed to be realizations of an unknown random field. In order to illustrate the effect of spatial variability of different storms on the raingauge network, data from individual storms are used rather than the annual average for all the storms. The purpose is to show that different storms may produce different designs and that the design based on the annual averages may not be representative of the storm characteristics. The readers are cautioned not to confuse such a situation with temporal analysis. Kriging does not treat temporal variation, and the two storms considered here should be thought of as two realizations of the stochastic rainfall process.

Symmetric Storm

A square catchment of 15 km^2 with a storm resembling a convective precipitation pattern, symmetrically distributed around the center of the catchment, is hypothesized. Due to the assumption of radial symmetry, the rainfall distribution is represented by concentric, circular contours with a contour interval of 0.5 cm. Fifteen raingauge stations are selected randomly on the catchment and the rainfall depth "recorded" at each station is obtained by linear interpolation between contours. To this true or "actual value," an error component is added to account for random measurement errors. The sum gives the "observed values" which represent the raw data obtained from the gauging stations in the real world. To avoid negative observed values the random error introduced is limited not to exceed 20% of

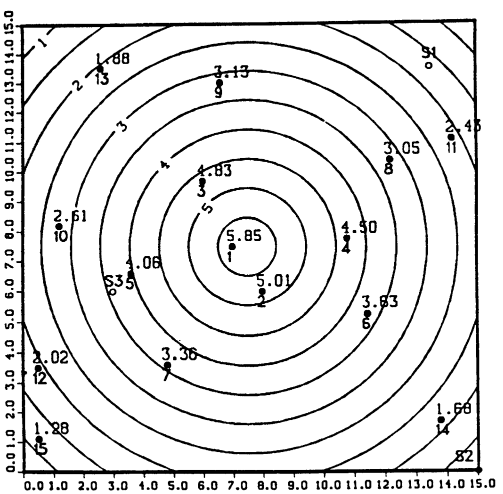


Fig. 1. Symmetric Storm.

the smallest actual value. The locations of raingauges and corresponding observed values are shown in Fig. 1.

The advantage in analyzing a set of hypothetical data like this is that, the true distribution of the storm is known beforehand. This provides a basis with which to compare the estimates. In the real world, the true areal distribution of any storm is never available. The hypothetical storm also provides the "true MAP value". This is computed by weighing the average precipitation between successive contours (average of two contour values) by the area between the isohyets, summing the weighted products, and dividing the sum by the total area. The true MAP value calculated in this way is 3.12 cm, and it will now serve as an index for examining the accuracy of the averages estimated by other techniques.

Table 1 – Variance reduction for the mean when fictitious stations are added (Symmetric storm). G.C.Model K(h) ≡ 1.08629 δ(h) (Order = 1)

Kriging variance with already existing 15 stations = 1,16018 cm^2

Fictitious station	X Coordinate (km)	Y Coordinate (km)	Kriging variance for the mean (cm^2)	Variance reduction %	Station rank
1	15.00	0.00	1.15631	0.33	S2
2	0.00	15.00	1.15767	0.22	
3	13.50	13.50	1.15449	0.49	S1
4	0.00	3.00	1.15804	0.18	
5	0.00	9.00	1.15752	0.23	
6	3.00	6.00	1.15687	0.29	S3
7	3.00	0.00	1.15789	0.20	
8	9.00	0.00	1.15690	0.28	

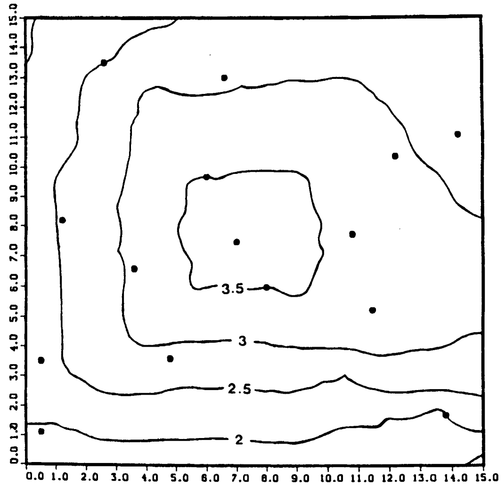


Fig. 2. Contour Map of Kriged Values (Sym. Storm).

