$$\theta = \theta_0 \left\{ \epsilon^{-\frac{t}{\tau_c}} \left[\frac{\cosh \sqrt{B - \frac{A}{\tau_c}}\, X}{\cosh \sqrt{B - \frac{A}{\tau_c}}\, L} \right] + \frac{4(B\tau_c - A)}{A\pi} \sum_{n=1,3,5\ldots}^{\infty} \frac{(-1)^{\frac{n+1}{2}} \epsilon^{-\alpha_n t} \cos \frac{n\pi x}{2L}}{(1 - \alpha_n \tau_c)n} \right\} \ldots\ldots [17]$$

where

$$\alpha_n = \frac{1}{A}\left[B + \left(\frac{n\pi}{2L}\right)^2\right] \ldots\ldots [18]$$

The solution can be derived as follows: Assume a product solution

$$\theta = XT \ldots\ldots [19]$$

where X is only a function of x, and T is only a function of t. From Equations [13] and [19]

$$T\frac{\partial^2 X}{\partial x^2} - BXT = AX\frac{\partial T}{\partial t} \ldots\ldots [20]$$

Dividing both sides by AXT gives

$$\frac{1}{AX}\frac{\partial^2 X}{\partial x^2} - \frac{B}{A} = \frac{1}{T}\frac{\partial T}{\partial t} \ldots\ldots [21]$$

Each side of Equation [21] must be a constant. Since all the temperature transients eventually must decay, the constant for the right-hand side of Equation [21] must be less than zero. Thus

$$\frac{1}{T}\frac{\partial T}{\partial t} = -\alpha_i \ldots\ldots [22]$$

Solving for T gives

$$T = c_1 \epsilon^{-\alpha_i t} \ldots\ldots [23]$$

where c_1 is an arbitrary constant. From Equations [21] and [23]

$$\frac{\partial^2 X}{\partial x^2} + (A\alpha_i - B)X = 0 \ldots\ldots [24]$$

Since $\alpha_i > 0$, and $A > 0$, and $B > 0$, Equation [24] will have two forms of solution

$$Y = c_2 \cosh \sqrt{B - A\alpha_m}\, X + c_3 \sinh \sqrt{B - A\alpha_m}\, X \quad \text{for } \frac{B}{A} > \alpha_m \ldots\ldots [25]$$

$$X = c_4 \cos \sqrt{A\alpha_n - B}\, X + c_5 \sin \sqrt{A\alpha_n - B}\, X \quad \text{for } \frac{B}{A} < \alpha_n \ldots\ldots [26]$$

The boundary condition of Equation [15] will make c_3 and c_5 equal to zero. Thus

$$X = c_2 \cosh \sqrt{B - A\alpha_m}\, X \text{ for } \frac{B}{A} > \alpha_m \ldots\ldots [27]$$

$$X = c_4 \cos \sqrt{A\alpha_n - B}\, X \text{ for } \frac{B}{A} < \alpha_n \ldots\ldots [28]$$

The boundary condition from Equation [14] will give a relationship between c_2 and c_4.

The boundary condition for Equation [16] will give the arbitrary constant out front and will also give

$$\alpha_m = \frac{1}{\tau_c} \ldots\ldots [29]$$

$$\alpha_n = \frac{B}{A} + \frac{1}{A}\left(\frac{n\pi}{2L}\right)^2,\ n = 1, 3, 5 \ldots\ldots [30]$$

Equation [29] assumes $1/\tau_c < B/A$; this is the case for the physical constants τ_c, A, and B.

The results from the foregoing analysis were given back in Equations [17] and [18].

Equation [13] was evaluated at $x = 0$, the thermocouple junction. Fig. 3 shows a comparison between the exact curve and an exponential curve. The lower portion of the exact curve is almost identical with the exponential curve. The time constant could be predicted within 5 per cent if a true exponential was assumed instead of using the exact curve.

This calculation shows that axial conduction effects are small for a representative case.

