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# Development and Application of a Storage Model for River Flow Forecasting

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A lumped sequential river flow forecasting model is outlined. It is shown to be flexible in both temporal and spatial scales, thereby allowing simulations to be undertaken for a wide range of practical purposes. In addition, the required data input is very low, and is restricted to topographic data for only small segments of the entire catchment. The model is successfully applied to the River Avon in England and the River Kako in Japan.

## Introduction

One of the most significant problems for the estimation of flood runoff is knowledge of the initial conditions in a basin related to each storm, because the runoff volumes from the same amount of rainfall often change greatly, depending on the initial moisture conditions. Approaches such as the antecedent precipitation index, or the initial flow method (Linsley et al. 1949) have been used in an attempt to estimate effective rainfall. However, these methods are not stable in the results they provide. Even when the estimation is done successfully, subsequent steps have other difficulties, i.e. the estimation of the exact timing and spatial distribution of effective rainfall, and the estimation of runoff hydrographs. This fact has made it difficult to use antecedent precipitation methods in actual control situations such as gates in a dam reservoir.



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The proposition here is that a calibrated lumped sequential model approach can be very parsimonious in its data requirement and, therefore, very appropriate for many practical purposes. The general approach taken here is that if such a lumped model can be efficiently calibrated on readily available data (e.g. daily precipitation and discharge), then it may be possible with appropriate model design to use the same calibration for much shorter flow forecasting periods (e.g. on an hourly basis). Such a sequential scheme has numerous attractions from a practical standpoint, and is an approach not yet attempted by existing lumped parameter models – forecasts for which are limited to the calibration time increment. Accordingly, such 'two-way' computations may be possible without requiring a change in the parameter values. Of course, as we have already observed, the key element here is the representation of storages within the model. The outline scheme has been discussed by Hata and Kira (1977) and Hata (1982). This paper extends that general approach and applies the model, which is described below, to the River Avon within the Wessex Water Authority in England and to the River Kako in Japan. An additional feature of this lumped storage model is that it seeks to summarize the topographic data collection that is necessary, and an integral part of this approach is therefore the establishment of "representative slope areas" within the whole catchment, which are deemed to be representative of the catchment topography.

## Runoff Process and Model Building

The flow processes of water in a river basin are divided into hillslope and channel flow. The former includes surface flow (or overland flow), interflow, and also groundwater flow. The process is very complicated because of the inter-relationship of each flow component and its variation in time, so each slope is in a different flow stage at any one time, according to its characteristics such as topographic gradient, vegetation, and soil depth. There are many slopes in a river basin of course, and it is very difficult to average them meaningfully. Moreover, it is not clear if such an 'average' value expresses the real average appropriate to the slope hydrology.

### Hillslope Flow Equations

The flows on a natural slope are here expressed by the kinematic wave equations, described by Eagleson (1970). The equations of the relationships between storage on a slope and discharge are the basic ones used in the storage model, and these have been derived from the kinematic wave equations, under some assumptions, by Hata and Kira (1977). For the surface flow



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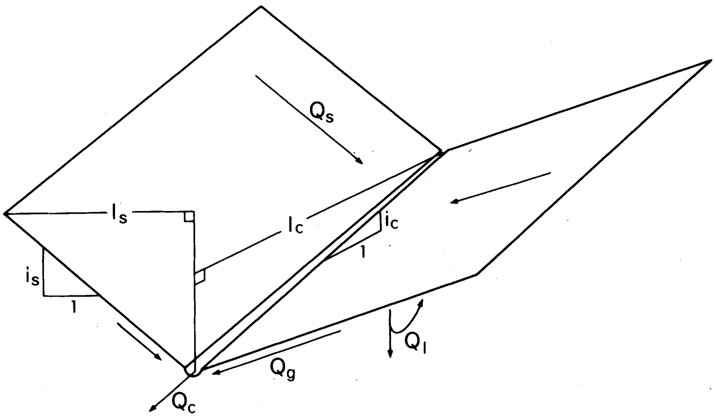


Fig. 1. Runoff processes in a representative slope area.

$$Q_s = \alpha_s S_s^{\frac{1}{p}} \tag{1}$$

$$\alpha_s = \frac{\sqrt{i_s}}{N} \left( \frac{1+p}{\ell_s \cdot \ell_c^{1-p}} \right)^{\frac{1}{p}} \tag{2}$$

where $Q_s$ = outflow from the surface storage $S_s$ on the rectangular surface with length $\ell_s$, width $\ell_c$, slope gradient $i_s$ as shown in Fig. 1, $N$ = equivalent roughness coefficient of the slope, and $p$ equals 0.6 according to Manning's equation. For the subsurface flow including interflow

$$Q_g = \alpha_g S_g \tag{3}$$

$$\alpha_g = 2K \frac{i_s}{\ell_s} \tag{4}$$

in which $Q_g$ = discharge from subsurface storage, $S_g$, in a surface soil layer under the above described rectangular slope, and $K$ = hydraulic conductivity of soil.

