

A fast approach for multiobjective design of water distribution networks under demand uncertainty

S. Sun, S.-T. Khu, Z. Kapelan and S. Djordjević

ABSTRACT

Water distribution system (WDS) design has received much attention lately from the point of view of uncertainties. Designers are generally interested in the Pareto optimal cost-robustness trade off curve. This paper aims to find a solution to the multiobjective problem in a computationally time-efficient way in comparison to previous methods from the literature. A parameter θ , which is linked to the system robustness through a derived analytic formula, is introduced. The robustness of the WDS can be approximated by one single model simulation; consequently a large amount of computational time is saved compared to using a sampling-based technique. The application of the method to the New York tunnels problem demonstrates that, although the resulting design is conservative on cost, the proposed method is very computationally efficient. This is of importance when high computational cost is the major obstacle for some real-world problems.

Key words | fast approach, multiobjective optimization, New York tunnels strengthening problem, robustness, uncertainty, water distribution system

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INTRODUCTION

Water distribution system (WDS) design is one of the most researched areas in water engineering. It is a complex optimization process involving a trade off between cost and robustness. Since a number of uncertainties are involved in the decision-making process (one of the most important uncertain quantities is the water demand), the need for considering uncertainties in WDS design problem is obvious (Kapelán *et al.* 2005).

Over the past few decades, in many cases, the WDS design has been formulated to minimize the cost without an uncertainty/robustness consideration. Schaake & Lai (1969), Alperovits & Shamir (1977) and Costa *et al.* (2001) solved the problem using non-evolutionary optimisation methods. Evolutionary algorithms especially genetic algorithms (GAs) are found to be very effective in solving this discrete and nonlinear problem (Halhal *et al.* 1997; Savic & Walters 1997).

Other than cost, robustness, which is about maintaining the required level of water service, is another important

issue in WDS design. Generally, there are two categories of robustness according to different drivers that cause an inadequate supply of water to customers. The first one is driven by pipe failure such as a pipe burst or pipe breakage. Rowell & Barnes (1982) and Kettler & Goulter (1983) proposed to address the problem by ensuring the network is looped. The second driver is the possible variations in water demands or uncertain network parameters. The discussion of this paper belongs to the latter category and considers water demand variations as the main source of the uncertainty. Previous research has been done within this category: the stochastic, single-objective least cost WDS design was first formulated and solved by Lansey *et al.* (1989) considering nodal heads as random normally distributed variables. Bao & Mays (1990) treated nodal pressures as functions of random nodal demands and pipe roughness and used Monte Carlo simulation (MCS) to estimate the robustness. Xu & Goulter (1999) developed an optimization model for robustness design of WDS using

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the first-order reliability method (FORM) for robustness evaluation and generalized reduced gradient (GRG2) for optimization. Babayan *et al.* (2005, 2007) presented redundant design and integration design approaches to solve the problem. “Safety margins” were added to uncertain parameters to transform the problem into a deterministic optimization problem in the redundant approach. The integration method includes within the objective function the influence of uncertainties on the system robustness.

All of the above robust WDS design methodologies, however, share one common limitation: the optimal design problem is solved to give a constrained single-objective design. Therefore, to identify the whole Pareto optimal cost-robustness trade off curve, which is normally of interests for decision makers, a series of single-objective optimisation problems need to be solved (Kapelán *et al.* 2005). Considering computational efficiency, Farmani *et al.* (2003) suggested using a multiobjective optimisation to solve the robust WDS design problem and recommended the Non-dominated Sorting GA-II (NSGAI) method (Deb *et al.* 2000) for WDS design problems. Kapelán *et al.* (2005, 2006) developed the multiobjective optimization framework to identify the optimal robust Pareto fronts of minimizing the cost and maximizing the robustness. The methodology is fundamentally a double loop process where a sampling loop is located within the optimization loop. The Robust NSGAI (RNSGAI) and Latin hypercube sampling (LHS) are respectively employed as the optimization method and the sampling method. The sampling method has the merit that it can easily handle water demands following any probability density function (PDF) forms including correlated demands. However, sampling based methods, even though more universal, straightforward and accurate, are significantly more time consuming. As Halder & Mahadevan (2000) and Zhao & Ono (2001) have highlighted, sampling methods are typically several orders of magnitude slower than analytical approaches, even when advanced sampling methods such as stratified sampling, LHS and so on are used.

