The auxiliary functions shown in Equations (24) are given by the following expressions:

$$\left.\begin{aligned}
k_{11} &= [\mu_1^2(\mu_2^2 - 1) + (\mu_2^2 + 1)^2 + 2\mu_2^{3/2}(\mu_2 + 1)]/q \\
k_{12} &= [\mu_1^2 + \mu_2^2 - 1 + 2(\mu_1^{1/2} + 1)(\mu_2^{-1/2} + 1)]\mu_2^2/q \\
k_{21} &= [\mu_1^2 + \mu_2^2 - 1 + 2(\mu_1^{-1/2} + 1)(\mu_2^{1/2} + 1)]\mu_1^2/q \\
k_{22} &= [\mu_2^2(\mu_1^2 - 1) + (\mu_1^2 + 1)^2 + 2\mu_1^{3/2}(\mu_1 + 1)]/q \\
l_{11} &= [\mu_1^{3/2}(\mu_2^{5/2} + 1) + (\mu_2^2 + 1)^2 \\
&\qquad + 2\mu_2^{3/2}(\mu_2 + 1)]/q \\
l_{12} &= [\mu_1^{3/2} + \mu_2^{3/2} + 1 \\
&\qquad + (\mu_1^{-1/2} + 1)(\mu_2^{-1/2} + 1)]\mu_2^{5/2}/q \\
l_{22} &= [\mu_2^{3/2}(\mu_1^{5/2} + 1) + (\mu_1^2 + 1)^2 \\
&\qquad + 2\mu_1^{3/2}(\mu_1 + 1)]/q \\
m_{11} &= 2[\mu_1^{5/2}(\mu_2^{3/2} + 1) + (\mu_2^2 + 1)^2 \\
&\qquad + 2\mu_2^{3/2}(\mu_2 + 1)]/q \\
m_{12} &= 2[\mu_1^{5/2} + u_2^{5/2} + 1 \\
&\qquad + (\mu_1^{1/2} + 1)(\mu_2^{1/2} + 1)]\mu_2^{3/2}/q \\
m_{22} &= 2[\mu_2^{5/2}(\mu_1^{3/2} + 1) + (\mu_1^2 + 1)^2 \\
&\qquad + 2\mu_1^{3/2}(\mu_1 + 1)]/q \\
p_{11} &= [2\mu_1^{1/2}(1 + \mu_1^{3/2} + \mu_2^{3/2}) \\
&\qquad + (1 - \mu_1^2 - \mu_2^2)]\mu_1^2/q \\
p_{12} &= 2[\mu_1^{1/2}(1 + \mu_1^{3/2} + \mu_2^{3/2}) \\
&\qquad + (1 - \mu_1^2 - \mu_2^2)]\mu_1^2/q \\
p_{21} &= [2\mu_2^{1/2}(1 + \mu_1^{3/2} + \mu_2^{3/2}) \\
&\qquad + (1 - \mu_1^2 - \mu_2^2)]\mu_2^2/q \\
p_{22} &= 2[\mu_2^{1/2}(1 + \mu_1^{3/2} + \mu_2^{3/2}) \\
&\qquad + (1 - \mu_1^2 - \mu_2^2)]\mu_2^2/q \\
r_{11} &= [\mu_1^{1/2}(1 - \mu_1^2 - \mu_2^2) \\
&\qquad + (1 + \mu_1^{5/2} + \mu_2^{5/2})]\mu_1^{3/2}/q \\
r_{12} &= [\mu_1^{1/2}(1 - \mu_1^2 - \mu_2^2) \\
&\qquad + 2(1 + \mu_1^{5/2} + \mu_2^{5/2})]\mu_1^{3/2}/q \\
r_{21} &= [\mu_2^{1/2}(1 - \mu_1^2 - \mu_2^2) \\
&\qquad + (1 + \mu_1^{5/2} + \mu_2^{5/2})]\mu_2^{3/2}/q \\
r_{22} &= [\mu_2^{1/2}(1 - \mu_1^2 - \mu_2^2) \\
&\qquad + 2(1 + \mu_1^{5/2} + \mu_2^{5/2})]\mu_2^{3/2}/q \\
q &= 2(\mu_1^{5/2} + \mu_2^{5/2} + 1)(\mu_1^{3/2} + \mu_2^{3/2} + 1) \\
&\qquad - (\mu_1^2 + \mu_2^2 - 1)^2
\end{aligned}\right\} \qquad (25)$$

DISCUSSION

D. J. Bergman[7]

This paper furnishes a much needed attack on one of the difficult problems of vessel design for process engineering; namely, temperature-transition skirt supports on hot vessels. There has been too little thought given to the effect of thermal stresses by the vessel designer. The solution of the very complicated three-intersecting-cylinder analysis for steady-state operation indicates thermal stresses far above the elastic limit for plain carbon steels, when high axial temperature gradients exist.

