The unidirectional subsurface shear stress, \( \tau_{uB} \), is considerably less for a hollow ball along the axis of symmetry.

**Interpretation of Life Estimates**

6 Estimates of the dynamic fatigue life of the hollow ball vary considerably depending upon the criterion or *decisive stress amplitude* which is selected. The 33.9 times life increase estimate for the hollow ball when using the maximum unidirectional subsurface shear stress, \( \tau_{uB} \), is non reasonable. It is not logical to expect that a weaker hollow ball (without inertia loadings) with significant bending stresses could be capable of such a life increase.

7 A reduction in the contact fatigue life for a hollow ball is not unreasonable; however, the 91.6 percent reduction estimate when using the Lundberg-Palmgren theory based upon the maximum subsurface shear stress, \( \tau_{s} \), is difficult to accept without confirming experimental data. The slight 9.3 percent reduction in contact fatigue life when using Hertz contact stress, \( \tau_{c} \), as the *decisive stress amplitude* is attractive but very optimistic.

8 Hollow ball bending stresses, especially the variation of stress on the inner wall surface, and repeated cyclic stresses at the hollow ball bond, are sources of competing modes of failure and cannot safely be neglected.

**Concluding Remarks**

The stress which should be used as the *decisive stress amplitude* has long been a subject of discussion among rolling bearing researchers. Classical solid-ball-type contact fatigue studies have not resolved this question since classical elastic half-space contact solutions of linear-elastic theory yield maximum values of the various subsurface stresses which are proportional to the Hertz contact stress. The subsurface stresses in this paper vary widely in their relation to the surface Hertz stress and this type of contact of a hollow ball with a flat plate offers a unique opportunity to conduct fatigue life tests and shed additional light upon the most acceptable *decisive stress amplitude* for rolling element bearing fatigue studies.

The results of the specific analysis described in the paper should not be interpreted to imply that there are no advantages to be gained from the use of hollow balls in special rolling element bearings. The drastic life-reduction estimates do indicate the significance of the hollow ball bending stresses upon the complex three-dimensional stress field in the vicinity of the contact. Additional analytical studies and experimental verification is indicated before optimum use of hollow balls can be anticipated in practical engineering applications. The thin wall of the hollow ball used in this analysis is of academic interest but of little practical interest.

**Acknowledgment**

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**DISCUSSION**

**Eugene F. Finkin**

The area of contact and stresses resulting from hollow ball contact is a subject of growing importance, and the authors work may do much to inspire interest in the subject.

Many people have been under the impression that the mechanics problem, of a sphere having a concentric spherical cavity, had been solved analytically with the complete stress distribution given in series form by Golecki. Unfortunately, this is not the case. Pih and Vanderveldt have shown that Golecki's solution oscillates for a ratio of radial distance to outer radius greater than 0.5, and is actually divergent near the outer boundary.

Pih and Vanderveldt's experimental photoelastic technique is of great practical utility, and seems to have good applicability to the hollow ball bearing problem. It is suggested that workers in this subject consider the possibility of using it.

The differences in stress distribution, shown in Fig. 4, may be more apparent than real. The present work does not give a check on the errors inherent in this approximate type of computation. Until this is done one cannot come to conclusions based on differences between the two distributions. One way to check the solution is to calculate a hollow ball whose cavity goes to zero volume (i.e., the solid ball solution) and compare this to the Hertz solution.

**D. F. Greby**

The original and main concept of using hollow balls is to reduce the high stress level in the outer race due to centrifugal force of the balls, when operating at high speeds. In addition the centrifugal force component of the applied load results in a very high stress, high torque, etc.

Computer analysis has shown that there would be many advantages in the use of hollow balls in high speed bearings. Decrease in fatigue life, lower ball to spin to roll ratio and reduction in torque. All these concepts are valid providing that the fatigue life of a hollow ball is not lower than that of a solid ball.

A thorough and complete analysis of the mode of failure of hollow balls is desperately needed at this time. This paper made a substantial step in the direction of obtaining a more thorough understanding of the stress levels experienced in hollow balls, and expected mode of failure. That is, the two competing modes of failure, surface fatigue and inner fiber stress fatigue, related to the wall thickness of a hollow ball.

**Chief Tribologist, Colt Industries, Fairbanks Morse, Inc., Power Systems Division, Beloit, Wis.**


**Chief Engineer, Industrial Tectonics, Inc., Compton, Calif.**
This paper deals with the results of one case and has not established a general case for the relationship of inner fiber stress and Hertz stress relative to wall thickness.

