Phenomenological Analysis of Elastic $\pi^- - p$ Scattering at 1.4 Bev

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Pion-proton scattering at 1.4 Bev observed by Eisberg et al. is analyzed in a phenomenological way. Interpreting the peak in forward direction as "shadow scattering" and analyzing this angular distribution, we obtain the result which agrees fairly well with the experimental one. By this method, moreover, it is found that the partial waves of $l=0, 1, 2$ and $l=1, 2, 3, 4$ are mainly responsible to the elastic and production cross sections, respectively.

§ 1. Introduction

Recent experiments on pion-nucleon and nucleon-nucleon collisions have revealed that the shadow scattering plays an important role in these processes. Eisberg et al. have tried to analyze their experiment on elastic pion-nucleon scattering in terms of a phenomenological potential, as customarily employed in interpreting nuclear reactions.

As is well known, the shadow effect results from the opening of new channels which have been closed at lower energies. As it is closely related to the multiple production of pions, some knowledge about the multiple pion production is required for the analysis of the scattering data in a Bev region. The experiments at the present stage, however, do not seem to supply enough data to such an approach. In these circumstances, we intend to attempt another approach, that is, estimation of the cross section for the multiple pion production from the knowledge about the shadow scattering. When we notice the result of Eisberg et al., it will be expected that it may be possible to separate, without large error, the contribution of the shadow scattering from the angular distribution of elastic scattering. Taking advantage of this, we have tried to estimate the cross section for pion production through the analysis of this shadow scattering.

In § 2, the general theory of scattering including the shadow effect is developed for the most simplified model. In § 3, the shadow part of the differential cross section obtained by Eisberg et al. is analyzed in terms of partial waves. From this result, the cross section of pion production is calculated for every partial waves. In spite of this somewhat crude estimation, the obtained total cross section of pion production shows fairly good agreement with the observed one.

§ 2. Shadow scattering and S-matrix theory

Below the threshold of the pion production, the unitarity of S-matrix is guaranteed in
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the one pion subspace (0) of the pion configuration, and may be expressed as

$$S^+(0)S^{(0)} = 1.$$  \hspace{1cm} (2.1)

In a Bev region, however, the cross section for the multiple pion production $\sigma_{\text{prod}}$ is observed as slightly larger than that for the elastic scattering $\sigma_{\text{scatt}}$. In such a case, the unitarity has to be expressed as

$$S^+(0)S^{(0)} + S^+(0)S^{(0)} + \cdots = 1$$  \hspace{1cm} (2.2)

where, $S^{(0)}$'s represent the submatrices relevant to ($i+1$)-pions production.

In the first place, let us simply ignore the spin of nucleon and analyze (2.2) into partial waves. Then

$$S_i = S_i^{(0)} + S_i^{(1)} + \cdots = S_i^{(0)} + M_i,$$

$$|S_i|^2 = |S_i^{(0)}|^2 + |M_i|^2 = 1$$  \hspace{1cm} (2.3)

for every partial wave.

From this relation, $S_i^{(0)}$ may be written as

$$S_i^{(0)} = \sqrt{1 - |M_i|^2} e^{2i\bar{\theta}} = \cos \theta_i \exp(2i\bar{\theta}),$$  \hspace{1cm} (2.4)

where

$$M_i = \exp(i\vec{\bar{\theta}}) \cdot \sin \theta_i.$$

Then the scattering and the production cross sections are expressed respectively as

$$d\sigma_{\text{scatt}}/d\Omega = (1/4k^2) |\sum_i (2l+1) |\exp(2i\bar{\theta}_i) \cdot \cos \theta_i - 1| \cdot P_i(\cos \theta)|^2,$$

$$d\sigma_{\text{prod}}/d\Omega = (1/4k^2) |\sum_i (2l+1) \exp(i\vec{\bar{\theta}}_i) \cdot \sin \theta_i \cdot P_i(\cos \theta)|^2.$$  \hspace{1cm} (2.5)

Here it must be noticed that the phase factor $\exp(i\vec{\bar{\theta}}_i)$ is not important so far as the total cross section $\sigma_{\text{prod}}$ is discussed.

Even when the spin of nucleon is taken into account, a similar procedure can easily be carried out. In this case, $R_i (= S_i^{(0)} - 1)$ becomes as follows:

$$R_i \rightarrow R_i^+ \frac{l+1 + (i\sigma)}{2l+1} + R_i^- \frac{l - (i\sigma)}{2l+1}.$$  \hspace{1cm} (2.7)

§ 3. Analysis of experiment

Eisberg et al. have obtained the following cross sections

$$\sigma_{\text{scatt}} = 10.0 \pm 0.8 \text{ mb}$$

$$\sigma_{\text{prod}} = 24.6 \text{ mb}$$  \hspace{1cm} (3.1)

and the angular distribution for the elastic scattering illustrated in Fig. 1.

As they have pointed out, the strong forward

* Here we discard weakly interacting processes such as the radiative capture of pions.
peak may be attributed to the shadow scattering. Adopting this interpretation, the angular distribution is analyzed. For simplicity, we shall set the crude assumptions of $R^z = R^- = -T$, and of $\phi' = 0$.

With such a simplification, the scattering amplitude can be expressed as follows:

$$f(\theta) = (i/2k_0) \sum_l (2l + 1) T_l P_l(\cos \theta) = (i/2) \sum_l (2l + 1) Q_l/\cos \theta,$$

(3.2)

$$T_l = 1 - \cos \theta_l$$

and $Q_l' = T_l/k_0$.

