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SEASONAL VARIATIONS AND STATIONARITY

G. GUDMUNDSSON

Science Institute, University of Iceland

The paper presents a method for testing second order stationarity against the alternative that seasonal variations take place in the autocovariance function. This is a modification of a test suggested by Priestley & Rao (1969) based on the concept of "evolutionary spectra". The method was applied to actual series of monthly data of discharge, temperature and precipitation. Seasonal variations in mean and variance were eliminated before the test was applied, but highly significant seasonal variation was still present in some series.

Most hydrological data consist of time series, and methods using the statistical theory of stochastic processes and time series analysis are used increasingly in the analysis of hydrological data. It is a basic assumption in this analysis that the observations can be regarded as a sample from a second order or weakly stationary process. We call a discrete valued stochastic process $\xi(k)$, where the index k denotes a unit of time, e.g. a day or a month. The definition of second order stationarity is that the autocovariance function,

$$\Phi(k, l) \equiv E\,[\xi(k)\ \xi(k+l)]$$

is independent of k for all l.

Actual observations only cover a finite interval of time. Necessary conditions for a meaningful practical application of the theory of stationary stochastic processes can be loosely described as follows:

1. The observations must be carried out over an interval of time long enough to enable satisfactory estimation of the relevant statistical properties of $\xi(k)$.

2. The second order properties of the series must be almost constant during the interval of observations.

In certain conditions it is possible to abandon the assumption of second order stationarity and estimate time dependent spectra, covariances and weights in autoregressive or moving average models. Apart from the time dependence, these estimates can be interpreted in much the same way as estimates from stationary series (Priestley 1965, Jones & Brelsford 1967).

Seasonal variation is a major factor in producing non-stationarity in hydrology. In the analysis of variations connected with wavelengths of the order of 1 month or less, this obstacle can be avoided by analyzing separately observations from different parts of the year. An estimate of a power spectrum in wavelengths between 2 and 30 days can be representative for statistical properties of daily observations in April. An example of such an analysis was given by Gudmundsson (1970). But if we cannot assume that a series of monthly data is approximately second order stationary there is little to be gained by estimating power spectra. It may sometimes be possible to associate estimates at wavelengths in the range from 2–6 months with a given season, although in Iceland I would not like to combine results from observations in April–June with other months.. A further drawback of such estimates would be that regardless of the length of the series, their bandwidths could not be narrower than about 4π radians/year. At wavelengths over 6 months, the possibilities of associating spectral estimates with a given season diminish further still and their interpretation loses all meaning unless we can assume that the observations are from a second order stationary process.

Seasonal variations in the mean and variance are described by the equation

$$\xi(k) = \mu(k) + \pi(k)\,\eta(k)$$

where $\mu(k)$ and $\pi(k)$ are strictly periodic functions with the period being 1 year. The process $\eta(k)$ has no seasonal variation in mean and variance. If $\xi(k)$ is stationary apart from seasonal effects we can define $\mu(k)$ and $\pi(k)$ so that $E[\eta(k)] = 0$ and $E[\eta(k)^2] = 1$ for all k. But generally $E[\eta(k)\,\eta(k+l)]$ will vary periodically with k for $l \neq 0$. An incorrect statement to the effect that $\eta(k)$ was second order stationary was made by McMahon et al. (1972).

A test for stationarity

A test for the stationarity of time series was published by Priestley & Rao (1969). We shall now adapt their method to the testing of the hypothesis that the process $\eta(k)$, defined in the previous section, is stationary against the alternative of seasonal variation in the second order properties.

Suppose we have a series of N years of monthly observations, X(k), of the process $\xi(k)$. We calculate the average and standard deviation for each month separately. The series Y(k) with zero mean and unit variance is obtained by subtracting the average and dividing by the standard deviation from the respective month into each value of X(k).

Consider now the estimates

$$F(k) \equiv \frac{1}{N} \sum_{j=1}^{N-k} Y(j)\, Y(j+k)$$

and

$$F_I(k) \equiv \frac{1}{4N} \sum_{I}^{N-k} Y(j)\, Y(j+k)$$

where I indicates summation of the products where j is one of the first 3 months of the year, II indicates j from April–June, etc.

