Nordic Hydrology 6, 1975, 14–27
Published by Munksgaard, Copenhagen, Denmark

SYNTHESIS OF BASIN RESPONSE WITH INADEQUATE DATA

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The present study is focussed on the problem of developing a basin response function with inadequate data. The case of a catchment with some measurements of output and with input data either totally absent or unemployable is specifically taken for the analysis. The method proposed by Delaine (1970) and demonstrated by him for an experimental drainage area of around 50 acres has been suitably extended to natural catchments from 25 mi.^{2} to 750 mi.^{2}. It is shown that the synthesis is subjected to uncertainties and errors when the method is applied to larger catchments, though reasonably good results are obtained for smaller catchments. The authors suggest an alternate method applicable for all sizes of catchments assuming linearity and time invariance and that the parameters of the model are correlated with the catchment characteristics.

The problem often faced during the hydrological analysis of a developing region is to obtain the basin response function without adequate data. The usual types of situations encountered fall under the following categories:

1) The catchment under investigation has neither the input nor the output data available, but there is adequate data available for catchments in a hydrologically homogeneous region so that their results could be extended to the catchment under study through some physically meaningful parameters.

2) The catchment has measurements of output for some length of time, but the input has not been suitably measured or no input data is available. Also, the same situation prevails over a large number of sub-basins in a region.

The procedures proposed by Snyder, Linsley, Taylor, etc. come under the first category. They give the coordinates of a few points as correlated to the basin characteristics. The complete description of the response through a suitable function and its correlation with watershed characteristics were proposed earlier by Wu (1965) and McSparran (1968) where complete input and output data have been required for the synthesis.

Many sub-basins of the large river systems in India seem to fall under the second category. The input is available only in the form of daily rainfall as there are very few recording type rain gauges. Also, with the inadequate density of the rain gauge network, the input data, though available, is not employable for synthesis of response functions.

SYNTHESIS OF BASIN RESPONSE: EVALUATION OF DELAINE'S APPROACH

The present paper aims at the critical evaluation of the applicability of the methods that could be exploited in the type of situation mentioned above and to propose a suitable procedure in tackling problems of that kind.

The method proposed by Delaine (1970) for deriving the unit graph without rainfall data deserves attention. Using the assumptions of linearity and time invariance, a procedure is given to obtain the basin response and input from the output sequences. A brief account of the method is given to facilitate further discussions.

If the input, response and output are continuous in time in the form x(t), h(t) and y(t), then by suitable convolution

$$y(t) = x(t) * h(t) \qquad \ldots (1)$$

The input, basin response and output are discretized with the same interval of sampling. By so doing, (m + n–1) parts of output occur with m parts of input and n parts of the basin response. The input, response and the output are represented by polynomials as below

$$x(k) = x_1 + x_2\, k + x_3 \cdot k^2 + \ldots . + x_m\, k^{m-1} \qquad \ldots (2)$$

$$h(k) = h_1 + h_2\, k + h_3\, k^2 + \ldots . + h_n\, k^{n-1} \qquad \ldots (3)$$

and hence

$$x(k) \cdot h(k) = y(k) = y_1 + y_2 k + y_3 k^2 + \ldots . + y_{m+n-1} k^{m+n-2} \qquad \ldots (4)$$

The factors of the polynomial (4) are the factors of (2) and (3). The allocation of factors is finalised by factorising (4) for two or more output sequences. The common factors are the factors of (3) and the discretized basin response can thus be computed from the polynomial obtained from the common roots. The remaining factors in each of the output sequences constitute the input polynomial x(k).

The method is demonstrated for an experimental catchment of 49.6 acres with two sets of runoffs lasting for $2^1/_2$ and $1^3/_4$ hr respectively. The unit graph and input ordinates at 15 minute intervals are evaluated by identifying the proper roots of x(k) and h(k). An interesting study on how the roots present difficulty in matching due to the presence of a random error of 5 percent is presented. A deviation as high as 30 percent is allowed in the roots in order to match them and the hydrograph ordinates still only show around 10 percent errors at the peak of the hydrograph.