Here the phase shift analysis is made considerably simpler by the fact that $Q_l'$s in eq. (3.2) are real numbers. Moreover, if the experimental values of $d\sigma/dQ'$ are expressed by an empirical formula, the analysis may be carried out more easily.

As an empirical formula, we adopt

$$d\sigma/dQ' = 0.2/(1.13 - \cos \theta)^2 \text{ mb/sterad},$$

(3.3)

which fits fairly well to the experiment as is shown in Fig. 1**. From (3.3), the scattering amplitude can generally be written as

$$f(\theta) = i^{2l} \sqrt{0.2/(1.13 - \cos \theta)}.$$

(3.4)

The unknown phase factor $i^{2l}$ may be determined on account of the dispersion relation holding at $\theta = 0^\circ$. (c.f. (3.2)). Thus

$$f(\theta) = i^{2l} \sqrt{0.2/(1.13 - \cos \theta)}.$$

(3.4')

From the eqs. (3.2) and (3.4')

$$\sqrt{0.8/(1.13 - x)} = \sum_l (2l + 1) Q_l P_l(x).$$

(3.5)

$Q_l'$ in eq. (3.5) is readily obtained by

$$Q_l' = \frac{1}{2} \int_{-1}^{1} \frac{\sqrt{0.8}}{1.13 - x} P_l(x) dx = \sqrt{0.8} Q_l$$

(3.6)

$$Q_l = \frac{1}{2} \int_{-1}^{1} \frac{P_l(x)}{1.13 - x} dx.$$

(3.6')

According to the well known Neumann-Heine's formula,*** the right hand side of (3.6)' is $Q_l(1.13)$, where $Q_l(x)$ is the Legendre function of second kind. Their numerical values are easily obtained from a table as listed in the 2nd column of Table 1. From these values of $Q_l'$, the partial scattering cross sections $\sigma_{l \text{ex}} = \pi(2l + 1) Q_l'^2$ are calculated and they are also listed in the 3rd column of the Table.

The scattering cross section thus obtained as $\sum_l \pi(2l + 1) Q_l'^2 = 9.1 \text{ mb}$ agrees fairly well with the experimental value (3.1).

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* Note that we consider only the shadow part of elastic scattering.

** As only the shadow scattering is taken into account, it will be rather natural that our empirical formula gives somewhat smaller value of cross section than the experimental value of elastic scattering.

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Table 1

<table>
<thead>
<tr>
<th>$l$</th>
<th>$Q_l'$</th>
<th>$\sigma_{l}^{\text{scatt}}$ mb</th>
<th>$\sigma_{l}^{\text{prot}}$ mb</th>
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<td>total</td>
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<td>9.1 mb</td>
<td>25.1 mb</td>
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The partial production cross sections

$$\sigma_{l}^{\text{prot}} = (\pi/k_0)^2 (2l+1) \sin^2 \delta_l$$  \hspace{1cm} (3.7)

are evaluated by inserting the values of $\delta_l$'s calculated from the relation $T_l = 1 - \cos \delta_l = k_0 Q'_l$. The results are listed in the 4th column of Table 1. The production cross section calculated up to $l=6$ ($\sigma_{\text{prod}} = \sum_{l=0}^{6} (\pi/k_0)^2 (2l+1) \sin^2 \delta_l = 25.1 \text{ mb}$) shows a fairly good agreement with the experimental value (3.1), in spite of the rather crude estimation. These results are illustrated in Fig. 2.

In order to examine our approximation of neglecting the contributions from the orbital angular momenta higher than $l=6$, let us evaluate the total cross section ($\sigma_p = \sigma_{\text{scatt}} + \sigma_{\text{prod}}$) by the well-known relation of

$$4\pi \text{ Im} f(0^\circ) = k_0 \sigma_p$$  \hspace{1cm} (3.8)

Inserting the value at $0^\circ$ of our empirical formula (3.4)' into the left hand side of (3.8), we obtain the result of $\sigma_p = 36.8 \text{ mb}$. This value is not so different from that of 34.2 mb in Table 1.

§ 4. Concluding remarks

Although the above results have to be regarded as qualitative, it may be said that dominant contributions come from $s$, $p$ and $d$ waves in the case of the elastic scattering, and from $p$, $d$ and $f$ waves in the case of the production.

Since the mean value of $l$'s responsible for production in the case of $1/k_0 = 2.7 \times 10^{-4} \text{ cm}$ is estimated to be 3 or 4, we may consider that the collision is taking place in the region of 4~5 times nucleon Compton wave length (mean value of collision parameter). This result is consistent with the experimental fact that the momentum transfer in the reaction is small.
It is important to examine whether or not the above conclusions depend sensitively upon the assumed form of the empirical formula. Within the range of the experimental errors, we have examined it in two extreme cases; that is, the steepest case $0.1/(1.08 - \cos \theta)^2$ and the broadest case $0.36/(1.2 - \cos \theta)^2$. It may be seen that our qualitative conclusions are not sensitively affected by the assumed forms of the empirical formulae.

The authors would like to express their thanks to Prof. T. Miyazima, Prof. S. Hayakawa and Dr. K. Nishijima for their valuable discussions.

<table>
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<tr>
<th>empirical formula</th>
<th>$d\sigma/d\Omega = 0.1/(1.08 - \cos \theta)^2$</th>
<th>$d\sigma/d\Omega = 0.36/(1.2 - \cos \theta)^2$</th>
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<td>$l$</td>
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References