If the process $\eta(j)$ is second order stationary we have $E[F_I(k)] \equiv E[F_{II}(k)] \equiv E[F_{III}(k)] = E[F_{IV}(k)] \equiv E[F(k)] \equiv E[F(-k)]$ independent of j for all $k < N$. The power spectrum of $\eta(j)$ is

$$\gamma(u) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} E[\eta(j)\, \eta(j+k)]\, e^{-iuk}$$

if the process is weakly stationary. We estimate it by

$$H(u) \equiv F(0) + \frac{1}{\pi} \sum_{1}^{M} C(k)\, F(k) \cos uk$$

where C(k) denotes the weights known as the Parzen Window (Jenkins & Watts 1968). These estimates are strictly positive. It is customary to carry out this estimate for $u \equiv i\pi/M$ where $i \equiv 0, 1, 2, \ldots, M$. In the following we shall indicate the frequencies with i rather than u. The variance of these estimates at $i \equiv 1, 2, \ldots, M-1$ is approximately $\sigma^2 \gamma(u)^2$ where $\sigma^2 \equiv 0.539$ M/N (Jenkins & Watts 1968) and the variance of $G(u) \equiv \ln H(u)$ is therefore approximately σ^2.

If the process is stationary the estimates

$$H_m(i) \equiv F_m(0) + \frac{1}{\pi} \sum_{1}^{M} C(k)\, F_m(k) \cos \pi ik$$

for m = I, II, II and IV

have much the same properties as H(i). They are, however, not strictly non-negative and the variance of $G_m(i)$ is about $4\sigma^2$.

Let us suppose that M is even and define

$$G.(i) \equiv \frac{1}{4} \sum_{m=I}^{IV} G_m(i),$$

$$G_m(.) = \frac{2}{M} \sum_{j=1}^{\frac{M}{2}} G_m(2j\text{-}1) \text{ and}$$

$$G.(.) = \frac{1}{4} \sum_{m=I}^{IV} G_m(.).$$

We leave out the estimates at $i = 0, 2, 4, \ldots, M$ because $G_m(i)$ is heavily correlated with the estimates at $i \pm 1$, but practically independent of the others.

In accordance with Priestley & Rao (1969) we then construct the following analysis of variation table for two-factor design:

Item	Degrees of freedom	Sum of squares
Between seasons	3	$S_s \equiv \frac{M}{2} \sum_{m} (G_m(.)-G.(.))^2$
Between frequencies	$\frac{M-2}{2}$	$S_f = 4 \sum_{i} (G.(i)-G.(.))^2$
Interaction + residual	$\frac{3M-6}{2}$	$S_{i+r} \equiv \sum\sum_{mi} (G_m(i)-G_m(.)-G.(i)+G.(.))^2$

The sums divided by σ^2 are χ^2 distributed with respective number of degrees of freedom. If the values of Y(j) are very highly correlated it is advisable to reduce this by suitable filtering before calculating F_m.

Examples

We shall now describe the use of this method for testing the hypothesis of second order stationarity of normalized monthly flows from Icelandic rivers and meteorological observations.

There are three main types of rivers in Iceland:

A. The drainage area consists of porous rocks, young Quaternary and post-glacial formations. The river receives little surface runoff and the discharge is relatively stable.

B. The drainage area consists of impermeable rocks, Tertiary basalts or early Quaternary formations. Meltwater and precipitation in the form of rain rapidly find their way to the river.

C. The drainage area consists of a glacier. A description of the effect of geographical conditions on the characteristics of Icelandic rivers was given by Kjartansson (1965). Analysis of the water balance, meteorological conditions and runoff of Icelandic glaciers was given by Björnsson (1971) and Gudmundsson & Sigbjarnarson (1972).

Laxá and Grímsá are good examples of the first and second types of rivers respectively. No series of observations of unmixed glacier runoff long enough for our purpose was available. We therefore included Jökulsá á Fjöllum, where part of the drainage area is in Vatnajökull but the rest is of type A. Similar analysis was carried out for monthly averages of temperature and precipitation in Akureyri and Reykjavík. Table 1 shows the estimates of mean and standard deviation for each month.

The main purpose of our study was to investigate this method of testing for stationarity. The series included were selected from this point of view and no detailed hydrological or meteorological interpretation of the results will be attempted.

