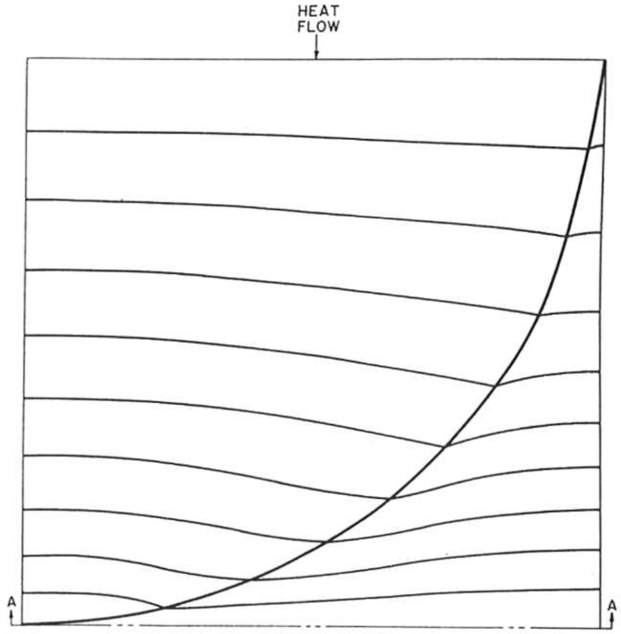
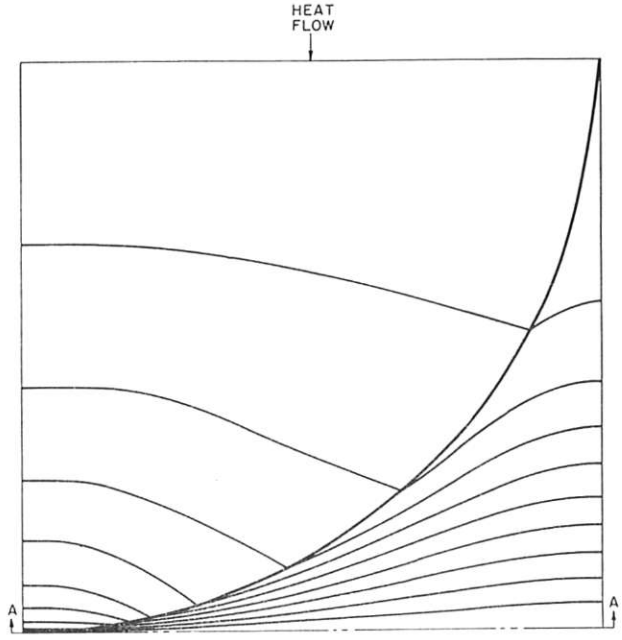


(a) k_s/k_g, 3



(b) k_s/k_g, 30

Fig. 9 Constant temperature lines in representative sample of spheres in cubical array as obtained by relaxation solution

Bibliography

1 "Reaction Rate, Heat Transfer, and Temperature Distribution in Fixed-Bed Catalytic Converters," by R. H. Wilhelm, W. C. Johnson, R. Wynkoop, and D. W. Collier, *Chemical Engineering Progress*, vol. 44, no. 2, 1948, pp. 105–116.

2 "Investigation of Effective Thermal Conductivities of Powders," by R. G. Deissler and C. S. Eian, NACA RM E52C05, 1952.

3 "Effective Thermal Conductivities of Magnesium Oxide, Stainless Steel, and Uranium Oxide Powders in Various Gases," by C. S. Eian and R. G. Deissler, NACA RM E53G03, 1953.

4 "Measured Effective Thermal Conductivity of Uranium Oxide Powder in Various Gases and Gas Mixtures," by J. S. Boegli and R. G. Deissler, NACA RM E54L10.

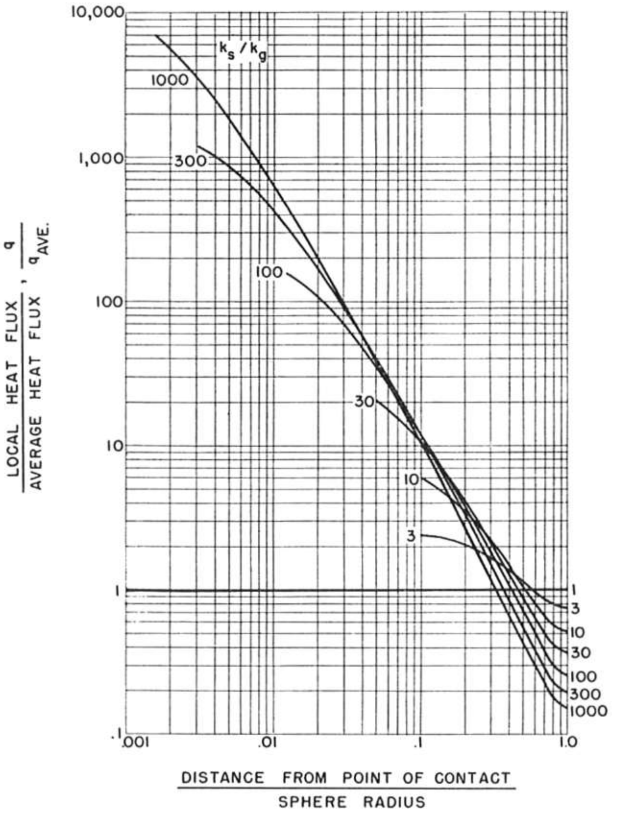


Fig. 10 Variation of ratio of local heat flux (per unit area) to average heat flux across plane A-A in Fig. 9 for various values of k_s/k_g