This method was hoped to yield the discretized response of the basin with only the few sets of runoff which may be readily available. But the following initial difficulties were obvious even before the application of the method to bigger catchments:

1. As the intervals of time of sampling have to remain the same for the input, response and output, the degree of the polynomial increases greatly as the size of the catchment increases. The roots are numerous and evaluation of the common roots between two sequences of runoffs may be difficult.

2. If the roots of the input happen to be the same for the sequences, they are likely to be mistaken for the roots of the response and errors are likely to be introduced in the evaluation of x(k) and h(k) simultaneously.

Still, the study was proposed to be undertaken for small and medium watersheds (see Fig. 5 to Fig. 10) where the basic assumptions of linearity and time invariance could still be taken to hold good. Larger accuracy was to be imposed on the extraction of roots by the computer and more storms could be taken for the analysis to partially take care of the difficulties envisaged above.

The catchments tried have the following characteristics:

Name of the basin	Area	Date of occurence of the storms considered
Coshocton, Ohio	27.24 mi.^{2}	June 28–29, 1957 April 25–26, 1961
Jalput, India	755 mi.^{2}	June 2, 1968 March 15, 1967

For the sake of demonstration, the catchment W-994, Coshocton Ohio, of area 27.24 mi.^{2}, was taken with a view to extending the results from an experimental watershed to a field situation. The roots of the storms 1 and 2 are given below, indicating the matched roots.

Roots of runoff 1		*Roots of runoff 2*
–0.026535193		–0.1334058
–0.30477031		
–1.2710956	←...→	–1.4404985
–0.46776999 + 1.3408828 i	←...→	–0.46698728 + 1.3460143 i
–0.46776999 – 1.3408828 i	←...→	–0.46698728 – 1.3460143 i
–1.1717574 + 0.74421778 i	←...→	–1.1432371 + 0.8616774 i
–1.1717574 – 0.74421778 i	←...→	–1.1432371 – 0.8616774 i
1.2797198 + 0.97428862 i	←...→	1.2196052 + 0.84152857 i
1.2797198 – 0.97428862 i	←...→	1.2196052 – 0.84152857 i
0.44183011 + 1.3390941 i	←...→	0.45257129 + 0.14000285 i
0.44183011 – 1.3390941 i	←...→	0.45257129 – 0.14000285 i

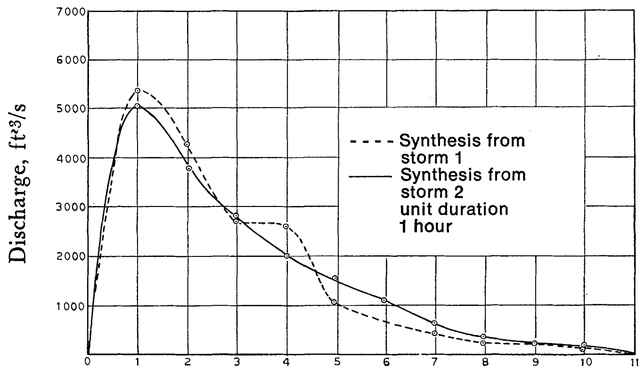


Fig. 1.
Response for Coshocton basin.

The unit graphs computed from the matched roots are plotted in Fig. 1. They have a very small amount of noise in them and are consistent when evaluated from either of the runoffs. The unit duration is one hour. To compare the results with the response obtained from the unit hydrograph theory another storm was chosen and a 3 hr unit hydrograph derived. The response derived earlier for one hour unit duration of input was reduced corresponding to 3 hr unit input by

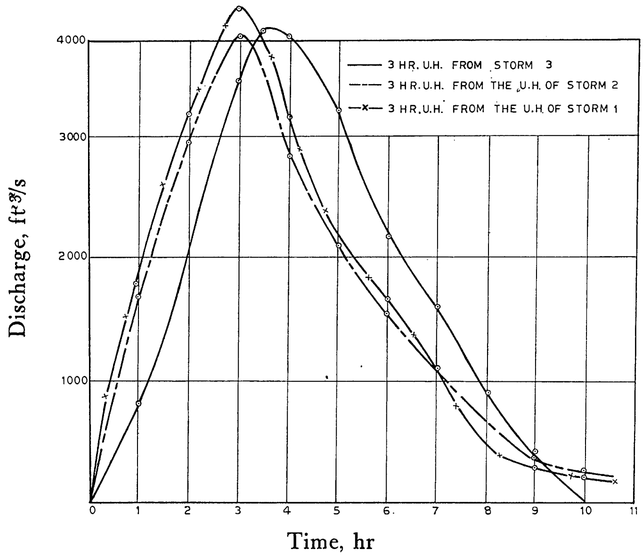


Fig. 2.
Comparison of observed and Synthesised Responses.