Based on these equations, the runoff phenomena on a slope area can be expressed in a similar manner to the tank model of Sugawara (1961). It is difficult in the tank model to relate the parameters to the geographical features of a watershed, but by using these equations the parameter values can be computed from the topographical data, such as slope gradient and length. The loss of water ($Q_\ell$) from a surface soil layer is expressed as

$$Q_\ell = C \alpha_g S_g \tag{5}$$

where $C$ is a constant which depends on the percolation rate to subsurface layers



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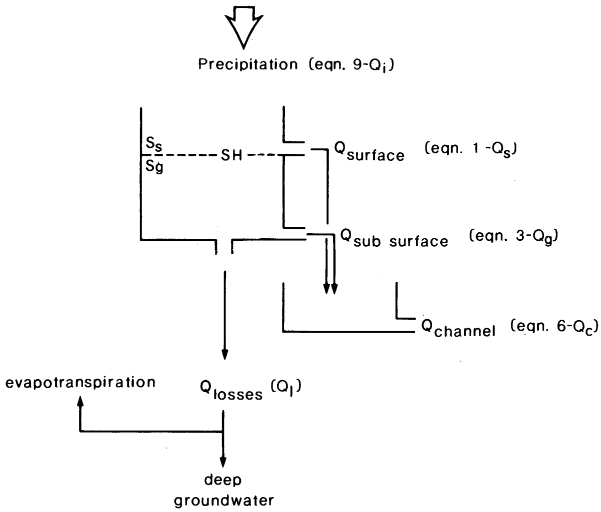


Fig. 2.
Representation of sequential forecasting model.

and on the evapotranspiration rate. When the effect of evapotranspiration is greater, $C$ will vary with the season. $Q_g$ and $Q_\ell$ are at their maximum when $S_g$ also reaches its maximum value ($SH$) – the storage capacity of soil layer. When the storage of water exceeds $SH$, surface flow occurs (see Fig. 2).

**Channel Flow Equations**

The channel flow, which has lateral inflows, is given simply as follows (Hata and Kira 1977)

$$Q_c = \alpha_c S_c^{\frac{1}{u}} \qquad (6)$$

$$\alpha_c = F^{-\frac{1}{u}} \frac{\sqrt{i_c}}{n} \left(\frac{1+u}{\ell_c}\right)^{\frac{1}{u}} \qquad (7)$$

$$F = \frac{a}{(aR^{\frac{2}{3}})^{u}} \qquad (8)$$

where $Q_c$ = discharge from a channel reach with length, $\ell_c$, gradient, $i_c$ and storage $S_c$, as shown in Fig. 1, $n$ = Manning's roughness coefficient, $R$ and $a$ = hydraulic radius and cross-sectional area of the flow respectively. The value of $F$ is about 2.5 when $R$ and $a$ are measured in m and $m^2$ respectively, and $u$ is approximately 0.7.

**Representative Slope Area and Computation Procedure**

A river sub-basin can be considered to comprise a large number of slopes, having a stream with lateral slope inflow, as shown in Fig. 1, and here this unit is consi-



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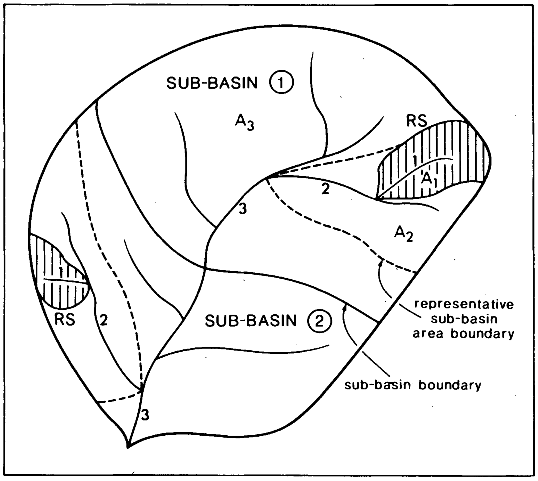


Fig. 3.
Explanation of basin structure and representative slope areas.

dered as the basic element of the basin for computational purposes. Such a basic area of runoff is shown as a representative slope with a channel ordered 1 in Fig. 3. The channel ordered 2 in Fig. 3 receives outflows from channels ordered 1 as lateral inflows, and the channel ordered 3 in the figure receives outflows from channels ordered 2 as lateral inflows. In this way, the discharge from a whole basin or from a sub-basin can be expressed.