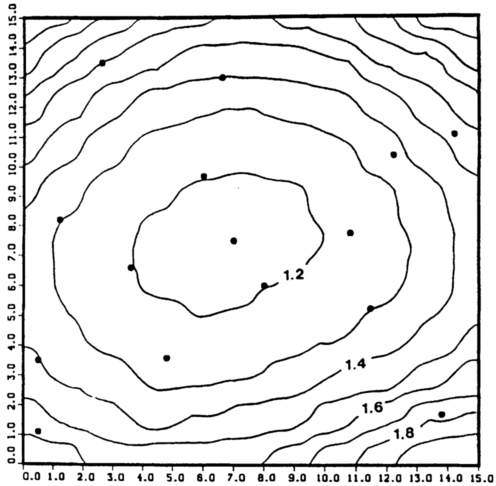


Fig. 3. Contour Map of Kriging Variance (Sym. Storm).

The kriged value and corresponding variance of estimation are calculated at each node of a 1.5 km × 1.5 km square grid. These are the point estimates interpolated between the observed values using an analogous kriging system (Delfiner and Delhomme 1973) called point kriging system. The contour maps of these values are shown in Figs. 2 and 3. In Fig. 3, it is noted that the kriging variance is minimum in the central area where the gauge density is higher.

On the variance map (Fig. 3) it is easy to determine the high variance regions. It is obvious that in order to increase precision, additional gauges need to be installed in these regions. To illustrate this point, eight fictitious stations were added one at a time in poorly estimated regions for the symmetric storm and the variance reductions for the mean were calculated. The results are presented in Table 1. Three stations with the highest variance reductions were then chosen and ranked S1, S2, and S3 depending on the amount of variance reduction. These stations were introduced in the kriging system one after the other. The percent relative variance reduction was calculated each time and the results are presented in Table 2.

Table 2 – Reinforcement of the network for Symmetric storm.

New stations introduced	Total number of stations	Kriging variance for the mean (cm^2)	Variance reduction %
None	15	1.16018	---
S1	16	1.15449	0.49
S1+S2	17	1.15071	0.82
S1+S2+S3	18	1.14940	0.93

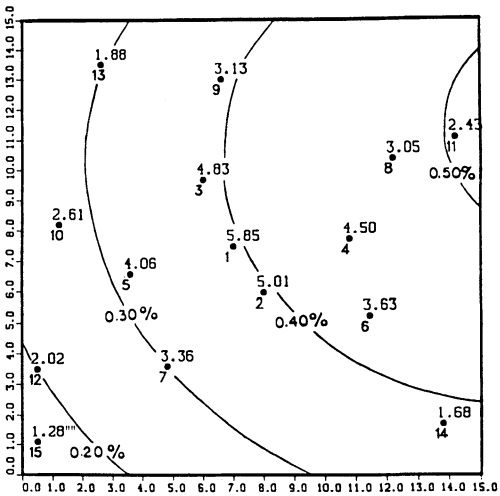


Fig. 4. Fictitious Point Method (Sym. Storm).

The variance reduction increased from 0.49% to 0.82% as the data points increased from 16 to 17. However, the reduction in variance due to the addition of S3, was marginal: from 0.82% to 0.93%. Therefore, it does not seem advisable to add S3. Hence, the recommended network design is a total of 17 stations. Alternatively, optimal site selection by successive installation of a gauge at every grid node of the domain and the resulting variance reduction contours are shown in Fig. 4. From this figure, it is obvious that installation of new gauges on the eastern side of the catchment should bring the maximum variance reduction. Fig. 4 also confirms the ranks assigned to eight potential reinforcement sites in Table 1. We see that variance reduction at S1 is more than 50%; at S2, it is about 35%; at S3, about 30% and so on. This also verifies that the results remain the same, whether the fictitious locations are chosen from the variance map (Fig. 3) or from a grid. However, the former approach saves computation time, since non optimal sites are ruled out in advance.