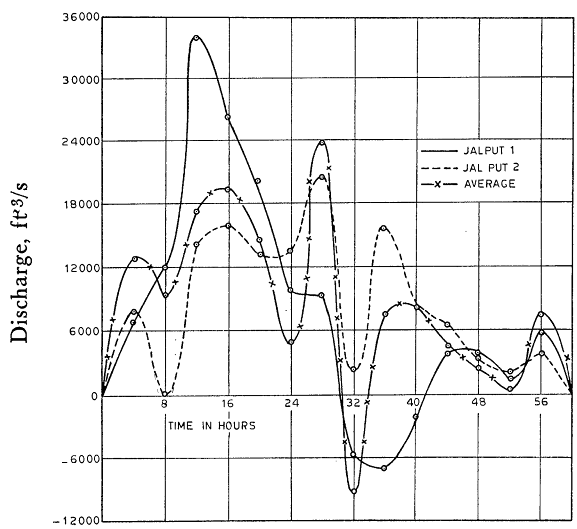


Fig. 3.
Jalput-1,2 and Average (Synthesis with 13 roots, unit duration 4 hours).

suitable convolution. They are compared on Fig. 2. There is very good conformity with respect to both peak value and lag times.

Extending the method to a larger catchment where the basic assumptions still hold good, Jalput catchment was considered for the analysis. For Jalput, the polynomial for storms 1 and 2 had the degrees 16 and 25 respectively. The roots were extracted by an IBM 7044 located at the Indian Institute of Technology, Kanpur, India, with an accuracy of 10^{-9}. A deviation of up to 30 percent in the real or imaginary parts was tolerated. From storm 1 and storm 2, 13 roots were found to match and the response functions computed from the roots matched are given in Fig. 3. Though the storm 1 gives the shapes of the hydrograph reasonably acceptably in the rising limb, the instability is seen on the falling limb. Negative ordinates are present which are not admissible. The response function from the common roots of the storm 2 appears with lots of noise in it. The average of the roots from the storms yielded the response function also shown in Fig. 3. There is no definite improvement in the result.

For the purpose of studying the sensitivity of the solution to the truncation level of the acceptable roots, one pair of roots having the greatest deviation from among the 13 roots accepted earlier was suppressed and the response computed from the remaining 11 roots for the storms is given in Fig. 4. The response can be seen to consist of noise only.

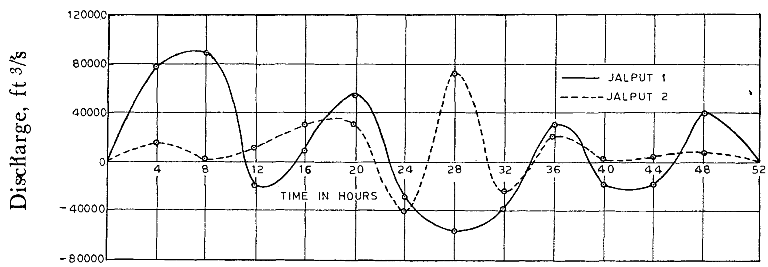


Fig. 4.
Jalput-1,2 (Synthesis with 11 roots, Unit duration 4 hours).

With the increased size of the catchment and its other characteristics, many roots have to be matched. Any shift in the allowable percentage of deviation can produce anything from a meaningful unit graph to a noise which cannot be used at all. This seems to be the critical drawback of the method.

Although a definite limit with respect to the area of the catchment, allowable

2*

error in runoff measurement and tolerance while matching roots cannot be recommended at this stage, work is under progress in this direction.