The change of storages in channels or in the surface soil layer is computed by the above mentioned equations and the following continuity equation

$$\frac{dS}{dt} = Q_i - Q_0 \tag{9}$$

in which $S$ = storage in soil layer or in channel, $Q_i$ = inflow, $Q_o$ = outflow. For the subsurface storage tank in a representative slope ($RS$), observed rainfalls are directly applied to $Q_i$ in Eq. (9); and $Q_s$, $Q_g$, $Q_\ell$ (see Fig. 1) in Eqs. (1), (3) and (5) respectively are applied to $Q_0$. The values of $N$, $SH$, $K$ and $C$ could all be estimated by field determinations in each sub-basin. However, if this were done, it would be difficult to decide upon the appropriate values to be employed in the model on account of spatial variability. Accordingly, these four variables are estimated by an iteration procedure. Starting values of these variables are selected and values of $S_g$ are set which yield the initial discharge in the long term calibration data set (e.g. daily data for 1 year or more) – see Eq. (3). Subsequently, $S_c$ values in each channel are obtained in the flow routing computations, whilst the initial $S_s$ values usually become zero in the long term calibration analysis. The four variables of $N$, $SH$, $K$ and $C$ are then changed within their physically meaningful limits, and the iterated results of their estimation are easily evaluated by using a graphical display in connection with convergence indices. In this manner, the



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simulated discharge in the calibration period can be accurately matched to the observed values by repetition of this sequence.

In terms of the discharge summation in the entire catchment, the storage in a channel ordered 1 in *RS* (Fig. 3) receives its lateral inflows from slopes located on both sides. These are expressed as follows

$$Q_{i1} = 2(Q_s + Q_g) \tag{10}$$

The outflow $Q_{o1}$ from the channel is computed using a finite difference scheme of Eq. (9). The storage of the channel ordered 2 receives its inflows as mentioned above

$$Q_{i2} = \frac{A_2}{A_1} Q_{01} \tag{11}$$

where $A_1$ = area of representative slope region, and $A_2$ = catchment area of the channel ordered 2 in Fig. 3. The outflow from the channel, $Q_{oz}$, is also computed with Eq. (9). The storage of the channel ordered 3 in the same figure receives its inflows as follows

$$Q_{i3} = Q_{03a} + \frac{A_3}{A_2} Q_{02} \tag{12}$$

in which $Q_{o3a}$ = inflow from an upper sub-basin (should one be present) and $A_3$ = area of sub-basin. The outflow from the channel storage is computed in the same way, and the discharge from a channel in any order can be computed in this way; thus it becomes possible to simulate the runoff from basins of any scale. Fig. 2 summarises the essential elements of the model as configured by use of Eqs. (1), (3), (5), (6) and (9).

## Method of Flood Forecasting

The simulation of runoff processes by using the above-mentioned equations is possible in any time unit, and the hourly flow routing is done in the same program as daily flow routing by simply changing the time scale in $\alpha_s$, $\alpha_g$ and $\alpha_c$ in Eqs. (1), (3), (5) and (6), because these parameters have a dimension, $T^{-1}$. The transformation from daily to hourly routing, for example, is as follows

$$\alpha_h \equiv \frac{1}{24} \alpha_d \tag{13}$$

in which $\alpha_h$ = parameter values in hourly routing, $\alpha_d$ = parameter values in daily routing.



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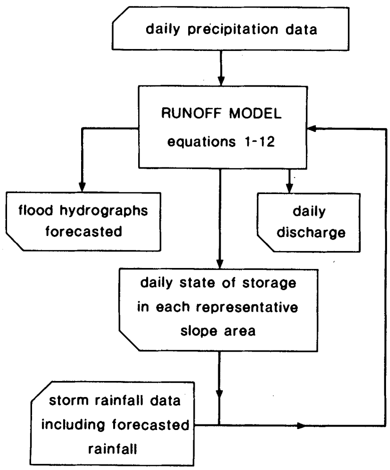


Fig. 4. Procedure of flood forecasting in conjunction with long-term simulation.