Optimization efficiency is currently an essential issue in the robust WDS design optimization. The high computational cost, associated with the system simulation and optimization process, remains the major obstacle in real-world applications, especially for complex and large-scale

design problems. This paper aims to propose a new fast approach to address the multiobjective WDS design problem. Making use of the analytic analysis, the new approach approximates the network robustness by a single WDS model simulation. It is therefore considerably more computationally efficient than using sampling based methods which require thousands of model simulations.

MULTIOBJECTIVE DESIGN PROBLEM DEFINITION

The multiobjective design of a WDS under water demand uncertainty is considered under the following assumptions: (1) the network configuration (i.e. pipe layout, connectivity, etc.) is known; (2) minimum pressure head constraints at pipe junctions (nodes) are given; (3) the diameters of the pipes are decision variables; (4) uncertain nodal demands are assumed to be independent random variables with normal distributions.

For assumption (4), due to the limitation of the analytical derivation, the fast approach is constrained to the resolution of problems where water demands follow normal distributions. When correlated water demands need to be taken into account, the Nataf transformation and Cholesky decomposition (Meulchers 1999) can be implemented to make the transformation from dependent variables to independent variables. Therefore, this assumption does not invalidate the proposed procedure.

The two objectives in WDS design considered are: the minimization of total cost and the maximization of robustness.

Robustness of the network can be defined as the ability of the WDS to provide adequate water supply to customers despite fluctuations in nodal water demands. Failure is considered based on a criterion of insufficient heads, assuming that nodal demands are always met. A node that does not satisfy its minimum pressure head constraints is regarded as a failure. In general, two different robustness definitions are employed. One is the probability of all nodes in the network simultaneously satisfying their minimum pressure head constraints, which is given as

$$P(H_j \geq H_j^{\min}, \quad \forall j = 1, 2, \dots, m) \quad (1)$$

where $P(\cdot)$ is the probability of some event; H_j and H_j^{\min} are the head and the minimum allowable head at node j , respectively; and m is the number of nodes. The second definition is from a nodal perspective, following Xu & Goulter (1999). Robustness is calculated for every node in the network and the smallest value is adopted as the system's robustness, which is given as:

$$\min \left(P(H_j \geq H_j^{\min}) \right) \quad (j = 1, 2, \dots, m) \quad (2)$$

Formula (1) measures the reliability of the whole system assuming that failure of a single node can lead to the failure of the whole network, while Formula (2) assumes that a nodal failure is a local failure. In this paper, Formula (2) is used for robustness calculation due to the analytic derivation limitation. Generally, the robustness from the system view is smaller than that from the individual node view. The reason for this is: the failure violating a certain nodal pressure constraint definitely causes a failure from the system view, but the inverse is not always true.

The multiobjective optimization problem is formulated as follows:

$$\min f(D_1, D_2, \dots, D_n) = \sum_{i=1}^n c(D_i, L_i) \quad (3)$$

$$\max \left(\min \left(P(H_j \geq H_j^{\min}) \quad j = 1, 2, \dots, m \right) \right) \quad (4)$$

where $f(\cdot)$ denotes a function of decision variables (diameters D_i , chosen from a discrete set of available diameters), $c(D_i, L_i)$ is the cost of pipe i with diameter D_i and length L_i , n is the number of pipes in the system, the diameters of which need to be decided.

The water distribution network system should satisfy the continuity equations at all nodes and the energy conservation equations around each elementary loop:

$$\sum Q^{\text{in}} - \sum Q^{\text{out}} = Q \quad (5)$$

$$\sum h_f - \sum E_p = 0 \quad (6)$$

where Q^{in} is the flow into a junction; Q^{out} is the flow out of a junction; Q is the demand at a node; h_f is the pipe head loss; and E_p is the energy input to the system by a pump. h_f has the term of the Hazen-Williams formula,

which is a nonlinear function of pipe diameters and volumetric demands:

$$h_f = \omega \left(\frac{Q}{C} \right)^a \frac{L}{D^b} \quad (7)$$

where ω is the numerical conversion constant depending on the units used; C is the pipe Hazen-Williams roughness coefficient. The chosen constants a and b are 1/0.54 and 2.63/0.54 respectively.