The sub-surface fatigue of hollow balls in our experimentation shows in the worst case a reduction in life from solid balls of 37.5 percent, when tested in a single ball test machine. In a four ball fatigue test machine, the life of the solid and hollow balls was almost identical.

We are extremely interested and will investigate the relief of the Hertz in the zero axis location of the stress pattern. In a normal curved raceway/ball relationship that exists in a ball bearing, it is highly probable the relief will not occur, changing the orthogonal shear stress. In addition the stress level studied (500,000 psi) was much greater than normally seen in turbo-machinery bearings (250,000 psi).

It is my opinion that the extreme interest in the use of hollow balls for high speed bearings should require additional analysis in the development of a general case for use by Engineers to determine the proper wall thickness for various low levels as experienced by a bearing. Normal Hertz stress level in turbo-machinery ball bearings is approximately 200,000 mean Hertz stress, not the 500,000 psi range at which this analysis was performed. In addition this general case should be confirmed by more experimentation and a method should be developed to confirm the theory for checking the actual stress level developed for stresses within a hollow ball.

J. Y. Liu

The authors present in this paper the numerical results of a digital computer program which uses the finite-element method to determine the stresses and displacements in the contact between a hollow ball and a half space. While the finite-element method has been used by many authors in solving problems of axisymmetric solids, its application to a contact stress problem is still new.

In the paper only one set of solutions corresponding to one specific case (ball size, wall thickness, load, and number and arrangement of elements) has been presented. It will be useful if the authors could produce solutions of at least three appropriate cases so that convergence of the stiffness matrix built in the computer program can be established. The specific load selected for the solutions given is quite high (maximum Hertz stress of 580 ksi) compared to normal rolling bearing practice, particularly at very high speeds when hollow balls are needed to reduce centrifugal loading.

It is interesting to note that the solution given for the hollow ball indicates that the maximum Hertzian stress does not occur at the center of the contact and that its magnitude is even higher than that of a solid ball of the same size and under the same load. In fact, the shape of the stress profile given in Fig. 4 of the paper is quite peculiar. It is noted that in the authors' reference [5], (Fig. 8), these contact stresses do not go to zero at the contact edge, as shown in Fig. 4 of the paper, and the stress profiles are shown much smoother at greater depths below the surface (Figs. 9–15 of the authors' reference [5]). Is it possible, therefore, that this stress distribution peculiarity is a spurious effect of the finite-element method used?

The comparison of the fatigue life between the hollow ball and the solid ball bearing would reflect greater life for the hollow balls if provision was made for the reduced centrifugal loading and increased loading zone of the hollow ball bearing, which must result in a much lower maximum contact load than that of a solid ball bearing.

H. E. Munson

The authors are to be commended for developing a technique to determine shear stresses in a hollow ball. Previous work with the hollow ball concept has limited itself to weight saving, flexibility, or the effect of the ball on the fatigue life of bearing rings. Information relating to the fatigue endurance life of the ball itself is highly significant.

The authors do not attempt to estimate endurance life from bending fatigue, but note that this mode of failure cannot be safely neglected. We believe that consideration must be given to the possibility that the mechanisms of rolling contact fatigue and bending fatigue may be not simply competing, but interacting. If this proves to be the case, it may severely limit the permissible hollowness in balls for high speed bearings. A substantial amount of test data is necessary to determine if an interaction does exist, as well as to establish which one of the life estimates from this analysis is realistic.

We question the statement that "engine life requirements of these anticipated speeds [3.0 X 10^9 DN] cannot be met by present solid ball or conventional ball bearing designs," because it implies that bearings with solid balls will not perform satisfactorily at 3.3 million DN. Actually, present-day bearings have operated at "3 million DN," and we believe it is premature to say that they cannot meet engine life requirements. There is, of course, a problem in this area because of the ambiguity included in the definition of "DN," which makes no provision for cross-section variations for a given bore diameter. We suggest instead the use of "TAC factor," DN/TAC, as a much more reliable means of evaluating a bearing's capability of running at very high speed.

H. W. Schibbe and H. H. Coe

In this paper the authors have determined, using a finite-element method and a high-speed digital computer, a numerical solution for the complex problem of three-dimensional stress distribution between two contacting elastic bodies. From the results, it appears that the finite-element method can be a useful tool for determining the stresses and deflections in high-speed bearing components, such as hollow balls. Of particular interest was the determination of the subsurface stress gradient and the magnitude of the stresses at the ball inner diameter. Such determinations have heretofore eluded solution.