PROPOSED PROCEDURE FOR SYNTHESIS OF BASIN RESPONSE

Raghavendran and Jayarami Reddy (1971) developed the correlation between the basin characteristics and the parameters of the response function in the form proposed by Edson (1957) as a gamma function, the theory of which is given below.

If the translation and storage effects are considered together, the function can be expressed as $Q \propto T^x e^{-yT}$ where Q = the output at time T and x and y are two parameters. For a unit input the expression can be derived to be

$$Q \equiv \frac{CNy}{\sqrt{(x+1)}} (yT)^x e^{-yT}$$

where $C \equiv 242/9$ the conversion constant, N the watershed area in $mi.^2$ and T the time in days. The parameters are related to the peak and the time to peak which can both be obtained from just the output. So the response function can be described by the characteristics of the output alone, though the unit duration of the input has to be estimated. The duration of the storms has been found from trial and error so that the correct duration gives the least hunting in the S curve and the equilibrium discharge compares well with the inflow rate. The list of the storms in a few catchments in the Krishna-Godavary river basins are given in Table 1. The standard duration of 2 hours is adopted for the synthesis. The perfect reproduction of known hydrographs is shown, leading to the conclusions that the proposed response function is quite valid to represent basin response.

The function compares itself very well with the function proposed by Nash, in that the parameters x and y are analogous to k and n. The input is instantaneous in the case of the Nash Model while it has a standard duration in the present investigation.

CORRELATION OF X AND Y WITH BASIN CHARACTERISTICS

Taking the parameter $LL_c/\sqrt{s}$, where L = length of the main stream, $L_c \equiv$ the distance from the outlet to a point on the stream nearest to the centroid of the basin measured along the stream and s = the slope of the main stream, the equation of the best fit in the range of available data is found to be

$$x \equiv 1.087 \; (LL_c/\sqrt{s})^{0.164}$$

$$\text{and } y \equiv 7.32 \; (LL_c/\sqrt{s})^{-0.158}$$

Table 1.
Correlation of x and y with catchment characteristics.

Catchment	Area ($mi.^2$)	Date of storm	Q_p of 2 hour-UH (ft^3/s)	$q_p = Q_p/A$ $ft^3/s/mi.^2$)	T_p (hr)	x	y	$LL_c/\sqrt{s}$
Cherla	1280.0	14.5.1969	21600	16.90	24	2.6	2.7	570.0
Palleru Bridge	1130.0	19.9.1965	17700	15.65	20	1.6	2.0	760.0
		27.6.1967	17000	15.00	28	3.0	2.5	
		29.8.1967	17400	15.40	32	4.0	2.8	
		24.7.1966	25300	22.60	24	4.3	4.8	
		13.7.1965	22500	19.90	24	3.5	3.6	
		12.1.1966	19800	17.40	24	2.9	2.7	
Jalput Dam	755.0	2.6.1968	14600	19.30	30	4.6	4.1	237.0
		15.3.1967	16900	21.90	20	3.4	3.8	
Salivagu	196.0	1.9.1964	3220	16.40	18	1.5	2.0	40.0
		23.9.1964	2400	17.30	20	2.0	2.4	
Laknawaram Lake	116.4	25.8.1967	1330	11.50	26	1.7	1.4	29.4
Pakhal Lake	104.8	22.9.1964	1800	17.30	20	2.0	2.4	26.6

Table 2.

Characteristics of a few American watersheds.

Name of the Watershed	Area (mi.²)	$LL_c/\sqrt{s}$	1 hour unit hydrograph		x	y	Date of storm
			Q_p (cf ³/s/mi.²)	T_p (days)			
Oxford, Mississippi W-35	11.8	2.03	364	0.0417	2.2	53	May 22, 1959
Oxford, Mississippi W-12	35.6	4.40	277	0.0625	2.6	43	August 31, 1961
Oxford, Mississippi W-17	50.2	6.60	230	0.0834	3.2	39	August 31, 1961
Oxford, Mississippi W-34	117.2	24.50	167	0.1200	3.6	30	August 31, 1961

VERIFICATION OF THE SYNTHESIS PROCEDURE

A few of the storms with known input and output data from Oxford, Missisipi watersheds have been taken. The x and y parameters are obtained as