METHODOLOGY

Framework for solving the problem

Figure 1 shows the framework of the proposed approach for solving the multiobjective optimization problem, which is based on a multiobjective optimization loop. In this paper, the NSGAII is applied as the multiobjective optimizer. The fitness, which denotes how good each candidate network is with respect to the objectives, needs to be evaluated for all the candidate networks. As a result robustness evaluations are performed in the optimization loop. Instead of evaluating robustness by traditional sampling-based methods, a parameter θ , which has a one-to-one relationship with robustness, is introduced and approximated by an analytic formula. In this way the network robustness can be identified by one deterministic simulation of the network flow.

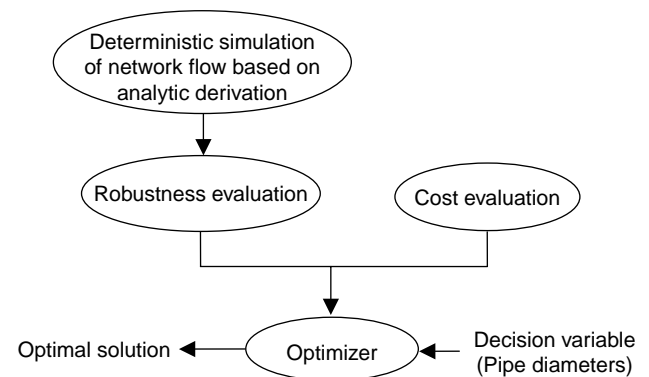


Figure 1 | General framework for WDS design under uncertainty.

Approximation of network robustness

This section derives the analytical expression of the network robustness to consider the probability expressed objective (4).

From the energy conservation Equation (6), the pressure head H_j at node j can be written as:

$$H_j = H + E_p - \Delta H - \sum_{i=1}^m h_f^i \quad (8)$$

where H is the original water head at source, ΔH is the altitude difference between the source and the pipe node j ; m is the number of pipes located upstream of the j th node. In a looped network, whether a node is upstream of another node can be judged from the simulation of flow direction.

As the node water demands \tilde{Q} are uncertain variables, from Equation (7), the head loss h_f along a pipe is:

$$h_f^i = \varpi \left(\frac{\tilde{Q}_i}{C} \right)^a \frac{L_i}{D_i^b} \quad (9)$$

\tilde{Q}_i is the water quantity flowing through the i th pipe and is a sum of several node water demands. $\tilde{Q}_i = \sum_{j=1}^m \tilde{Q}_j$, where m is the total number of nodes downstream of the pipe. Since \tilde{Q}_j follows a normal distribution, the sum \tilde{Q}_i follows the same distribution.

For any i , make (subscript i is omitted for writing briefness):

$$\tilde{q} = \frac{\tilde{Q} - \bar{Q}}{\bar{Q}} \quad (10)$$

where \bar{Q} is the mean of variable \tilde{Q} . So \tilde{q} is a normally distributed variable with mean 0 and variance $\sigma_{\tilde{q}} = \sigma_{\tilde{Q}}/\bar{Q}$. Generally the variance $\sigma_{\tilde{q}}$ is a value much smaller than 1.

Substituting Equation (10) into (9) gives:

$$h_f = \varpi \left(\frac{\tilde{Q}}{C} \right)^a \frac{L}{D^b} = \varpi (1 + \tilde{q})^a \left(\frac{\bar{Q}}{C} \right)^a \frac{L}{D^b} \quad (11)$$

Applying Taylor's first order expansion to Equation (11) at $\tilde{q} = 0$, h_f is approximated as a linear function of \tilde{q} :

$$h_f = \varpi (1 + \tilde{q})^a \left(\frac{\bar{Q}}{C} \right)^a \frac{L}{D^b} \approx k + ka\tilde{q} + O(\tilde{q}^2) \quad (12)$$

where $k = \varpi (\bar{Q}/C)^a (L/D^b)$ for expression clarity.

Substituting (12) into (8) gives:

$$H_j = H + E_p - \Delta H - \sum_i (k_i + ak_i\tilde{q}_i) \quad (13)$$

Then for any node j , the robustness in Formula (4) becomes:

$$P \left(H + E_p - \Delta H - \sum_i (k_i + ak_i\tilde{q}_i) \geq H_j^{\min} \right) \quad (14)$$

$$j = 1, 2, \dots, m$$

with a newly introduced coefficient θ' , adding a redundancy $\theta'\sigma_{\tilde{q}}$ on the mean \tilde{q} to replace \tilde{q}_i in the inequality in (14), the following inequality is formed:

$$H + E_p - \Delta H - \sum_i (k_i + \theta'ak_i\sigma_{\tilde{q}_i}) \geq H_j^{\min} \quad (15)$$