The storage ($S_g$) of each representative slope, a surrogate for the antecedent moisture condition of each sub-basin, is estimated from an application of the model based on the observed daily discharge and rainfall data. When a storm occurs, hourly or shorter-time flood routing starts on the initial storage computed from daily routing. The flood forecasting could be done by using all the possible rainfall data including forecasting rainfalls as inputs to the basin which could be divided into sub-basins according to the rainfall stations. The method is diagrammed in Fig. 4. Alternatively, just a single raingauge site may be employed, as in one of the applications.

## Applications

### Single Representative Slope Area – River Avon, England

The model as described above was applied to the River Avon in Wessex, England. The catchment area at the Bathford gauge station is 1,552 km$^2$ (see Fig. 5). Approximately 15 % of the total area is urban, and the rest is developed agriculturally with about 50 % grassland and, typically, the hydrograph peak lags the rainfall by some twelve hours or so. Whilst three topographic features dominate the general area (Southern Cotswold Scorp, Oolitic limestone; Salisbury Plain, chalk; Mendip Hills, Carboniferous limestone) the catchment studied here comprises predominantly clay-loams, with the catchment hydrographs being clearly of the 'storm response' type – the groundwater component being relatively minor. Currently, the flood forecasting in the Wessex Water Authority is undertaken by using the unit-hydrograph method with the initial flows routed through an empirical relationship to estimate effective rainfalls. However, there are some difficulties



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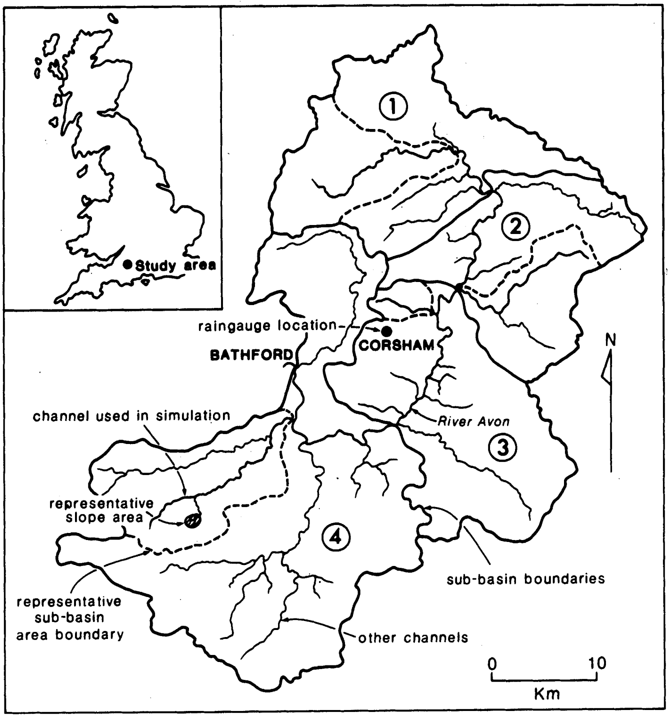


Fig. 5. Catchment area of the River Avon at Bathford and its sub-basins.

in both the estimation of effective rainfall and its distribution, as we have mentioned above. The current paper, therefore, seeks to assess the appropriateness of using the lumped sequential model for such forecasts.

The study basin is divided into four sub-basins as shown in Fig. 5. Only one representative slope area is used here for simplicity and for checking the viability of the method. The precipitation data measured at Corsham (Fig. 5) are used to test the model, and the data are applied to all sub-basins. According to the map of average annual rainfall, published by the Meteorological Office, rainfall approaches 1,200 mm in the south-west part of the basin (the Mendip Hills), 950 mm in the north-west, and is less than 750 mm in most of the eastern part of the basin. Thus the rainfall measured at Corsham (average annual rainfall 800 mm), which is around the average value of the basin, is used as being 'representative' of the rainfall in the whole basin.

