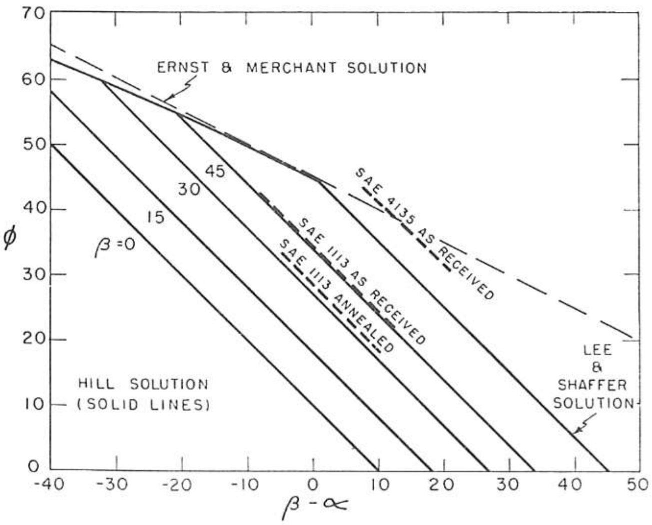

Fig. 19 Comparison of Hill's Solution With Experiment

Equation [11] cannot be applied to the results of the present investigation, since the angle relationships $\phi = \alpha$ is only approximately true, while τ_S/τ_T is not equal to unity.

Hill's analysis, mentioned earlier, which yields solutions lacking definite uniqueness, can be represented by areas on ϕ and $\beta - \alpha$ co-ordinates. Such possible solutions are areas bounded by the solid lines of Fig. 19, one of which is shown as the Lee and Shaffer line, and the curved line merging into the dotted line given by the Ernst and Merchant solution. A third bounding line depends on the friction angle β, as indicated in the figure. In order to permit comparison, the experimental data of the present investigation are given as average values by the heavy broken lines. It is at once evident that the results of the SAE 1113 steels now lie in regions conforming with the theory but that those of the SAE 4135 alloy steels remain in a forbidden region. Lack of conformity here is probably due to the fact that the stress distribution in the cutting zone is not uniform, as pointed out earlier, and other factors such as work hardening, variable angle of friction, and others may influence the results. However, of the various analytical solutions examined, the Hill solution appears to be most promising and warrants further study.

From the foregoing it is apparent that none of the theories proposed is entirely satisfactory for predicting the cutting angles, and some of the physical concepts appear to be incorrect. The results of the present investigation, intended to test the current theories on angle relationships, although yielding some qualitative agreement, indicate that no universal functional relationship has yet been established.

Conclusions

1 Several published theories on the angle relationship $\lambda = \phi + \beta - \alpha$ in orthogonal metal cutting have been compared with experiments reported in this paper, and it was found that none agreed with all of the experimental observations.

2 Hill's solution of an ideal plastic based on a new theorem is qualitatively correct. It assumes, however, that the friction process along the tool face does not occur in the maximum shear direction.

3 The friction process on the tool face appears to be a metal-shearing process as was correctly postulated by Finnie and Shaw. The exact mechanism was not determined although average shearing stresses calculated from force and contact-area measurements were not abnormally high.

4 The calculated shearing stresses in the metal on the tool face were found to be lower than those on the shear plane. The reason for this was thought to be nonuniform temperature distribution and nonuniform adhesion of chip to tool (i.e., nonuniform lubrication). It is suggested that velocity could influence the ratio of these shear stresses.

5 MnS_2 in SAE 1113 free-machining steels acts as a self-lubricant during the cutting process at a surface speed of 27.3 fpm. Additional lubrication in the form of externally applied cutting fluids appeared to have no effect on the cutting forces or angle relationships.

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Discussion

I. Finnie.[5] The discussion and extension of the various angle theories together with the large amount of experimental data make this paper a valuable addition to the metal-cutting literature.

[5] Shell Development Company, Emeryville, Calif.