By letting the WDS network satisfy the inequality (15), the robustness in (14) can be approximated as:

$$P \left(H + E_p - \Delta H - \sum_i (k_i + ak_i\tilde{q}_i) \geq H_j^{\min} \right)$$

$$\geq P \left(H + E_p - \Delta H - \sum_i (k_i + ak_i\tilde{q}_i) \geq H + E_p - \Delta H - \sum_i (k_i + \theta'ak_i\sigma_{\tilde{q}_i}) \right)$$

$$\geq P \left(\sum_i k_i\tilde{q}_i \leq \theta' \sum_i k_i\sigma_{\tilde{q}_i} \right) = P \left(\frac{\sum_i k_i\tilde{q}_i}{\sqrt{\sum_i k_i^2 \sigma_{\tilde{q}_i}^2}} \leq \theta' \frac{\sum_i k_i\sigma_{\tilde{q}_i}}{\sqrt{\sum_i k_i^2 \sigma_{\tilde{q}_i}^2}} \right)$$

$$= P \left(Z < \theta' \frac{\sum_i k_i\sigma_{\tilde{q}_i}}{\sqrt{\sum_i k_i^2 \sigma_{\tilde{q}_i}^2}} \right) = \Phi \left(\theta' \frac{\sum_i k_i\sigma_{\tilde{q}_i}}{\sqrt{\sum_i k_i^2 \sigma_{\tilde{q}_i}^2}} \right) \quad (16)$$

where Z represents a variable applying to the standard normal distribution $N(0,1)$, Φ is the cumulative density function (CDF) of the standard normal distribution

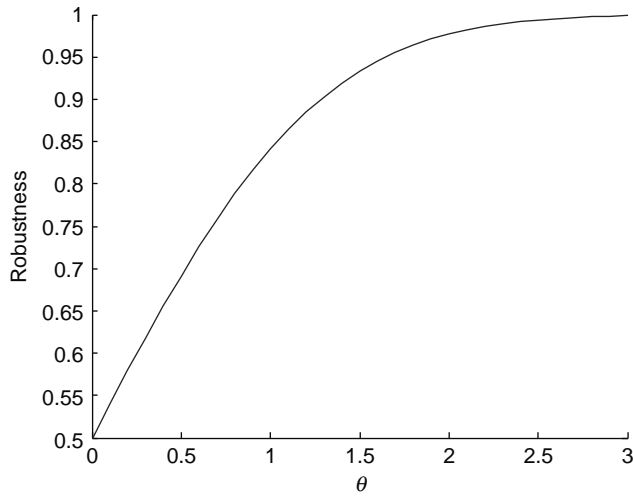


Figure 2 | Theoretical relationship between robustness and θ .

$\Phi(\theta) = (1/\sqrt{2\pi}) \int_{-\infty}^{\theta} \exp(-z^2/2) dz$. Let

$$\theta = \theta' \frac{\sum_i k_i \sigma_{\bar{q}_i}}{\sqrt{\sum_i k_i^2 \sigma_{\bar{q}_i}^2}} \quad (17)$$

Figure 2 shows the one-to-one relationship between the robustness φ and the parameter θ . It is a part of the CDF curve of the standard normal distribution. To implement this robustness calculation using the analytic analysis, a water demand redundancy $\theta' \sigma_{\bar{q}_i} \bar{Q}_i$ is added to the average value \bar{Q}_i at every node i and the WDS is then simulated. All redundancies are under the same-ratio-to-variance redundancy assumption, i.e. the redundancies added on the water demands of all nodes are obtained by multiplying their variance with the same ratio θ' . The node with the smallest surplus of water head is chosen as the critical node that determines the network robustness under node view. θ is calculated by (17) along the path from the water source to the critical point and mapped to the network robustness.

However, $\sigma_{\bar{q}_i}$ in Equation (16) is the variance of the sum of several nodes' demand. It is smaller than the sum of the variance of the nodes $\sigma_{\sum \bar{q}_i} \leq \sum \sigma_{\bar{q}_i}$. Therefore the added redundancy node water demand will make the design conservative, i.e. the "real robustness" of the designed system is greater than the robustness mapped from θ . In a looped WDS, there are several possible paths from the water source to the critical node. The robustness presented

by θ is only an approximate value which may change with the chosen path according to (17). Considering that the robustness evaluation is generally conservative according to the previous analysis, the largest value of θ s from all possible paths is adopted in the optimization.