Despite the above simplification, the results of the long-term simulation and flood forecasting are quite acceptable, as shown in Figs. 6, 7, 8 and 9, in which the daily rainfall and discharge data in 1981 are used for the calibration of the parameters ($N$, $K$, $C$ and $SH$ – see Eqs. (2), (4) and (5)). The 1982 flows are then simulated, being based on the computed initial storages ($S_s$, $S_g$, $S_c$), and the



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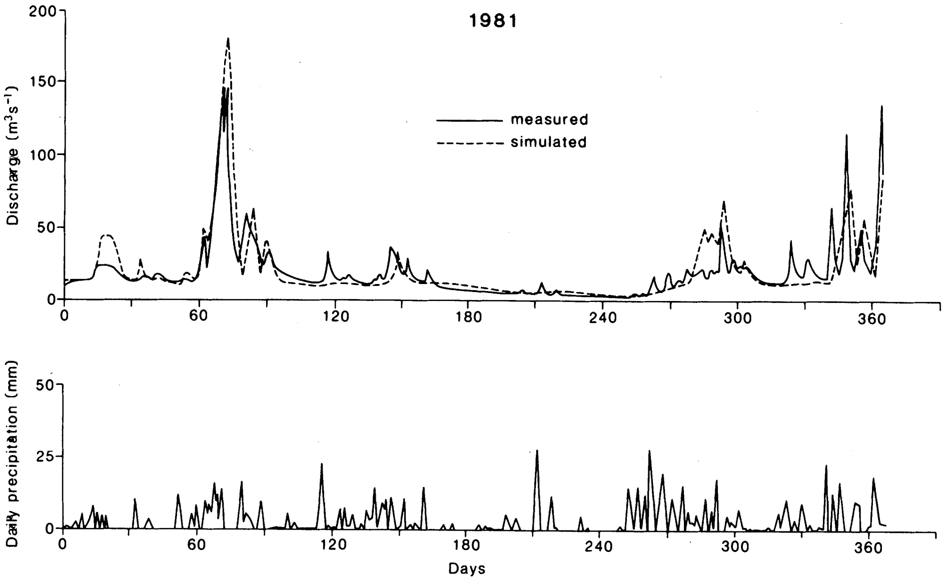


Fig. 6. Long-term streamflow simulation (with start date 1 January 1981), together with measured discharge at Bathford and precipitation at Corsham.

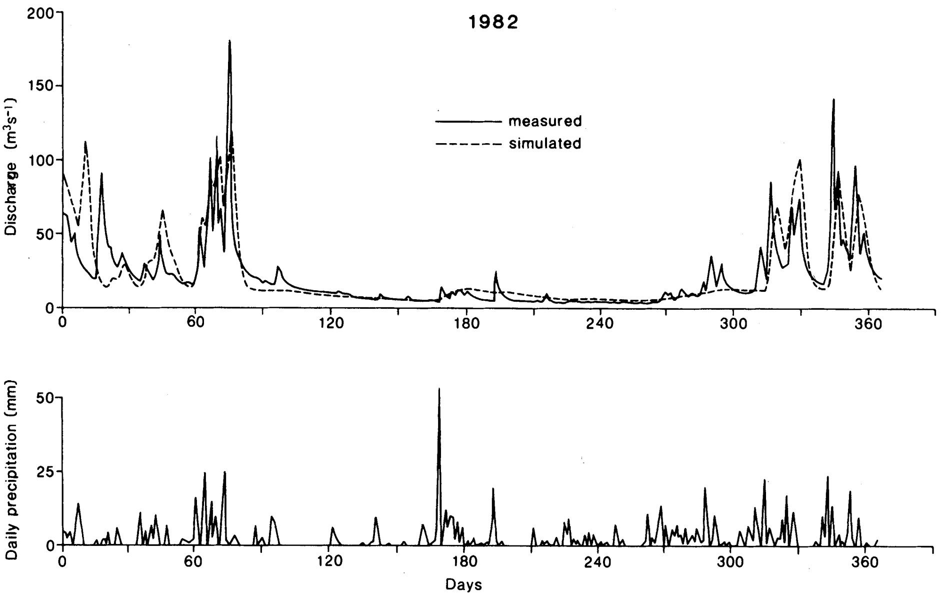


Fig. 7. Long-term streamflow simulation in the verification period of time with start date 1 January 1982 (discharge at Bathford and precipitation at Corsham).



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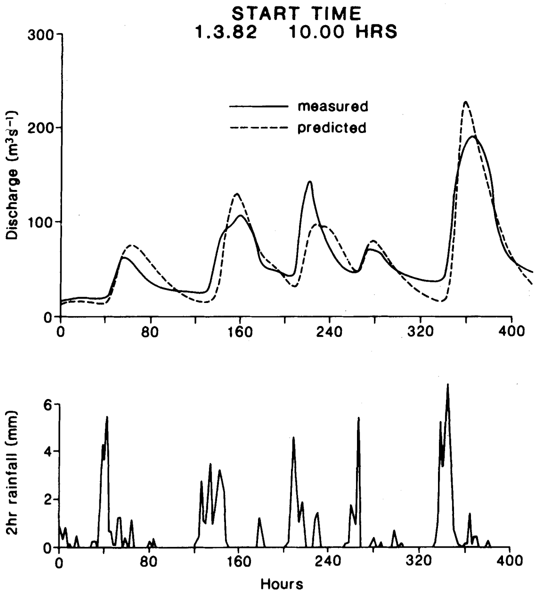


Fig. 8.
Result of forecasting flood with start at 10:00 1 March 1982 (discharge at Bathford and rainfall at Corsham).