We examine next the effect of this reinforcement. Let us assume that stations S1, S2 and S3 have been installed and data are available at these stations (obtained from the hypothetical contours). With the values of observed rainfall available at

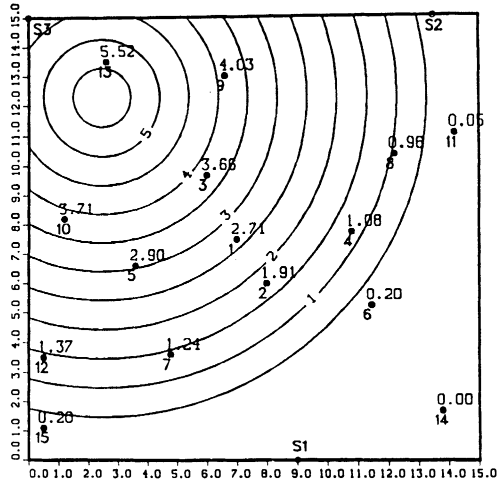
Table 3 – Improvement for the mean after the new stations are installed in the real world (Symmetric storm).

New stations introduced	Total number of stations	Kriged mean (cm)	Kriging variance for the mean (cm^2)	Estimation error
None	15	3.33	1.16	0.21
S1	16	3.18	1.05	0.06
S1+S2	17	3.08	0.36	0.04
S1+S2+S3	18	3.08	0.34	0.04

these stations, the current G.C. Model can be computed for different configurations to provide the kriged mean, as shown in Table 3. Fixing the allowable estimation error at 0.05, it is again seen that a design using 17 stations gives satisfactory results.

Asymmetric Storm

Another set of hypothetical data was generated by shifting the storm center to the north-west fringe of the basin. The locations of raingauges along with the observed values are shown in Fig. 5. The contour maps of kriged values and kriging variance are shown in Figs. 6 and 7, respectively. The true MAP value is 1.95 cm.



2.71
● : Rainfall Depth in cm at Gauging Station # 1
1

Fig. 5. Asymmetric Storm.

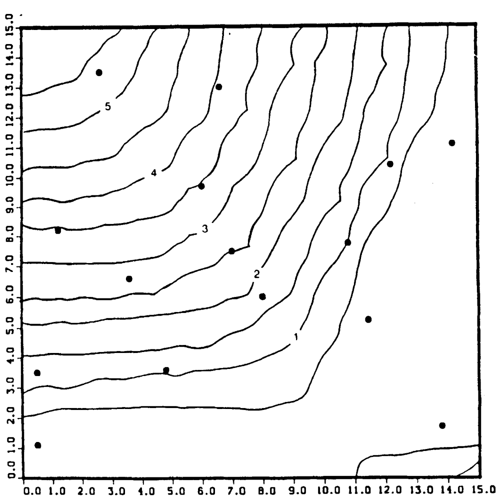


Fig. 6. Contour Map of Kriged Values (Asym. Storm).

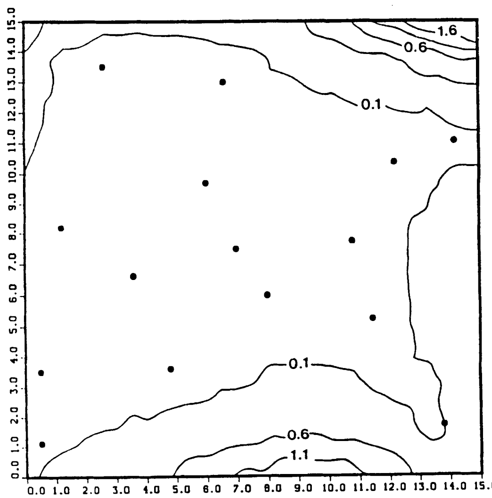


Fig. 7. Contour Map of Kriging Variance (Asym. Storm).