Procedure for fast approach of WDS design

Utilising the robustness obtained by the analytic derivation, the whole procedure of solving the WDS design problem is mainly a loop of NSGAIII optimisation. The hydraulic performance of the WDS network is simulated by EPANET (Rossman 2000). The procedure is as follows:

- (1) Create the initial GA population randomly: the decision variables include pipe diameters and θ' . Let θ' belong to the interval $[0, 3]$, considering $\theta' < \theta$

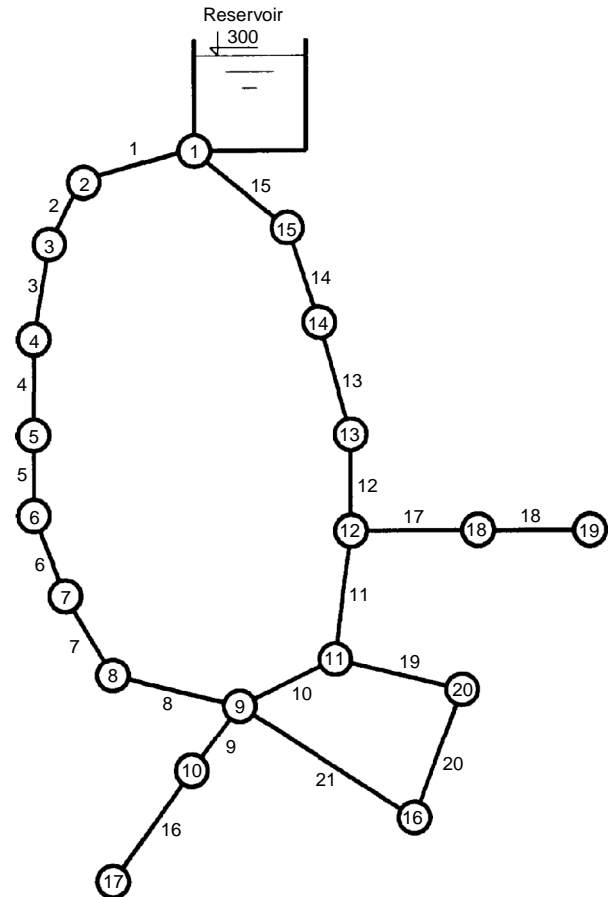


Figure 3 | Layout for New York City tunnel water network.

Table 1 | Main characteristics and parameters within NSGAI

| Population | Generations | Selection | Genetic operator | Crossover rate | Mutation rate |
|------------|-------------|---|---|----------------|---------------|
| 200 | 500 | Tournament selection (Parent chromosome = 100; Tournament number = 2) | Simulated binary crossover & Polynomial mutation | 0.9 | 0.1 |

according to (17). When $\theta = 3$ the robustness corresponds to 99.87% (sufficiently high).

- (2) For each chromosome, run the WDS simulation once: the nodal water demands are simulated as $\bar{Q}(1 + \theta'\sigma_{\bar{q}})$ for each node. Evaluate the fitness of each chromosome by calculating the objective values defined in (3) and (4). The node with the least head surplus is selected and believed to be the critical node with smallest robustness. Instead of using objective (4), θ is calculated from (17). In a looped network, θ s are evaluated for all of the possible paths and the largest θ is chosen as the fitness value. If a head deficit appears at the critical node, both objectives of the cost and θ are given penalty values. θ needs to be maximized according to the requirement of maximizing the robustness.
- (3) Sort the chromosomes using the NSGAI algorithm.
- (4) With genetic operators, create the next generation of chromosomes. Repeat steps 2–4 until a convergence criterion is met.
- (5) Map θ to the robustness with the curve in Figure 2 and finally the Pareto optimal cost-robustness front is obtained.

APPLICATION OF THE METHOD TO THE NEW YORK CITY TUNNEL PROBLEM

The New York City tunnel problem (Schaake & Lai 1969) is a well known case for testing WDS design methods. The original objective of this study was to determine the most economically effective design for additions to the existing system of tunnels, given in Figure 3. The same input data was used in this paper.

To demonstrate the methodology presented in this paper, node water demand distributions are assumed to be normally distributed with means equal to the deterministic

demand values and standard deviations equal to 10% of the corresponding mean values, as adopted by Kaplan et al. (2005).

