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The Replacement Effect and Comparisons of Per Capita Income Across Countries: A Short Note

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This article is intended to clarify two aspects of Leibenstein's replacement effect that have not been adequately addressed in print. First, formal analysis of the replacement effect demonstrates that it is not determined by the quality of entering and departing workers alone but, rather, by the entire age schedule of labor quality. Second, a simple model is used to demonstrate the absence of welfare implications associated with the replacement effect and to quantify the magnitude of the replacement effect.

The recently released report on population growth and economic development sponsored by the National Academy of Sciences (NAS) (National Research Council, 1986) updated many of the changes in thinking that have occurred in the 15 years since the publication of *Rapid Population Growth: Consequences and Policy Implications* (Leibenstein, 1971). In that study, Leibenstein pointed out, quite correctly, that the average quality of the labor force will vary with the population growth rate if the quality of individual workers is systematically related to their age. Calling the phenomenon the *replacement effect*, he stated that "to the extent that entrants into the work force are of higher quality than those who leave through retirement or death, the average quality of the labor force improves more rapidly if the rate of population growth is higher (other things equal) rather than lower" (p. 188). The "Son of NAS" reiterates the existence of the replacement effect while noting that it is a transitional phenomenon and that "the welfare implications of Leibenstein's argument are unclear" (p. 60).[1]

Many would challenge the specific issue of whether the more recent education and greater vitality of young workers enhances their productivity more than greater experience enhances the productivity of old workers. This article, however, is intended to clarify other aspects of the replacement effect that have not been adequately addressed in print. First, the direction of the replacement effect is not determined by the quality of entering and departing workers alone but, rather, by the entire age profile of labor quality. Second, a simple model is used to demonstrate the absence of welfare implications associated with the replacement effect and to quantify the magnitude of the replacement effect.

The first point is easily demonstrated by using well-known mathematics (for applications see Arthur and McNicoll, 1978; Lee, 1982; Mason, 1981; Preston, 1982). Define terms as follows:

$Q(a)$ = quality of workers aged a;[2]
$L(a)$ = number of workers aged a;
n = rate of growth of the labor force;
D = duration of employment.

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Designating the age of entry to the labor force as 0, the average quality of the labor force is defined as

$$\bar{Q} = \int_0^D L(a)Q(a)\,da \Big/ \int_0^D L(a)\,da. \tag{1}$$

Analysis is confined to stable populations, and with no loss in generality, it is assumed that all workers survive to age D.[3] Average quality is given by

$$\bar{Q} = \int_0^D e^{-na}Q(a)\,da \Big/ \int_0^D e^{-na}\,da. \tag{2}$$

Taking the derivative with respect to the population growth rate n, we have

$$\frac{\partial \bar{Q}}{\partial n} = \bar{Q}[A^L - A^Q], \tag{3}$$

where $A^L = \int ae^{-na}\,da / \int e^{-na}\,da$ is the average age of workers and $A^Q = \int ae^{-na}Q(a)\,da / \int e^{-na}Q(a)\,da$ is the average age of workers weighted by their quality. The percentage change in average quality equals the difference between the two mean ages. It is in this sense, then, that population growth leads to higher average quality if young workers are more skilled than old workers, or conversely, that population growth leads to lower average quality if young workers are less skilled. The sign and magnitude of the *replacement effect* depend on both the quality and the number of workers at every age, not just at entry and departure ages.[4]

The second point is that changes in per capita income related to the replacement effect do not mirror changes in the standard of living; they are purely compositional. Consider two countries. In both, entering workers earn $Y(0)$ and, because older workers have more experience, those aged a earn $Y(a) = e^{\lambda a}Y(0)$. Thus as far as income goes, any individual is as well off in one country as he or she would be in the other.

