Numerical Simulations for Traffic Flow in Two-Dimensional Network with Obstacles

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We investigate effect of obstacles on formation of a traffic jam for a traffic flow in a network of city roads, by numerical simulations. The network of city roads is implemented in a double-bond square lattice, instead of a usual single-bond square lattice; we assume that each bond has five decorated sites. A car or an obstacle is assumed to be located at one of those decorated sites or lattice sites. Two types (µ and ν types) of cars are assumed; cars of µ type (ν type) move mainly to the positive x (y) direction and change their direction to the positive y (x) direction with a probability. What we found is that a traffic jam is formed more easily for obstacles placed close to traffic signals in the traffic flow direction than for those in the other cases.

Control of traffic flow is an important subject in our modern society. Traffic jams appear in highways, in networks of city roads or in computer networks and should be avoided if possible. The nature of a transition between a free-moving phase and a traffic-jam phase has to be clarified in order to control the traffic flow.

Biham, Middleton and Levin (BML) proposed a two-dimensional traffic flow model on a network of city roads with one lane;1) there is a crossing at each lattice site and cars are located only at crossings. There are two types of cars in the BML model, each of which moves only to the positive x (y) direction or the positive y direction. They found that there is a phase transition of the first-order type between the free-moving phase and the traffic-jam phase. The BML model has been modified by Cuesta et al.2) so that cars can change their direction by a probability. In those models, cars move directly from a crossing to its nearest crossing if that moving is allowed. This direct moving from a crossing to its nearest crossing has been lifted by putting decorated sites.3), 4) In the present paper, we investigate effect of obstacles, such as illegally parking cars on roads, on the formation of a traffic jam, by using a network of city roads with two lanes, which is obtained by extending the model given by Horiguchi and Sakakibara.4)

We consider a network of city roads with two lanes implemented on a square lattice on which there are \(2L \times 2L\) lattice sites in the xy plane. Each lattice site is denoted by \((i, j)\) with \(i, j = 1, 2, \ldots, 2L\). We assume that a set of three lattice sites \{(2l – 1, 2m – 1), (2l, 2m – 1), (2l – 1, 2m)\} forms a crossing, denoted by \(r\); we delete a site \((2l, 2m)\) from the crossing for convenience and hence sites in the crossing are labeled by \((r, (1, 1)), (r, (1, 2)), (r(2, 1))\). The nearest four crossings to the crossing \(r\) are denoted by \(r \pm x, r \pm y\). Two lanes are given by a bond between \((2l – 2, 2m – 1)\) and \((2l – 1, 2m – 1)\), called the first lane, and a bond between \((2l – 2, 2m)\) and

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(2l − 1, 2m), called the second lane, in the x direction. We have two lanes in the y direction in a similar way. We put five decorated sites on each bond; those in the x direction are denoted by (r, h1,i), (r, h2,i), . . . , (r, hl,i), where l = 1 for the first lane and l = 2 for the second lane, in order from r − x to r. We use vs,i instead of hs,i for lanes in the y direction. We impose periodic boundary conditions.

We assume a one-way traffic to the positive x direction and to the positive y direction. We consider two types of cars (μ and ν types); a car of μ (ν) type moves mainly to the positive x (y) direction and changes its direction to the positive y (x) direction with a probability γ (0 ≤ γ ≤ 1/2). We put N/2 cars of μ type and those of ν type. We put obstacles at some of decorated sites on the first lanes and sites (r, (1, 2)), (r, (2, 1)). The cars run primarily on the first lane and use the second lane for passing an obstacle or a jamming car. A traffic signal is set at each crossing. We make microscopic equations of motion of cars. For example, an equation of motion for a car of μ type at the site h1,1 is given by

\[
\mu^t_{r,h1,1} = \mu^t_{r,h1,1} \left\{ \left( \mu^t_{r,h2,1} + \nu^t_{r,h2,1} \right) \sigma_{r,h2,1} + o_{r,h2,1} \right\} \left( \mu^t_{r,h2,2} + \nu^t_{r,h2,2} \right) + \mu^t_{r,h1,2} \left( \mu^t_{r,h1,2} + \nu^t_{r,h1,2} \right) \sigma_{r,h1,2} + o_{r,h1,2} \right\} \mu^t_{r,h2,2} \nu^t_{r,h2,2}
\]

Here the existence of a car of μ (ν) type and an obstacle at (*, *) is denoted by μs,νs, and os, respectively, and the state of traffic signal is denoted by σ; those are Boolean variables and hence \( \sigma = 1 - a \). We have 26 equations for cars of μ type and their corresponding equations for cars of ν type.

We have made numerical simulations for the network with \( L = 16 \) and calculated the average velocity of cars. We have obtained a phase transition of the first-order type between the free-moving phase and the traffic-jam phase as a function of the car-density. For example, we have found for γ = 0.1 that the transition occurs at \( n \simeq 0.36 \) when there is no obstacle, where the car density \( n \) is defined by \( n = N/23L^2 \). When we put 50 obstacles at sites very lose to the crossings, namely (r, (1, 2)) and (r, (2, 1)), we have the phase transition at \( n \simeq 0.25 \). On the other hand, when we put 50 obstacles at sites not close to the crossings, we have the phase transition at \( n \simeq 0.34 \). In this way, we have found that the obstacles put at decorated sites play not so important effect on the formation of the traffic jam. However the obstacles put very close to traffic signals in the traffic flow direction make big effect on the formation of the traffic jam. Details of the present investigation are given elsewhere.

References