The analyses presented herein, however, were limited to a hollow ball in contact with a flat plate. For this contact geometry the problem is more easily solved since the three-dimensional case reduces to a two-dimensional solution through the property of axisymmetry. A typical ball-race contact in a high-speed bearing has nonaxisymmetric geometry and, therefore, does not readily lend itself to solution by the finite-element method. A true three-dimensional solution is required which is far more difficult. Furthermore, the solution may exceed the capacity of the largest digital computers or may not be practical in terms of required computer time. In the writers' opinion, therefore, a three-dimensional solution for the stress distribution in a ball-race contact is beyond the present capability of the finite-element method described herein.

For purposes of comparing fatigue life, which is based on either Hertz compressive or subsurface shear stress, the authors used a 1000-pound contact load for both the hollow and solid ball cases. This seems unrealistic since the main advantage for
using hollow rather than solid balls in a high-speed bearing application is that of reducing outer-race contact load by lowering ball centrifugal force. Reducing contact load, of course, lowers the Hertz compressive stress and consequently fatigue life increases. Or, stated another way, lower inertia hollow balls permit a higher bearing speed (higher DN values) for the same contact load.

At the writers' laboratory, a current high-speed program uses 75 mm ball bearings with \( \frac{11}{16} \)-in. dia hollow balls. The OD/ID ratio of 1.2 is the same as for the 1-in. dia ball analyzed in this paper; therefore, a weight reduction of approximately 56-percent per ball is realized for both sizes. If the 75 mm bearing with the \( \frac{11}{16} \)-in. hollow balls were operating at a high speed (30,000 rpm) and load condition such that the outer-race contact load were 1000 pounds, use of a solid ball would result in a contact load of 1250 pounds. Using Hertz compressive stress as the criteria for fatigue life, a 1250-lb solid ball load results in a 50-percent reduction in fatigue life from that of the hollow ball.

This discussion indicates that more work has to be done before the finite-element method of stress analysis can be applied to the problem of hollow ball-race contact loading in an actual high-speed bearing application.

Authors' Closure

The authors appreciate the efforts of the discussers and their obvious interest in this initial analytical effort. All of the comments are constructive and will be helpful in the proposed additional work, which all concerned realize will be necessary.

It should be kept in mind that the analytical effort was a first approach to answer in a definitive way what the nature of the stress fields are when a hollow ball is pressed against a flat plate. No effort was made to simulate a bearing race. It was felt that this could come later, once the analytical techniques had been developed.

Initiation of the analytical effort was stimulated as a result of fracture failures of 3\( \frac{1}{2} \)-in.-dia balls in large bearing applications (Air Force search radar antenna azimuth bearings). Because of the difficulty of quench hardening a large diameter ball, it was felt that a hollow ball would offer better response to heat treatment and load condition such that the outer-race contact load were 1250 pounds. Using Hertz compressive stress as the criteria for fatigue life, a 1250-lb solid ball load results in a 50-percent reduction in fatigue life from that of the hollow ball.

The authors, however, take exception to the discussers' remarks concerning a three-dimensional solution of a typical ball-groove contact. The finite element type of representation is very general and does readily lend itself to the solution of a nonaxisymmetric geometry, such as a ball in a grooved bearing race. It is true that practical difficulties would be encountered at the present state-of-the-art because such a three-dimensional solution would require considerable costly computer time. Additional work in this area will undoubtedly reduce the required computational time to an acceptable level.

Mr. Munson's remarks concerning the probable interacting nature of the mechanisms of rolling contact fatigue and reversed bending fatigue are pertinent and most probably correct. The paper considers these two mechanisms to be competitive, in the sense that we are presently predicting useful life by two different failure theories. Present day bearings in test rig have reached \( 3.0 \times 10^6 \) DN, but none are currently in flight operation. The actual value of the limiting DN number is not as important as the fact that such a limitation will soon be reached. A ball bearing may be capable of operation at \( 5.0 \times 10^6 \) DN operation, but it must also exhibit a satisfactorily long endurance life. The DN designation is generally accepted and is useful in relating past experience with similar bearings, both as to size and type of application, lubrication, etc. The MRC TAC factor has not been widely accepted to date.

Dr. Liu's remarks regarding the finite-element analytical techniques are correct in questioning the Hertz stress profile. It is possible that there were some spurious effects or numerical instabilities at the contacting surface elements. Experimental verification of the stress profile of Fig. 4 would be most welcome. The noted smoothing-out of the subsurface stress patterns at greater depths lends confidence in their values. The shape of the actual surface stress curve may contain some error; however, the values and depths of the two maximum subsurface shear stresses are accurate and these are the stresses of interest in fatigue life calculations. Additional solutions are needed.