5 "Heat Flow Through Granulated Material," by T. E. W. Schumann and V. Voss, *Fuel in Science and Practice*, vol. 13, no. 8, 1934, pp. 249–256.

6 "The NBS-NACA Tables of Thermal Properties of Gases. Table 2.42 Dry Air Thermal Conductivity," by R. L. Nuttall, December, 1950.

7 "The NBS-NACA Thermal Properties of Gases. Table 19.42, Argon Thermal Conductivity," by R. L. Nuttall, July, 1951.

8 "A Summary of Viscosity and Heat-Conduction Data for He, A, H_2, O_2, N_2, CO, CO_2, H_2O, and Air," by F. G. Keyes, Trans. ASME., vol. 73, 1951, pp. 589–596.

9 "Thermal Conductivities of Gases," by F. G. Keyes, Trans. ASME, vol. 76, 1954, pp. 809–816.

10 "The Thermal Conductivity of Ceramic Refractory Materials. Its Calculation Based on the Thermal Conductivity of the Components," by A. Eucken, *VDI Forschung*, vol. 353, 1932. (Translated by General Electric Company, July 11, 1950).

11 "The Thermal Conductivity of Metals at High Temperature," by C. L. Hogan and R. B. Sawyer, *Journal of Applied Physics*, vol. 23, no. 2, 1952, pp. 177–180.

12 "Thermal Conductivity: X, Data for Several Pure Oxide Materials Corrected to Zero Porosity," by W. D. Kingery, J. Francl, R. L. Coble, and T. Vasilos, *Journal American Ceramic Society*, vol. 37, no. 2, Part II, Thermal Conductivity, February, 1954, p. 108.

13 "Measurements of the Heat Conductivity of Nitrogen-Carbon Dioxide Mixtures," by F. G. Keyes, Trans. ASME, vol. 73, 1951, pp. 597–601; discussion, pp. 601–603.

14 "On the Conductivities of Certain Heterogeneous Media for a Steady Flux Having a Potential," by C. H. Lees, *Philosophical Magazine*, vol. 49, 1900, pp. 221–226.

Discussion

B. E. Short.[6] The authors are to be congratulated for their paper. Possibly it is of particular interest to the writer since we were involved in a very brief and somewhat crude experimental

---

[6] Professor of Mechanical Engineering, The University of Texas, Austin, Texas. Mem. ASME.

study of this sort during the past summer with a fibrous insulating material. Our study was a comparison of the effective or apparent thermal conductivity of a fibrous material for high temperatures (1200 to 1500 F) used with air and with helium. Our study showed that the effective thermal conductivity of this material with helium was from 1.75 to 2.5 times that for the same material with air when at a temperature of 1000 to 1400 F.

The minimum conductivity to which the authors refer, that of parallel plates of the solid material (perpendicular to heat flow) separated by layers of the gas, according to Nusselt and quoted by Jakob, implies that the effective conductivity of the material should be less dependent on the type of gas as the temperature of the material increases. In the region of 800 to 1500 F this radiation effect should be appreciable. It is thought that the radiation effect should not be greatly affected by the type of system employed; that is, whether parallel plates or granular. Fig. 5(*a*) of the paper, for magnesium oxide, shows that the ratio of effective conductivity with helium to that with air is slightly above 4 at 200 F and slightly less than 3.5 at 800 F. Fig. 5(*b*) for uranium-oxide powder shows a ratio with helium to that with nitrogen of less than 2 at 200 F and slightly greater than 2 at 1200 F.

Nusselt's analysis with the flat plates would indicate a large reduction in the ratio with gases of like thermal-conductivity ratios over the temperature range of 200 to 1200 F.

It would be interesting to learn of the authors' evaluation of the difference between their results and the Nusselt analysis.

D. G. Stephenson[7] **and W. Woodside.**[7] The authors are to be commended on their clear and complete presentation of a fine piece of work. Although their concern is with heat transfer from nuclear reactor fuel elements, the results of their study are of interest in other fields of research. At the Division of Building Research of the National Research Council of Canada we have been studying heat flow and temperature distribution in soil particles. In this we also considered the idealized model of spherical particles stacked in cubical array and have determined, by experiments on a model, values of K/K_g for three values of K_s/K_g.