Table 4 – Variance reduction for the mean when fictitious stations are added (Asymmetric storm). G.C.Model K(h) ≡ −0.00003 $|h|^5$ (order ≡ 2)

Kriging variance with already existing 15 stations ≡ 3.93022 cm^2

Fictitious station#	X Coordinate (km)	Y Coordinate (km)	Kriging variance for the mean (cm^2)	Variance reduction %	Station rank
1	13.50	15.00	3.92596	0.108	S2
2	0.00	15.00	3.02973	0.012	S3
3	0.00	0.00	3.92574	0.114	S1

Table 5 – Reinforcement of the network for Asymmetric storm.

New stations introduced	Total number of stations	Kriging variance for the mean (cm^2)	Variance reduction %
None	15	3.93022	– – –
S1	16	3.92574	0.114
S1+S2	17	3.92358	0.169
SD1+S2+s3	18	3.92345	0.172

Table 6 – Improvement for the mean after new stations are installed in the real world (Assymetric storm).

New stations introduced	Total number of stations	Kriged mean (cm)	Kriging variance for the mean (cm^2)	Estimation error
None	15	1.91	3.93	0.04
S1	16	1.91	2.00	0.04
S1+S2	17	1.94	0.14	0.01

Last column ≡ |Column 3 − 1.95|

A similar analysis was performed for the asymmetric storm. The results are presented in Tables 4 to 6. It is seen that the variance reductions resulting from the introduction of the fictitious points are marginal, and the existing network of 15 stations is sufficient to produce the optimal averages, well within the allowable estimation error. The low values of variance reduction contours in Fig. 8, further support this conclusion.

Finally, the kriged averages were compared with the estimates using the traditional approaches namely, the arithmetic, isohyetal and Thiessen methods. For the two storms considered, the results are summarized in Tables 7 and 8. It is seen that for the symmetric storm, the arithmetic mean gives just as good results as kriging. For the asymmetric storm, the Thiessen method gives even better results. A probable reason for this may be that the arithmetic mean and the Thiessen methods give

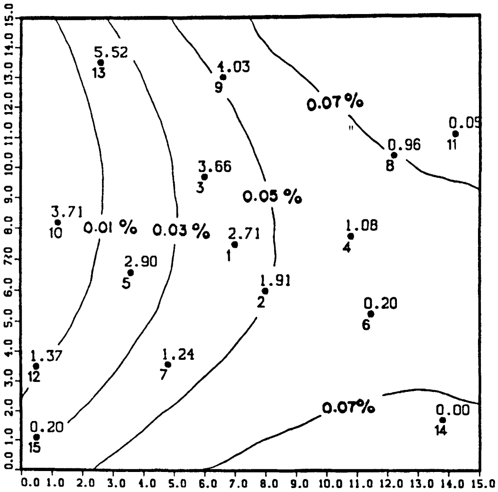


Fig. 8. Fictitious Point Method (Asym. Storm).

Table 7 – Comparison among different methods for spatial averaging of rainfall data (Symmetric storm).
Actual mean ≡ 3.12 cm

Number of Stations	15	16	17	18
Arithmetic mean (cm)	3.29	3.18	3.04	3.07
Thiessen mean (cm)	3.36	3.29	3.28	3.25
Isohyetal mean (cm)	2.96	2.97	2.98	2.98
Kriged mean (cm)	3.33	3.18	3.08	3.08

Table 8 – Comparison among different methods for spatial averaging of rainfall data (Asymmetric storm).
Actual mean = 1.95 cm

Number of Stations	15	16	17	18
Arithmetic mean (cm)	1.97	1.85	1.76	1.90
Thiessen mean (cm)	1.97	1.93	1.93	1.92
Isohyetal mean (cm)	2.21	2.27	2.28	2.28
Kriged mean (cm)	1.91	1.91	1.94	1.90

better estimates if the gauges are well dispersed in the catchment so that the spatial distribution of the storms is captured well. This is the case for the hypothetical network considered here and therefore, the traditional approaches give good estimates for MAP. Nevertheless, since the kriging estimates are accompanied by their corresponding estimation variances, which none of the traditional approaches provide, this is very useful in the network design.