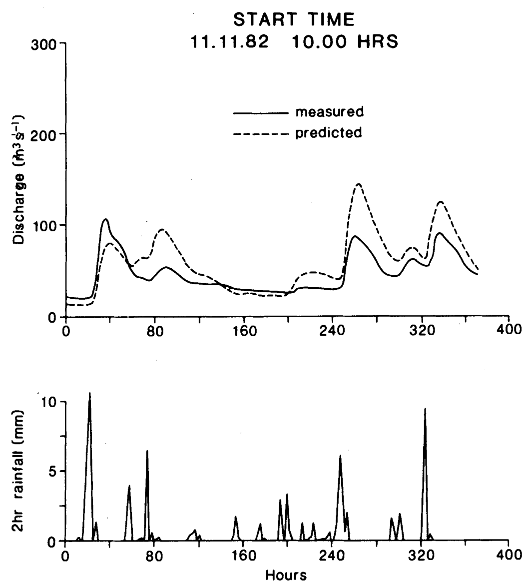


Fig. 9.
Result of forecasting flood with start at 10:00 11 November 1982 (discharge at Bathford and rainfall at Corsham).

calibrated values of the four parameters *N, K, C* and *SH*. Two hour rainfall data are used here for the computation but a smaller time increment can be used in the same way for higher resolution. With respect to the current application, there are two aspects that would enhance the simulation accuracy but which, of course,



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would necessitate a much increased initial data acquisition. Firstly, it would be beneficial to use the data of rainfall stations which were installed inside, or near to, each sub-basin. In this case, each rainfall station would have a weight equal to the ratio of the sub-basin area to the area of the whole basin (analogous to the Thiessen polygon method).

Secondly, in this application only one representative slope is used for the whole basin, as mentioned above, thus the storages on *all* slopes are equivalent throughout the whole basin. These factors may make it more difficult to simulate small fluctuations of the hydrographs, because such fluctuations result from the differential runoff on slopes. However, the following results of the application show the effectiveness of the method as employed, in which the single small area of a representative slope (Fig. 5) is used for the estimation of the water movement in a basin, and its outflows are routed through the channel system.

For the evaluation of the accuracy of the simulation results, the following measure can be used.

$$M_2 = \frac{\sum(|Q_c - Q_0|/Q_0)}{n} \tag{14}$$

where $Q_o$ = measured discharge, $Q_c$ = computed discharge, and $n$ = number of observations. The $M_2$ value in the calibration year of 1981 is 0.35 and the value in the verification year of 1982 is 0.44. There were heavy snowfalls in the basin in January 1982. Of course, the snowfalls are simply inputs in depths of water to the model and result in a too rapid response compared with the observed response, as shown in Fig. 7 (days 0-30). In addition, the numerical value of $M_2$ is increased in 1982.

The characteristic values of the basin which were measured for the model are listed in Table 1. The representative slope area and the channels are subjectively selected from the map of the basin as an area having the average characteristics of the basin. It is, of course, very difficult to make such a selection but it is sufficient to estimate such an area from the map. Of course, any inappropriate assignation of a representative area is modified by the parameter values through the calibration of the model. The estimated parameter values used in the calibration of the model are as follows: *SH* (the water capacity of the surface layer) = 180 mm, $N$ (in Eq. (2)) = 13.0 ($m^{-1/3}s$), $K$ (in Eq. (4)) = 0.0002 ($m\ s^{-1}$), and $C$ (in Eq. (5)) = 2.0. The value $C$ will change seasonally, as mentioned above, and it was varied as follows: $C_{1\text{-}2}$ (in January and February) = 0.1 $C$, $C_{3\text{-}6}$ (from March to June) = 2.0 $C$, $C_{7\text{-}8}$ (in July and August) = 3.0 $C$, and $C_{9\text{-}12}$ (from September to December) = $C$. The roughness coefficient of the channel bed ($n$ in Eq. (7)) is evaluated as 0.03 ($m^{-1/3}s$) throughout the basin.