In many cases this results in yield or creep in the vessel wall or skirt weld, but with long-time operation not enough cycles occur in the lifetime of the vessel to cause fatigue failure, consequently thousands of skirt-supported hot vessels are in satisfactory service.

With coke chambers operating on a daily cycle the case is very different, since the number of stress reversals and the stress range become important. Even more important is another factor that most seriously affects analysis of stress.

Anyone who has witnessed a complete cycle of operation of a thermal coking unit is convinced that the maximum stress conditions occur during the quench period. Following steaming, water is injected into the bottom of the coke chamber to cool the coke and vessel walls preparatory to coke removal. The writhing and bending of the vessel as the cooling progresses unevenly up the walls is unmistakable evidence of the tremendous force set up by transient thermal gradients.

Perhaps if cooling could be done much more slowly the conditions would be eased. But to complete cleaning of a coke chamber in time for reuse, it seems necessary to set up an operating schedule based on time rather than temperature. Further complications such as hard dense coke areas, fissures in the coke bed, and wandering of the transfer hole from the center of the coke bed toward the walls, all contribute to irregularities in cooling and complications to the heat-transfer and stress problems.

With digital-computer facilities readily available to ease many of the laborious calculations required by engineers there is a tendency to forget that coke chambers solve heat-transfer and stress problems which are complicated by transients, as well as variable heat-transfer rate, conductivity, and terminal temperature. Since they are not equipped with print-out typewriters or oscilloscopes, too little attention is paid to their solutions of the problems they are fed until internal distress causes flashing of an alarm signal in the shape of bulging side walls or cracked welds.

Some temperature data taken on the outside of the 2-in-thick walls of a 10 × 40-ft coke chamber in 1930 furnish interesting background to the coke-chamber problem. This vessel was supported upon a cast-steel cradle rather than a skirt.

The rate of cooling of the 2-in. wall as shown by skin couples at a number of points was a maximum in the range of 550 F to 400 F, presumably where boiling water at the pressure in the vessel wet the steel walls. The maximum rate determined by several couples was about 105 deg F in 8 min, or 790 deg F per hr. This indicates a heat loss over 6800 Btu per hr per sq ft from the 2-in-thick wall based on *outside* temperature. The rate on the inside would be higher corresponding to boiling heat transfer. The radial gradient on the outside would be 23 deg F per in. with a correspondingly higher rate on the inside.

The transient temperature difference between inside and outside faces of the wall at the onset of wetting probably causes the most severe stress. When this occurs the inside face is suddenly cooled almost to the temperature of the boiling water, while the outside and middle are still very much hotter. The inner fibers try to contract but are restrained by the remainder of the thick wall. So stretching of the inner wall occurs. This is aggravated by the internal pressure of steam, and by pressure against the coke as the steel contracts. Shortly after, as the wave of cooling passes deeper into the wall, the whole wall contracts, putting the still hot outer fibers into compression. When the whole wall cools down to uniform temperature the inner fibers are in compression while those outside are in tension. This mechanism seems to be responsible for the bulging of the walls of coke chambers. Fortunately, the lower part of the vessel, including the skirt-weld area has been cooled somewhat due to heat loss

---

[7] Universal Oil Products Company, Des Plaines, Ill. Mem. ASME.

through the insulation before quenching takes place. However, radial gradients may still be the critical ones at the skirt weld.

At the time the skirt-weld area is subjected to this fast cooling, the axial temperature gradients are confused both as to value and direction. Particularly, the gradient in the skirt reverses and for a time heat flows both ways from the mid-portion to the intersection and the bottom.