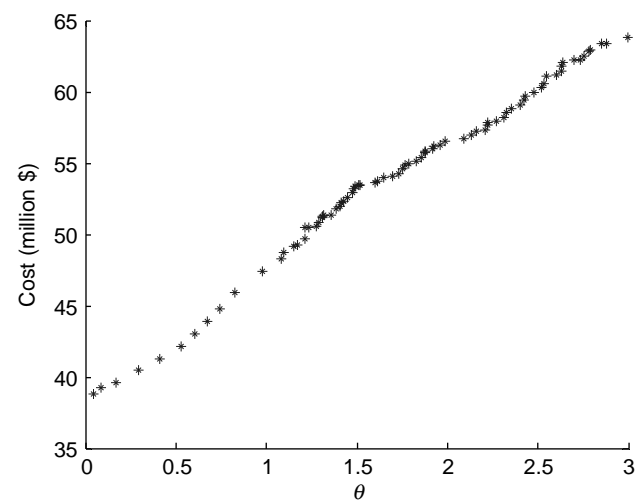
RESULTS AND DISCUSSIONS

General results

The main parameters used in the NSGAI were determined through limited sensitivity analysis. The final adopted parameters as well as the main characteristics are shown in Table 1.

Figure 4 shows the Pareto front of the last generation. When θ equals 0, the problem becomes the minimization of the construction cost with fixed water demands according to (16). The cost of the design obtained from the multi-objective optimization is \$38.8 million, which is consistent with previous work (Murphy et al. 1993).

θ is mapped to robustness to obtain the cost and robustness trade off curve. As the robustness evaluated from

**Figure 4** | Pareto front by proposed fast method.

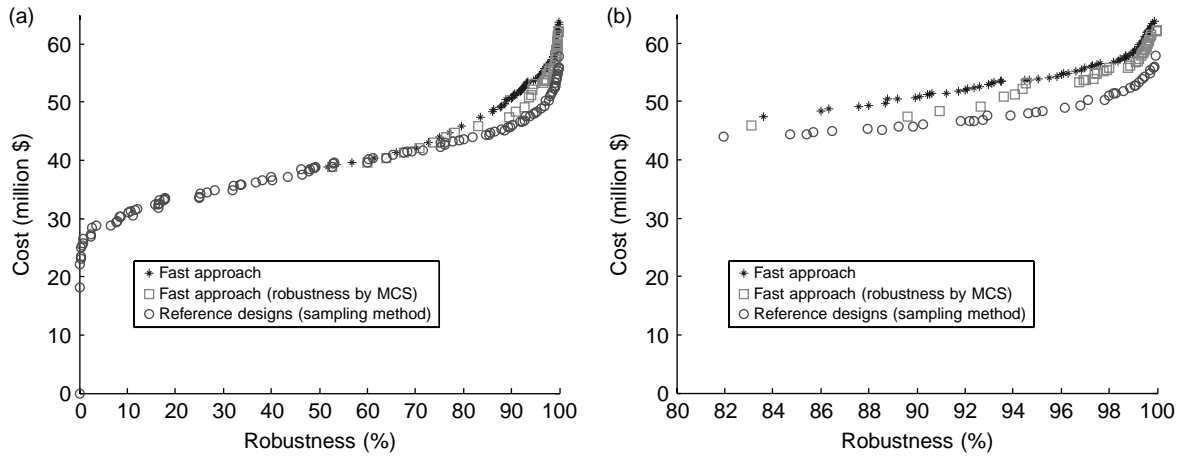


Figure 5 | Pareto front by fast method and its comparison to reference designs.

θ is an approximate value, a MCS of 10^5 times (as used by Kapelan et al. 2005; Babayan et al. 2007) is executed for all the obtained designs in order to calculate the more accurate robustness for comparison. The reference Pareto front of minimization of the cost and maximization of the network robustness is identified by optimization using a pure sampling method (1,000 LHS) to compute the robustness. The robustness of the designs on the Pareto fronts is reevaluated using 10^5 MCS. The Pareto front identified by the fast approach and the front identified by using MCS to estimate the more accurate robustness of designs obtained by fast approach are shown in Figure 5(a) as well as the reference front. The robustness of designs mapped from θ is generally smaller than the more accurate robustness obtained by MCS. This result is in agreement with the conservative nature of the proposed fast method, as previously stated. Since designs with high robustness are usually of interest, Figure 5(b) shows the part of the fronts where the designs have a robustness greater than 80%. Using the pure sampling method in the optimization gives the front with the robustness varying from 0 to 1, while the fast approach utilising the analytic analysis only presents designs of robustness greater than 50%. This is due to the fact that in the optimization involving the analytic analysis, the introduced parameter θ is required to be positive, which corresponds to the robustness greater than 50% according to Equation (16) or Figure 2. The fast approach gives more expensive designs than when using a full sampling method, partly because of the conservativeness introduced by the