Conclusions and Recommendations

Based on the idealized and hypothetical study, we venture the following conclusions. The universal kriging technique produced two different designs for the symmetric and asymmetric storms analyzed. For the symmetric storm which covers the whole catchment area, two additional raingauges were required to make the estimated average rainfall depth very close to the actual value. In the case of asymmetric storm, where about one third of the catchment area received no rainfall, 15 existing raingauges were sufficient to produce an accurate estimate of the MAP. This shows that kriging takes into account the spatial variability of the storm within the catchment and not only the location of the gauges. It is noted however that these results are based on estimates of the G.C. Model which are subject to finite sampling errors. Chua and Bras (1980) suggest an alternative simulation procedure to mitigate this shortcoming.

In practice, historical records may show a variety of spatial storm variability patterns on the same catchment. According to the above conclusions each storm could result in a different network design. Which design, then will be the optimal: one that gives the maximum number of raingauges or one that corresponds to a storm that is centered at the catchment area or one that corresponds to a storm covering the maximum catchment area? Is it justified to base the design on the annual averages which mask the spatial variability of individual storms? These are questions that require additional work on different storm configurations.

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References

Bras, R.L., and Rodriguez-Iturbe, I. (1985) *Random Functions and Hydrology,* Addison-Wesley Publishing Company, Reading, MA.

Chua, S. H., and Bras, R.L. (1980) Estimation of Stationary and Nonstationary Random Fields: Kriging in the Analysis of Orographic Precipitation, Tech. Rept. 255, Ralph M. Parsons Lab. for Water Resources and Hydrodynamics, M.I.T., Cambridge, MA.

Delfiner, P., and Delhomme, J.P. (1973) Optimum Interpolation by Kriging. In: *Display and Analysis of Spatial Data,* John Wiley & Sons, London.

Delfiner, P. (1976) Linear Estimation of Nonstationary Spatial Phenomenon. In: *Advanced Geostatistics in the Mining Industry,* D.Reidel Publishing Co., Boston.

Delhomme, J.P. (1978). Kriging in Hydrosciences, *Advances in Water Resources, Vol. 1,* pp. 251-266.

Delhomme, J.P. (1979) Kriging in the Design of Streamflow Sampling Networks, *Water Resources Research, Vol. 15, (6),* pp. 1833-1840.

Huijbregts, C.J. (1973) Regionalized Variables and Quantitative Analysis of Spatial Data. In: *Display and Analysis of Spatial Data,* John Wiley & Sons, London.

Kafritsas, J., and Bras, R.L. (1981) The Practice of Kriging, Tech. Rept. 263, Ralph M. Parsons Lab. for Water Resources and Hydrodynamics, M.I.T., Cambridge, MA.

Krige, D.G. (1951) A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand, *J. Chem. Metall. Min. Soc., Vol. 52,* pp. 119-139.

Linsley, R., Kohler, M., and Paulhaus (1975) *Hydrology for Engineers,* McGraw Hill, New York.

Matheron, G. (1971) *The Theory of Regionalized Variables and its Applications,* Cahiers du Centre de Morphologie Mathematique, Fontainebleau, France.

Shamsi, Uzair M., and Yoganarasimhan, G.N., (1986) Optimal Interpolation of Rainfall Data by Kriging, *Journal of The Institution of Engineers (India) Vol. 67,* pp. 134-138.

Shamsi, Uzair M. (1984) Kriging in Spatial Analysis of Hydrologic Data. Master Th., Asian Institute of Technology, Bangkok, Thailand.

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