If the foregoing analysis be valid and radial gradients are the troublesome ones, it appears that the open crotch suggested in the paper may serve but poorly in protecting the skirt weld at the most critical period of cooling. Temperature tests on vessels with an internal insulation of 2 in. of ganister reduced the rate of wall cooling to 120 deg F per hr. This served as excellent protection against bulging of coke chambers, but had to be abandoned because of damage by the jets when hydraulic decoking was used.

E. R. Slater[8]

The authors have presented a very valuable paper giving a method for arriving at stresses in high-temperature vessels and their skirt supports which should be extremely useful to the design engineer. However, there appear to be certain discrepancies which occur in equations (12). It is found that, neglecting terms due to temperature, the equilibrium conditions imposed by equations (11) are not satisfied by the values of M_1, M_2, M_3 and Q_1, Q_2, Q_3 as obtained from equations (12). This is easily confirmed by letting $\beta_1 = \beta_2 = \beta_3$, $D_1 = D_2 = D_3$, and $\varphi_1 = \varphi_2 = 0$. It would seem that the error occurs in the terms containing K within the square brackets. By inspection it is found that by multiplying these terms by 2 the conditions of equations (11) can be met, but there has not been time to carry out a rigorous check to determine if this is indeed the answer, nor has any examination of the temperature terms been made. However, it would be reassuring to know that these discrepancies have not been carried forward to equations (24) and hence back into equations (1) from which the shell and skirt stresses are obtained.

It is said in the paper that the variables k, l, m, p, and r can be reduced to chart form using μ_1, as an independent variable and μ_2 as the free parameter. If the authors have already done this, it is hoped that the charts will be published since they would save much tedious calculation.

Authors' Closure

Before commenting in detail upon the discussions submitted for this paper, the authors wish to acknowledge the valuable suggestions and important points raised by both of the discussers.

Dr. Bergman's emphasis on the importance of thermal fatigue in cyclic service serves to underline the strong point made about this subject in the text of the paper. His data on the transient values of axial and radial thermal gradients in delayed cokers add further substance to the fact that thermal causes are the most important design considerations for these severely cyclic units. While the authors readily agree that the radial thermal gradients may contribute significantly to the development of bulging, their influence on skirt-weld cracking is doubted. Had these gradients been the cause of skirt-weld cracking, one should anticipate the cracks to develop perpendicular to the run of the weld seam. Instead, all field experience to date indicates that the cracks run in the lengthwise direction, that is, parallel to the weld seam, as shown in Fig. 5. This leaves little doubt in the authors' minds that the moments resulting from axial thermal gradients are the major cause of skirt-weld cracking, as explained in more detail in the paper.

Dr. Bergman's observation regarding the reversal of axial gradients in the skirt during the quenching operation is entirely correct. Offhand, however, one would expect that these gradients would not be as severe as those developed during the operating cycle, since the net temperature difference (ΔT) influencing the heat flow (Q) would tend to be far smaller when the drums are brought down to ambient temperature. This prediction could be upset if the surface-boiling film coefficients are so high as to permit a greater quantity of heat to flow out of the skirt (which will act as a heat reservoir of small volume during the quenching operation), even though the temperature levels corresponding to this cycle are substantially lower. The general question of the influence of temperature differentials and film coefficients upon gradients and thermal stresses is dealt with in detail in Reference [11].

The authors agree with Dr. Bergman that an internal insulation would be the ideal solution to the question. Such, in fact, was the case in the days of mechanical ball-and-chain decoking apparatus, when the roughly 2-in. thick layer of coke remaining on the walls after decoking served as an excellent internal thermal shield for the vessel. Unfortunately from this viewpoint, hydraulic decoking will not permit any insulation to stay in the vessel, regardless of whether such insulation is the result of accidental coke deposition or was intended to be used as a thermal shield for the structure.

Mr. Slater's comments on the discrepancies in equations (12) are sincerely appreciated. This discrepancy was the result of an inadvertent error made in the transformation of equations for purposes of preprinting, and has been corrected in the final publication of the paper. Even in the preprint, the error does not go beyond equations (12), so that all other equations are correct exactly as given.

In response to Mr. Slater's question, the authors did prepare chart-form solutions for all of the auxiliary functions. The resulting charts, 18 in number, when reproduced in sufficient size to permit their accurate use, would have made the paper so bulky that the authors deemed it wiser to refrain from their inclusion in this work. With the background information and equations fully presented, these charts can be prepared by any individual suitably interested in this subject.

---

[8] Berkeley, Calif.