The predicted flood hydrographs for both the long (Fig. 7) and shorter term simulations (Figs. 8 and 9) have similar shapes, although there are differences



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Table 1 – Characteristic values of the representative slope area (see Fig. 5) and sub-basins.

| Sub-basin | Channel order | Catchment area $A$ (km$^2$) | Channel length $\ell_c$ (km) | Channel slope $i_c$ | Hillslope gradient $i_s$ |
|---|---|---|---|---|---|
| (1) | 2 | 112.17 | 19.98 | 0.0028 | |
| | 3 | 304 | 7.30 | 0.00096 | |
| (2) | 2 | 104.89 | 18.80 | 0.0036 | |
| | 3 | 253 | 14.82 | 0.00094 | |
| (3) | 2 | 13.96 | 7.25 | 0.0125 | |
| | 3 | 312 | 21.76 | 0.00069 | |
| (4) | 2 | 151.07 | 24.16 | 0.0072 | |
| | 3 | 683 | 20.30 | 0.00053 | |
| Representative slope area | 1 | 3.18 | 3.54 | 0.030 | 0.052 |

based on the different time unit for the input of rainfall. This suggests that the continuity of the computation from the long-term simulation to the short-term one is accurate and that the overall accuracy of the flood forecasting depends chiefly upon the results of the long-term simulation, i.e. the initial water storage in the slopes. A typical example showing the importance of the initial storage condition appears in the storm of the 18 June 1982 (Fig. 7), which has the highest daily rainfall, 52 mm/day in the year, but has very little runoff.

**Multiple Representative Slope Areas – River Kako, Japan**

A more complex application to the model to reveal its most complete inclusion of spatial variability was also undertaken. The model was applied to the River Kako in Japan. The catchment area at the Kunikane gauging station is 1,679 km$^2$ (see Fig. 10). The basin was divided up into 14 sub-basins, each with their representative slope areas. The precipitation data at the station inside (or otherwise the closest station) each sub-basin was used as input. The storage in the surface soil layers ($S_g$ in Eq. (3)) of course differs in each sub-basin, because of the varying precipitation inputs and varying characteristics of the representative slopes which change $\alpha_s$ and $\alpha_g$. A two year calibration period was used (1 January 1973 – 31 December 1974), and Fig. 11 illustrates the observed and simulated hydrographs in the following year, serving as a verification period. An example of flood forecasting is also shown in Fig. 12. The $M_2$ value (Eq. (14)) in the calibration year of 1974 is 0.27 and the values in subsequent verification years are: 0.36 in 1975, 0.26 in 1976, 0.33 in 1977 and 0.30 in 1978. It is clear that the detailed streamflow fluctuations can be modelled relatively precisely by increasing the number of representative slope areas.



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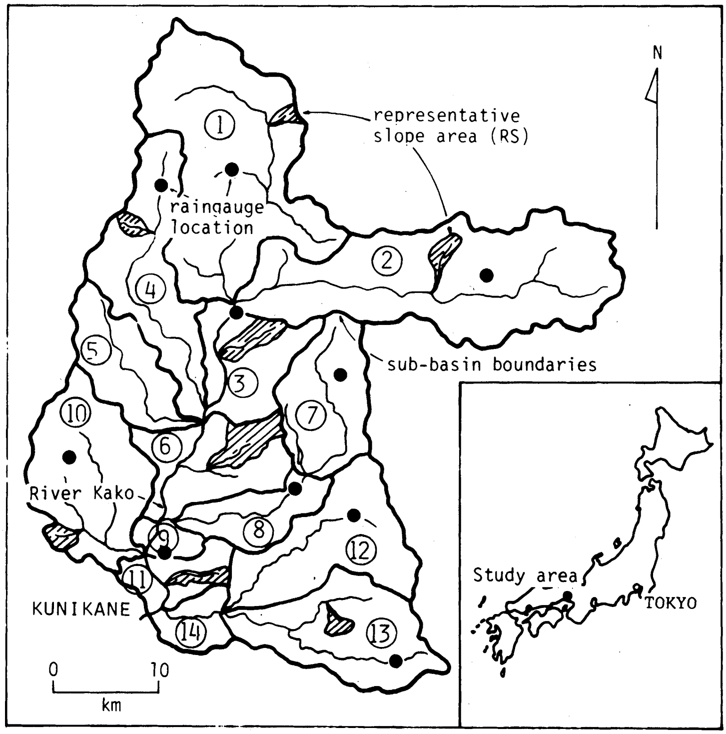


Fig. 10. Catchment area of the River Kako at Kunikane and its sub-basins.