It is obvious that the normalization does not eliminate all seasonal effects from rivers of type A or B. The following is an example of the arguments for this statement:

During the summer, precipitation usually falls as rain on the whole drainage area of Laxá. The water reaches the river at different times, leading to a certain correlation between the values of $\xi(k)$. In other seasons precipitation may consist of rain or snow, often simultaneously in different parts of the area. Thaw takes place later and is reflected partly by seasonal variation in mean and variance, but also in serial correlation with time-lags different from those induced by rainfall in the summer.

As a contrast to the highly correlated values of monthly discharge of Laxá

Table 1.

Series		J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.
Laxá Gl/month (22 years)	Mean	105	99	112	119	138	118	115	113	110	113	105	106
	S.D.	6	9	11	14	24	21	7	8	10	8	7	8
Jökulsá Gl/month (32 years)	Mean	278	264	319	368	576	552	840	893	605	428	324	296
	S.D.	33	70	83	96	156	156	174	296	144	82	36	37
Grímsá Gl/month (25 years)	Mean	43	39	47	47	104	144	78	44	59	57	65	48
	S.D.	47	30	42	27	46	53	46	35	41	39	52	35
Reykjavík, temperature °C (70 years)	Mean	–0.2	0.0	0.9	2.9	6.6	9.6	11.3	10.7	8.1	4.6	1.8	0.3
	S.D.	1.9	1.9	2.0	1.6	1.3	0.7	0.8	0.8	1.4	1.5	1.6	1.5
Akureyri temperature °C (71 years)	Mean	–1.9	–1.8	–1.1	1.2	5.5	9.2	10.7	9.6	7.2	3.0	0.0	–1.3
	S.D.	2.5	2.6	2.6	1.9	1.7	1.3	1.3	1.4	1.6	1.9	2.0	2.0
Reykjavík, precipitation mm (35 years)	Mean	90	62	64	52	39	43	47	61	72	98	83	79
	S.D.	43	36	40	24	24	18	21	36	37	39	46	34
Akureyri, precipitation mm (35 years)	Mean	45	39	40	32	16	24	34	38	45	56	46	53
	S.D.	23	19	28	20	12	18	18	28	31	30	25	23

and Jökulsá á Fjöllum, series of monthly precipitation are known to be relatively free from serial correlation. It was therefore expected that seasonal effects would be largely confined to variation in mean and variance.

The results of the test were fully consistent with these considerations. Highly significant values of "between frequencies" and "interaction" effects were obtained from Laxá and Jökulsá á Fjöllum, whereas the results from the precipitation indicated no significant correlation and accordingly no seasonal effects of normalized values.

There is less correlation between monthly discharge values of Grímsá than Laxá or Jökulsá. Different physical factors produce the correlations in different seasons, but the series is probably too short to exhibit highly significant inter-

Table 2.

Series	Item	Degrees of freedom	Estimate of χ^2
Laxá	between seasons	3	1,3
	between frequencies	8	185
	interaction	24	76
Jökulsá á Fjöllum	between seasons	3	0,8
	between frequencies	8	115
	interaction	24	78
Grímsá	between seasons	3	0,9
	between frequencies	8	51
	interaction	24	30
Reykjavík, temperature	between seasons	3	2,8
	between frequencies	11	199
	interaction	36	82
Akureyri, temperature	between seasons	3	0,5
	between frequencies	11	154
	interaction	33	42
Reykjavík, precipitation	between seasons	3	0,3
	between frequencies	11	14
	interaction	33	23
Akureyri, precipitation	between seasons	3	3,0
	between frequencies	11	16
	interaction	33	44

action from this relatively small effect. However, it is also possible that, for instance, correlations resulting from the connection between rain and groundwater in July–September happen to be rather similar to correlations due to correlated monthly temperatures in January–March.

To the author the most unexpected results of the analysis was the great difference between the interaction terms of normalized monthly temperatures in Akureyri and Reykjavík. There are certainly differences between the climates at the two places, but, at least to a layman, it is not obvious why these differences should entail significant seasonal variation in normalized monthly temperatures in Reykjavík and not in Akureyri.

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Address:

Science Institute,
University of Iceland,
Dunhaga 3,
Reykjavik, Iceland.