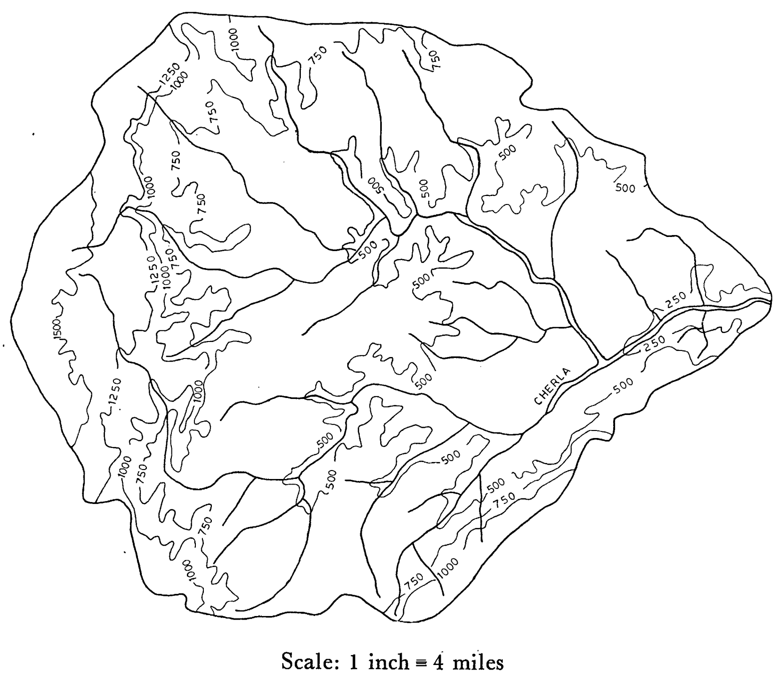
$$x = 1.95\ (LL_c/\sqrt{s})^{0.206}$$

$$\text{and } y = 62.4\ (LL_c/\sqrt{s})^{-0.236}$$

The details of the catchment and the storms are given in Table 2. The observed and computed hydrographs are shown by Raghavendran and Jayarami Reddy (1971) to conform very satisfactorily.

CONCLUSIONS

The method proposed by Delaine has intrinsic difficulties as the size of the basin increases. Further work is under progress to obtain some definite limitations for the application of the method.



Scale: 1 inch = 4 miles

Fig. 5.
Cherla Basin.

Under the assumption of the basin being a linear time invariant system, the response function could be easily synthesised with the procedure proposed by the authors for all sizes. The response function could also be computed for ungauged streams in the region.