The model consisted of four hemispheres of marble of radius $3 \pm {}^1/_{64}$ in., sandwiched between two plates of ${}^3/_8$-in-thick aluminum, these plates representing the isothermal planes through the centers of the spheres and the points of contact, respectively. The edges were closed in by ${}^1/_8$-in-thick lucite. The thermal conductivity of the marble (K_s) was determined by guarded hot-plate tests on two slabs of marble from the same source as the hemispheres. By filling the void space in the model with different materials of known conductivity (K_g), different values of the ratio K_s/K_g could be realized. The thermal conductivity K of the model was determined with an 18 × 18-in. guarded hot-plate apparatus, one of the aluminum face plates of the model fitting exactly over the 12 × 12-in. test area of the hot plate. Values for K_s/K_g of 6.4, 30, and 80 were obtained by using as fill materials dry sand, vermiculite, and silica aerogel, respectively. The corresponding values of K/K_g were found to be 3.2, 6.2, and 10.3. The maximum possible error in these values is estimated at less than 5 per cent.

Since these results differed from the authors' calculated results shown as the solid line in Fig. 8, we have tried to check by relaxation one point on their curve. For $K_s/K_g = 80$, the results in Fig. 11 were obtained. These data lead to a value for K/K_g of 8.2, whereas Fig. 8 shows a value of approximately 7.8.

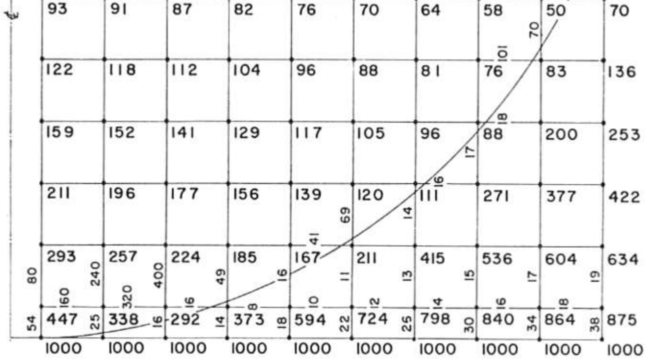


Fig. 11 Temperature distribution in a sphere which forms part of a cubical array, with $K_s/K_g = 80$. The numbers at each nodal point are the temperatures, the numbers along the grid lines are the conductances.

It would be of interest to know why our relaxation gives a larger value of K than is obtained by the authors, and also why the relaxation solutions differ so much from the experimental results.

Authors' Closure

The authors wish to thank Messrs. Short, Stephenson, and Woodside for their interesting discussions. In connection with Professor Short's remarks concerning the comparison of the present experiments and the analysis of Nusselt, it appears that the difference in the results is due to the simplified model and radiation effect considered by Nusselt. Although Nusselt's parallel plate model indicates that we might have appreciable radiation effects in some cases, his model overestimates the relative importance of radiation. In the actual case, where the powder is made up of particles which are in contact, rather than parallel plates separated by the gas, the heat transfer by conduction will be much greater due to the contacting particles, and the heat transfer by radiation will be lower due to the fact that the form factors will be lower than for parallel plates. A rough estimate indicated that the radiation effect in the experiments probably would not amount to more than a few per cent in the worst case, so that the trends predicted by Nusselt should not apply to the present experiments. In the case of the experiments on fibrous materials mentioned by Professor Short, it is possible that Nusselt's parallel plate model might give a closer approximation than it did for the powder particles, so that in that case Nusselt's trends might apply.

In comparing Messrs. Stephenson's and Woodside's relaxation solution with that of the authors, it appears that the agreement of the two sets of calculations to within 5 per cent is probably satisfactory. In order to obtain more accurate solutions, it would be necessary to use finer grids. It should be mentioned that the grid lines used by the authors were all shifted half a grid width from those in Fig. 11. This difference will not, of course, affect the results for sufficiently fine grids.

It is difficult to say why the experimental results of Messrs. Stephenson and Woodside should differ so much from the calculated relaxation solutions. The following suggestions are offered only as possibilities. As mentioned in the paper, the high

---

[7] Building Services Section, National Research Council, Ottawa Canada.

heat flux concentration near the point of contact of the spheres at high ratios of k_s/k_g makes the effective conductivity sensitive to slight irregularities near the point of contact. Thus if the marble or the filling material happened to be inhomogeneous near the point of contact or if the surface were slightly irregular at that point, the conductivity could be affected. Slight external pressure might increase the measured conductivity by producing a finite area of contact between the spheres as in the case of the authors' electrical analog mentioned in the paper before the summary of results. The aluminum plate near the point of contact with the sphere might not be isothermal due to the high concentration of heat flux at that point. It also seems possible that heat loss from the sides of the model or through the lucite plates could have affected the measured values, although a detailed analysis would be required to determine whether or not those effects could be significant.