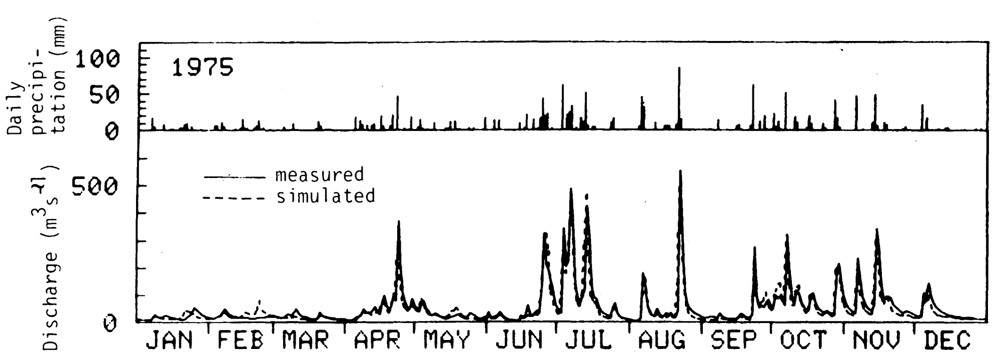


Fig. 11. Long-term streamflow simulation in the verification year of 1975 (discharge at Kunikane and areal precipitation in the Kako basin).



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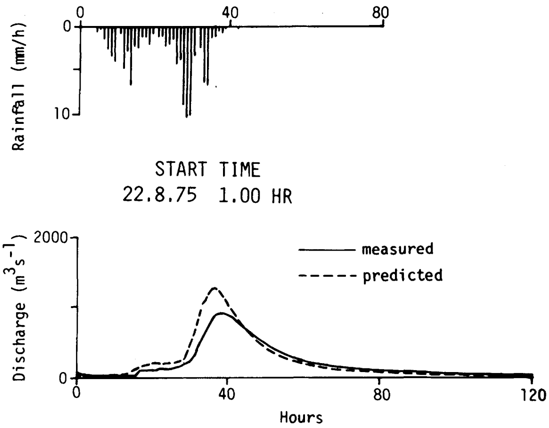


Fig. 12. Result of forecasting flood with start at 1:00 22 August 1975 (discharge at Kunikane and areal rainfall in the Kako basin).

## Discussions and Conclusions

It is obviously very important to evaluate the storage in a basin but, equally, it is difficult to estimate the storage adequately and to connect the results of the estimation with forecasting floods in the usual discrete runoff analyses (e.g. unit hydrograph), because such models are not continuous. Secondly, we have already discussed the problem of estimating effective rainfall in the unit hydrograph case. The current model circumvents both these problems of storage and effective rainfall estimation for individual events, by calibration of a sequential data set. This means that unlike discrete runoff analysis, sequential models of the form used here allow a judgement to be made of the precision of the estimated runoff just before the storm, and also of the estimated runoff at the same period of time in the other years. If the discharge before the storm, or the discharges at the same period of time in the other years are accurately simulated, then obviously the accuraty of the forecasted hydrograph of a flood will be high. Such statements represent most important advantages over discrete runoff analyses.

The sequential flood forecasting method outlined here in conjunction with the long-term simulation has some advantages compared with the discrete runoff analysis or direct runoff analysis method. One such advantage is the direct information about the storage of water in a river basin ($S_s$, $S_g$, and $S_c$) just before a storm which has a big influence on a flood runoff.

The change from long-term simulation to short-term simulation and vice versa is easy in the current procedure developed here, because of the use of the same



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model with the same parameters. It has been shown that the use of a small representative slope area for the estimation of runoff, and subsequent channel routing can facilitate flood forecasting on a range of basin scales (see Eq. (11)), with the proviso that there is a sufficient and appropriate time series of calibrating data available for the catchment. This procedure therefore minimizes the otherwise necessary requirement of taking topographic measurements of the whole basin, but simultaneously shows good resolution in its forecasting capability on both flexible time and spatial scales.

Accordingly, it is maintained that the sequential simulation model developed and illustrated here has a large number of practical advantages over discrete analysis methods. It is parsimonious in input requirements, flexible in simulation increment time, and yet incorporates an appropriate level of process component description (basin storages for example) to ensure accurate simulation results.

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