mathematical derivation in Equation (16) and partly due to the same-ratio-to-variance redundancy assumption as Babayan et al. (2007) described for the redundancy method. The approach used to turn the WDS design problem under uncertainties into a deterministic problem, by adding a same-ratio-to-variance redundancy to each node, may cause bias of the network capacity. This is due to the fact that demand fluctuations at different nodes exert different effects on the system robustness, depending on the system design characters. However, this behaviour is difficult to identify.

Table 2 | Optimal robust solutions with fast approach and full sampling method

| Pipe | Duplicated pipe diameter D_i (cm) | | | | | |
|-------------------|-------------------------------------|-------|-------|----------------------|-------|-------|
| | Fast method | | | Full sampling method | | |
| | 90% | 95% | 99% | 90% | 95% | 99% |
| 1–6 | – | – | – | – | – | – |
| 7 | – | – | – | – | – | 274 |
| 8–13 | – | – | – | – | – | – |
| 14 | – | 305 | 427 | – | – | – |
| 15 | 518 | 427 | 427 | 457 | 518 | 488 |
| 16 | 305 | 244 | 213 | 244 | 244 | 274 |
| 17 | 244 | 244 | 244 | 244 | 274 | 274 |
| 18 | 244 | 244 | 244 | 244 | 213 | 213 |
| 19 | 213 | 152 | 213 | 152 | 183 | 183 |
| 20 | – | – | – | – | – | – |
| 21 | 183 | 213 | 183 | 213 | 213 | 213 |
| Cost (10^6 \$) | 50.58 | 53.96 | 57.32 | 46.05 | 48.87 | 52.79 |

Table 3 | Optimal robust solutions from literature methods

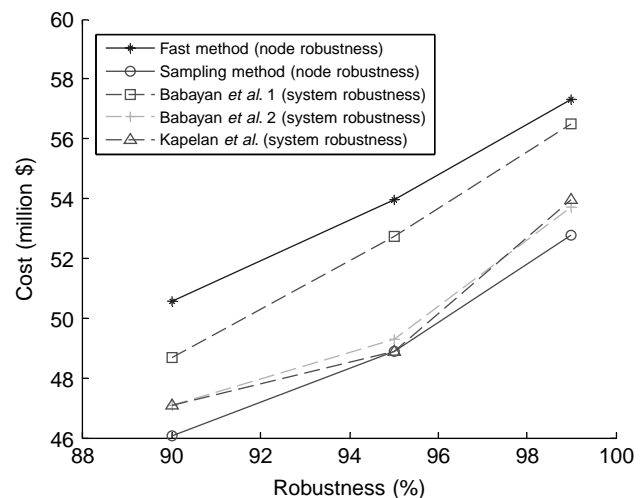
| Pipe | Duplicated pipe diameter D_i (cm) | | | | | | | | |
|-------------------|-------------------------------------|-------|-------|--------------------------------|-------|-------|------------------------------|-------|-------|
| | Babayan <i>et al.</i> (2007) 1 | | | Babayan <i>et al.</i> (2007) 2 | | | Kapelán <i>et al.</i> (2006) | | |
| | 90% | 95% | 99% | 90% | 95% | 99% | 90% | 95% | 99% |
| 1–6 | – | – | – | – | – | – | – | – | – |
| 7 | – | 274 | – | – | – | – | – | – | – |
| 8–13 | – | – | – | – | – | – | – | – | – |
| 14 | – | – | 366 | – | – | 305 | – | – | 335 |
| 15 | 488 | 488 | 396 | 457 | 488 | 427 | 457 | 518 | 396 |
| 16 | 274 | 274 | 244 | 244 | 274 | 213 | 244 | 244 | 213 |
| 17 | 274 | 274 | 274 | 274 | 274 | 274 | 274 | 274 | 274 |
| 18 | 213 | 213 | 213 | 213 | 213 | 183 | 213 | 213 | 213 |
| 19 | 213 | 183 | 244 | 183 | 183 | 213 | 183 | 183 | 183 |
| 20 | – | – | – | – | – | – | – | – | – |
| 21 | 183 | 213 | 183 | 213 | 213 | 213 | 213 | 213 | 213 |
| Cost (10^6 \$) | 48.67 | 52.73 | 56.48 | 47.08 | 49.28 | 53.72 | 47.08 | 48.87 | 53.96 |