Per capita income in country A is given by

$$\begin{aligned}(Y/L)^A &= \int_0^D e^{\lambda a}Y(0)e^{-na}L(0)\,da \Big/ \int_0^D e^{-na}L(0)\,da \\ &= Y(0) \int_0^D e^{(\lambda-n)a}\,da \Big/ \int_0^D e^{-na}\,da \\ &= Y(0)\,\frac{n}{\lambda - n}\,\frac{e^{(\lambda-n)D} - 1}{1 - e^{-nD}}.\end{aligned} \tag{4}$$

Per capita income in country B can be determined in a similar fashion, with country B's population growth differing from A's. The ratio of country A's per capita income to country B's per capita income is calculated and given in Table 1 with the duration of employment, D, set at 50 years and the rate of population growth for country B set at 0. As is evident from the table, the average income of workers varies substantially in response to changes in the age structure. Nonetheless, workers of the same age are paid identically in both countries, there

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Table 1. Per Capita Income in Country A Divided by Per Capita Income in Country B

	n			
λ	0.00	0.01	0.02	0.03
0.02	1	0.96	0.92	0.88
0	1	1	1	1
−0.02	1	1.04	1.08	1.12

are no gains to migration, and lifetime incomes are identical in both countries. Quite clearly, the replacement effect is a compositional effect that confounds analysis of the relationship of population growth to the standard of living. Ideally, per capita income comparisons between countries or between alternative computer simulations should be purged of such age structure effects if the impact of population growth on standards of living is to be assessed accurately.

Although the values of Table 1 indicate the possibility of a significant replacement effect, it is unlikely that it accounts for the great improvements in labor quality observed in today's developing countries. Advances in education, health, and nutrition; technological innovation; and other factors that bear directly on the labor quality of individual workers have undoubtedly played a more significant role. What, then, is the relationship between changes in average quality and the age schedule of quality for its members in a more dynamic setting? And how can changes in average quality associated with the replacement effect be distinguished from changes associated with shifts in the age schedule of quality?

Because of the comparative statics emphasis of the analysis carried out here, the likelihood that labor quality is growing over time is ignored. This simplification is easily rectified if quality at each age grows at a constant rate, ξ, that is, the entire age schedule of quality shifts up uniformly (in percentage terms) each year. In this case, the "cross-sectional" age schedule is proportional to the base year values, $Q(a)$. The path of quality for each *cohort* is proportional to $e^{\xi a}Q(a)$ and average quality grows at rate ξ. The replacement effect, which is a one-time shift in average quality accompanying transition from one demographic equilibrium to another, is still quantified by equation (3) because the average ages of labor, weighted and unweighted, are independent of the rate of growth of labor quality.

Notes

[1] *Population Growth and Economic Development: Policy Questions* (National Research Council, 1986) calls the replacement effect transitional because the *rate of growth* of labor quality depends on population growth only during transition from one demographic equilibrium to the next. The replacement effect is permanent, however, in the sense that the average *level* of labor quality differs between two populations with stable but different rates of population growth.

[2] The variation in quality by age may depend on a number of factors: (1) initial skill endowment, (2) gains from experience, (3) depreciation associated with aging, and if workers are not perfectly substitutible, (4) the magnitude of other factors of production and (5) the relative size of labor cohorts.

[3] Equation (1) can also be further generalized to allow across the board increases in labor quality associated, for example, with labor's augmenting technological change. None of the conclusions are affected by such a generalization.

[4] Given sufficiently restrictive assumptions, Leibenstein's (1971) characterization of the replacement effect is adequate. If the age quality schedule is monotonic, as might realistically be the case for average years of schooling in developing countries, the Leibenstein condition is sufficient. But for more realistic characterizations of quality, which include experience and depreciation of human capital, a more general formulation is clearly desirable.

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References

Arthur, W. B., and G. McNicoll. 1978. Samuelson, population and intergenerational transfers. *International Economic Review* 19:241–246.

Lee, R. D. 1982. Age structure, intergenerational transfers and economic growth: An overview. *Revue Economique* 33:1129–1155.

Leibenstein, H. 1971. The impact of population growth on economic welfare—Nontraditional elements. Pp. 175–198 in *Rapid Population Growth, Consequences and Policy Implications*. Baltimore, Md.: National Academy of Sciences.

Mason, A. 1981. An Extension of the Life-Cycle Saving Model and Its Application to Population Growth and Aggregate Saving. East–West Population Institute Working Papers 4, East–West Population Institute, Honolulu, Hawaii.

National Research Council, Working Group on Population Growth and Economic Development. 1986. *Population Growth and Economic Development: Policy Questions*. Washington, D.C.: National Academy Press.

Preston, S. H. 1982. Relations between individual life cycles and population characteristics. *American Sociological Review* 47:253–264.

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