Discussion

H. H. Chamberlain.[4] This paper is of considerable interest to those making and using aircraft gas-turbine thermocouples. The authors have covered the subject of long-life fast-response thermocouples quite thoroughly and in addition, their points in favor of thermocouples as primary detectors of temperature rather than the other possible forms, such as resistance detectors or optical methods, are believed to be well taken.

In point 4 under Advantages of the Loop Junction Construction, the authors state that a superior junction weld is produced by the butt-welding possible with the new construction. This seems to the writer to be questionable, and having personally made both types of welds, he has not found any inherent superiority of one over the other. In addition, he knows of no test data which would tend to indicate any superiority in either thermal emf or life; rather, there are considerable data which would indicate no appreciable difference between the two types of of junctions given equal handling and operating conditions, and there is obviously some superiority in the twisted-type junction in so far as damage from poor handling. In point 5 the authors claim a reduced working of the wires in fabrication. Since in forming the loop junction, the wires are flat-dropped and then formed in a punch press before welding, it seems that there is at least as much stress and strain involved here as in the twisting of wires to form a twisted junction.

Under the heading, Forced Vibration of Thermocouples, the authors claim: "It appears that no part of the tailcone will remain inactive, and hence there is no ideal location for placement of thermocouples or similar instrumentation." Actually, measurements indicated considerably less vibration at points on the tailcone skin near the front or back, as compared with regions in the middle of the cone. It would seem that this was due to the stiffening effect of the relatively heavy metal rings at the front and back used to bolt or clamp the tailcone into place.

The section on Accuracy of Parallel Arrangement seems to be of prime importance. The classical method of paralleling thermocouples, namely, equal-resistance thermocouples, all brought to a common junction point, is, of course, capable of theoretical perfection in averaging. However, this system when

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[4] Meter and Instrument Department, General Electric Company, West Lynn, Mass.

used with more than a few thermocouples becomes very cumbersome mechanically, and difficult from a manufacturing standpoint. Although the manifold-type harnessing arrangement described by the authors has in theory a greater error than the equal-resistance paralleling method, in practice, as the authors point out, this system can be so designed as to reduce these errors to a completely negligible value. Some of the advantages of the manifold or ladder-type paralleling arrangement would include the following:

1 Thermocouples all identical electrically and mechanically.

2 Much cleaner and neater physically.

3 A weight advantage which begins to be felt at about the 4 to 5 thermocouple level and becomes increasingly important as the number of thermocouples is increased.

4 Simplicity of manufacture. Simplified mountings since the harness can be mounted entirely from the thermocouples themselves and each will bear its equal share of the load.

5 Flexibility, since simply by changing the harness itself, the same-type thermocouples may be used in any number from about three up to as many as can be crowded in around the tailcone.

S. S. Stack.[5] The authors of this paper have indicated the vast amount of engineering and test work involved in the development of fast-response thermocouples to meet the present-day requirements for temperature control of aircraft gas turbines.

The basic calculations involved relative to time response, gas velocity, conduction and radiation errors, the averaging of measurements at several points, and so on, are applicable for other types of temperature detectors. Therefore, testing time and the evaluation of future designs and operating characteristics at other than test conditions can be simplified.

In line with the continual changes in design of engines and operating characteristics, we are confronted continuously with corresponding changes in measuring equipment.

One major factor, which has been found difficult to determine in a relatively short time, is the prediction of expected life under various operating conditions from short-time life tests for durability with respect to temperature, heat shock, vibration, and thrust.

The investigation of the vibration of the thermocouple mounting bosses attached to the tailcones, is probably indicative of acceleration due to vibrations in the tailcone itself, and not necessarily that occurring at the junction end of the thermocouple. It is indicated that circular stiffening bands and insulating blankets have been found effective in improving the life of some types of thermocouples. This would indicate that further improvements could be obtained in this direction by means of other than rigidly mounting the thermocouples in bosses which are directly attached to the tailcone.

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[5] General Engineering Laboratory, General Electric Company, Schenectady, N. Y.