For given target robustness levels of 90, 95 and 99%, the designs obtained from the proposed fast method and from the full sampling method are given in Table 2, where the robustness is defined from the node view. Table 3 gives designs from the literature (including two methods from Babayan *et al.* 2007 and one from Kapelán *et al.* 2005). These designs present the same target robustness level but are defined from a system view. Figure 6 presents the cost-robustness trade off curves obtained from different methods. This result is in agreement with the previous analysis regarding the relationship existing between the robustness from the node and system views (section 2) that the robustness from the node view is generally smaller than that from the system view.

Computational efficiency

The attractive advantage of the new proposed approach using analytical derivation to compute the robustness of the WDS network is its computational efficiency. In Kapelán *et al.* (2005), RNSGAI is also employed for saving computational time: the total number of Epanet2 model evaluations was $2N_{\text{pop}}N_{\text{gen}}N_s$, (where N_{pop} is the GA population size, N_{gen} is the number of generations and N_s is the LHS size). While in the present study using the fast

approach resulted in only $N_{\text{pop}}N_{\text{gen}}$ Epanet2 model evaluations. If the additional computational effort required when generating samples for LHS rather than for MCS, as well as the posterior evaluations of the WDS for the designs of the Pareto front are excluded, even for values of N_s considered very small (for example 10), the calculation of Kapelán *et al.* (2005) is about 20 times of that of the proposed fast method. Compared to the full sampling method, the proposed fast method is several orders of magnitude more efficient.

**Figure 6** | Comparison of cost-robustness curves from different methods.

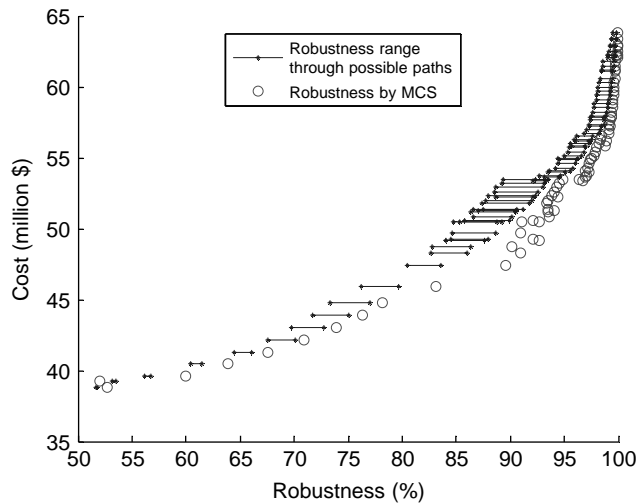


Figure 7 | Robustness represented by θ .

Sensitivity analysis of the chosen path

In the optimization process of the above approach, θ is calculated according to Equation (17) along the path of water travelling from the water source to the critical node. If the WDS is looped, there are several paths available. As the robustness represented by θ is only an approximate value which may change with chosen path from Equation (17), the influence of the chosen path on the robustness represented by θ is now studied. The designs on the Pareto front obtained from the fast approach are used. Figure 7 presents the range of robustness mapped from θ when all the possible paths are computed for each design. The more accurate robustness obtained by MCS is also presented in Figure 7. The largest difference of robustness for the same network obtained using different paths is about 5%. When the robustness of the design is high, for example, more than 90%, the difference is less than 2%. The robustness obtained by mapping from θ is generally less than the more accurate robustness obtained by MCS. In the optimization process, the use of the largest value of θ to evaluate candidate networks is reasonable because it is the closest value to the more accurate value.

CONCLUSIONS

A new fast approach for the multiobjective design of WDS under water demand uncertainty is formulated and

demonstrated using the New York City tunnel case study. By analytical derivation, the robustness is linked to a newly introduced parameter θ , which can be expressed by an analytical formula. The robustness evaluation of the new approach is free of sampling technique; hence it saves a large amount of computational time compared to traditional methods from the literature. NSGAI is employed as the optimizer to solve the multiobjective problem. Although the proposed method identifies somewhat more expensive designs in comparison to previous approaches in the literature, its advantage is attractive. It is computationally efficient, which is of importance especially when a WDS is large and a sampling-based technique for robustness evaluation nested in optimisation is impossible.

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