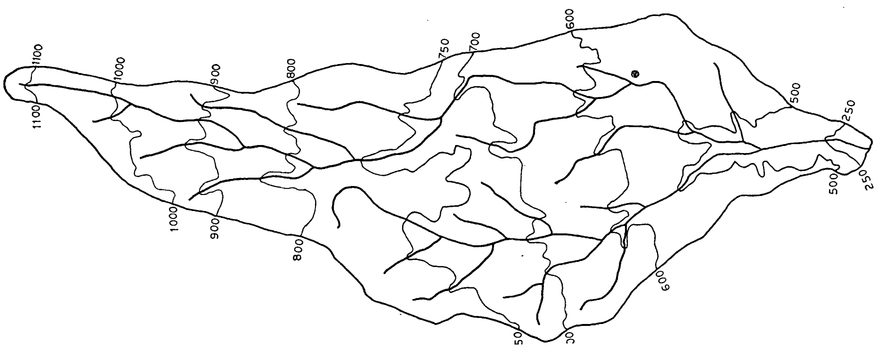


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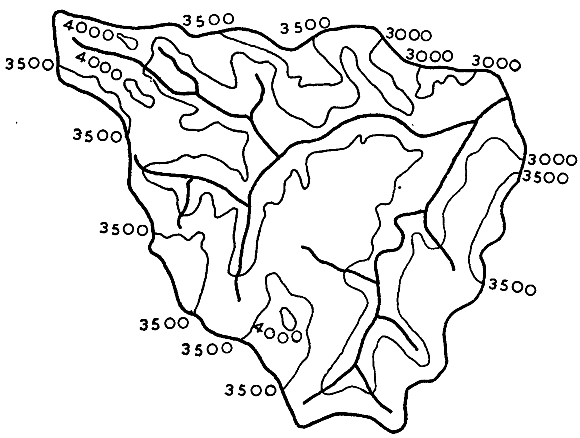
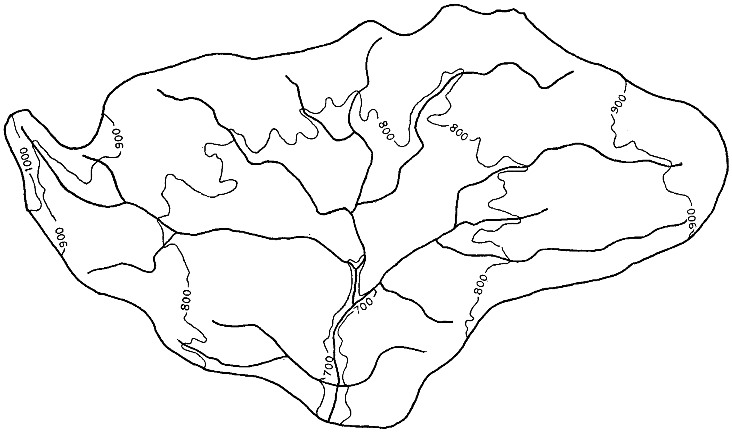
Fig. 6.
Palleru Bridge basin.



Scale: 1 inch = 10 miles

Fig. 7.
Jalput dam basin.

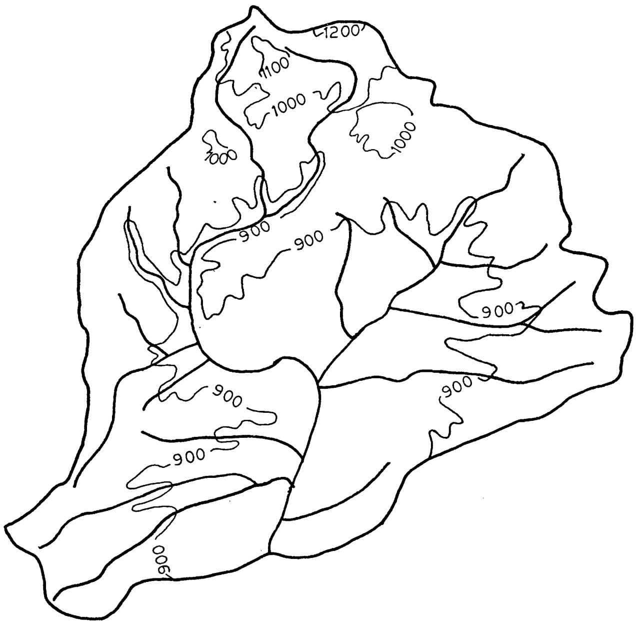


Scale: 1 inch ≡ 2 miles

Fig. 8.
Salivagu Basin.

Scale: 1 inch ≡ 2 miles

Fig. 9.
Laknawaram lake basin.



Scale: 1 inch = 2 miles

Fig. 10.
Pakal Lake basin.

ACKNOWLEDGEMENT

The authors wish to thank Dr. P. G. Sastry, Professor in Civil Engineering, Regional Engineering College, Warangal, for his constant encouragement and constructive suggestions in the preparation of this paper.

REFERENCES

Delaine, R. J. (1970) Deriving unit graph without using rainfall data, *J. Hydrol., Vol.* X, No. 4, pp. 379–390.

Edson, C. G. (1957) Parameters relating unit graph to watershed characteristics, *AGU* Vol. 31, pp. 591–596.

McSparran (1968) Design Hydrographs for Pennsylvania Watersheds, A.S.C.E., *J. Hydraul. Div.*, Vol. 94, No. HY3, pp. 937–960.

Raghavendran, R. and Jayarami Reddy, P. (1971) Synthesis of system response for small watersheds in Krishna-Godavary river Basins, International Symposium in Water Resources, Bangalore.

Wu, I. P. (1963) *Design Hydrographs for small watersheds in Indiana, A. S. C. E., J. Hydraul. Div.,* Vol. 89, No. HY6, pp. 35–66.

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Received May, 1973.