Referring to the authors' Table 1, it is of historical interest that Equation [3] was derived as long ago as 1896,[6] and again in 1907[7] with the same assumptions as given in the paper.

Methods of analysis based on the concept of an ideal plastic material have been valuable in analyzing operations such as extrusion and rolling but have been disappointing when applied to the cutting of real metals. A recent example is a paper by Hill[8] which gives certain permissible regions for the various angles in cutting. Comparison of this theory and the authors' data show the SAE 1113 to lie inside and the SAE 4135 outside the permissible zone.

Professor Thomsen has recently shown the type of accuracy with which the ideal plastic solutions can be applied to the extrusion of lead and aluminum. It would be of value to have his comments on the use of ideal plastic solutions for the analysis of problems such as cutting in which large shear strains, and hence strain hardening, occur in a very short distance. This would appear to be related to the suggestion made by the authors that the shear angle depends strongly on the plastic-flow properties of the steel.

The authors' interpretation of λ in terms of τ_s/τ_T, l_s/l_T, and $\sin \beta$ is interesting but unfortunately, as they point out, none of these quantities is easily predictable in advance. Values of l_T were measured during the experiments described in reference (1) of the authors' bibliography. Owing to their uncertain nature, they were not reported but for dry cutting appeared to be from 2.5 to 4 times the depth of cut. These values are larger than those given by the authors and may reflect the influence of cutting velocity or of a difference in materials. The term l_s/l_T which involves the shear angle as well as the uncertain quantity l_T would seem to be undesirable in any expression which is to be used for shear-angle prediction.

The authors' experience that τ_T is less than τ_s can be confirmed by the writer. This is also true at very low cutting velocities (1 ipm) and hence is not likely to be due to a difference in temperature between the shear plane and tool face.

[6] Section written by G. Hermann in *Ingenieur und Maschinenmechanik*, edited by H. Weisbach, part 3, 1st half, second edition, 1896, p. 825.

[7] Book review on Taylor's "The Art of Cutting Metals," by G. Lindner, *Zeitschrift VDI*, vol. 51, 1907, p. 1070.

[8] "The Mechanics of Machining: A New Approach," by R. Hill, *Journal of Mechanics and Physics of Solids*, vol. 3, 1954, p. 47.

The most direct physical interpretation of λ is that of the angle between the resultant cutting force and the shear plane. It is interesting that this stays reasonably constant for a given material through a range of cutting conditions.

The authors mention that the stresses on the shear plane and tool face are probably not uniform. This is confined by the variation in metal transfer along the tool face and by the fact that chips curl.

It remains to be seen whether a successful angle relationship can be obtained without considering the effects of nonuniform states of stress.

Authors' Closure

The authors wish to thank Dr. Finnie for his valuable discussion of the paper and, in particular, for pointing out that Equation [3] was actually derived quite early but that the derivation apparently remains undisclosed to later research workers.

The use of the concept of an ideal plastic solid in metal-deformation analysis leads to solutions which, at least in some cases, are correct to a first approximation. The restriction placed on this solid, however, that it should transform from a rigid to a plastic material over an infinitesimal region may often be too drastic; real metals appear to change their state over a finite elastic-plastic transition region. Failure of the theory of plasticity of ideal plastic solids to hold is probably due to the neglect of elastic strains, requirement of a definite yield condition, absence of work hardening, homogeneity, isotropy, and others. While the idealizations may not be serious, however, when analyzing deformation processes in which appreciable metal volume participates in the deformation, it may become entirely unrealistic when analyzing a region as minute as a metal-cutting zone. The authors feel, however, that treating the metal-cutting zone as an ideal plastic solid is valuable and leads to a better understanding of metal-deformation processes, even though agreement between theory and experiment is not always attained.

The reason for introducing l_T, the length of metal deposit on the tool face, was to focus attention on the fact that a thin layer of metal on the tool face appears to undergo plastic deformation. If this is the case, then ordinary concepts of sliding friction are incorrect and should be discarded. The fact that l_T at present cannot be predicted should not detract from its use in the interpretations of the significance of metal-cutting data.