An optimization model for water resources allocation risk analysis under uncertainty

Y. L. Xie and G. H. Huang

ABSTRACT

In order to deal with the risk of low system stability and unbalanced allocation during water resources management under uncertainties, a risk-averse inexact two-stage stochastic programming model is developed for supporting regional water resources management. Methods of interval-parameter programming and conditional value-at-risk model are introduced into a two-stage stochastic programming framework, thus the developed model can tackle uncertainties described in terms of interval values and probability distributions. In addition, the risk-aversion method was incorporated into the objective function of the water allocation model to reflect the preference of decision makers, such that the trade-off between system economy and extreme expected loss under different water inflows could be analyzed. The proposed model was applied to handle a water resources allocation problem. Several scenarios corresponding to different river inflows and risk levels were examined. The results demonstrated that the model could effectively communicate the interval-format and random uncertainties, and risk aversion into optimization process, and generate inexact solutions that contain a spectrum of water resources allocation options. They could be helpful for seeking cost-effective management strategies under uncertainties. Moreover, it could reflect the decision maker's attitude toward risk aversion, and generate potential options for decision analysis in different system-reliability levels.

Key words | conditional value-at-risk, inexact two-stage stochastic programming, risk analysis, uncertainty, water resources allocation

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INTRODUCTION

Water resources are critical for human survival, and human society would be unable to prosper or even exist without them. The ever-growing conflicting demand for water resources supplies threaten the sustainability of this essential resources recycling. Coupled with rapid increasing water demand, decreasing usable water supplies and poor management have led to inefficient water resources allocation, and the unsustainable use of water resources with significant economic, social, and environmental ramifications. Moreover, in water resources systems, many system parameters and their inter-relationship may appear uncertain. Such uncertainties, that would affect the related exercises for generating desired water resources management schemes, may be caused by the errors in acquired data, variations in spatial

and temporal units, and incompleteness or impreciseness of observed information (McIntyre *et al.* 2003; Maqsood *et al.* 2005). Therefore, it is desired that the uncertainties should be considered in water allocation planning programming.

Over the past decades, inexact optimization models have been widely used to tackle uncertainties and complexities in water resources allocation problems, and a majority of them were based on fuzzy, stochastic, and interval-parameter programming (abbreviated as FMP, SMP, and IPP), as well as their combinations (Slowinski *et al.* 1986; Wagner *et al.* 1994; Huang 1996; Chang *et al.* 1996; Russell & Campbell 1996; Wang & Du 2003; Li *et al.* 2006, 2007; Li & Huang 2008; Cetinkaya *et al.* 2008; Simonovic 2009; Guo & Huang 2010; Xu & Qin 2010; Lv *et al.* 2012). For example, Huang (1998) developed

an interval chance-constraint programming model for water quality management in a Chinese city, which allowed probability distributions and discrete intervals to be incorporated within the optimization process. Jairaj & Vedula (2000) optimized a multi-reservoir system through using a fuzzy mathematical programming method, where uncertainties existing in reservoir inflows were treated as fuzzy sets. Faye et al. (2003) proposed a long-term water resources allocation model for an irrigation management problem of a reservoir system, where the fuzzy logic presented as a particularly adequate means to refine on-line the formulation of the objective function of the recurrent optimization problem. Teegavarapu & Elshorbagy (2005) proposed a fuzzy mean squared error measure to evaluate the performance of time series prediction models in water resources, where membership functions derived from a number of modeler preferences could be easily aggregated to obtain a single integrated membership function. Chaves & Kojiri (2007) applied a stochastic fuzzy neural network model for the optimization of reservoir monthly operational strategies considering maximum water utilization and improvements on water quality simultaneously, where the stochastic fuzzy neural network was defined as a fuzzy neural network model stochastically trained by a genetic algorithm. Zhang et al. (2009) introduced an inexact-stochastic dual water supply programming model for regional water resources management, which was based on analysis of the inexact characteristics in demand and supply subsystems of a dual water supply system and their dynamic interactions. Lu et al. (2010) advanced an interval-valued fuzzy linear-programming method based on infinite α-cuts for an agricultural irrigation problem, where a two-step infinite α-cuts solution method is communicated to the solution process to discretize infinite α-cuts to interval-valued fuzzy membership functions. Tran et al. (2011) developed a stochastic dynamic programming model for reservoir water management strategy planning in southern Vietnam, where multi-users, stochastic water level, the timing and quantity of water release, and climatic conditions were considered. Liu et al. (2012) proposed an interval-parameter chance-constrained fuzzy multi-objective programming model for assisting water pollution control within a sustainable wetland management system, where the proposed approach can effectively handle the uncertainties and complexities in the water pollution control systems. However, in water resources planning practice, when it comes to

the violation of some overriding policies, those methods/models would fail to analyze the economic consequences; also, none of the above methods could facilitate the analysis of various policy scenarios that were associated with different levels of economic penalties when the promised targets were violated in the water resources management process.

Inexact two-stage stochastic programming (ITSP), coupled with two-stage stochastic programming (TSP) and IPP, is an attractive technique to help overcome the above shortcomings. In the ITSP, a decision is first undertaken before values of random variables are known; then, after the random events have happened and their values are known, a second-stage decision can be made in order to minimize 'penalties' that may appear due to any infeasibility (Loucks et al. 1981; Birge 1985). ITSP methods have been widely explored in water resources management in the past decades (Ferrero et al. 1998; Huang & Loucks 2000; Seifi & Hipel 2001; Luo et al. 2003; Maqsood et al. 2005; Li et al. 2007, 2009; Guo et al. 2010; Huang et al. 2010; Wang & Huang 2011). For example, Maqsood et al. (2005) developed an interval-parameter fuzzy two-stage stochastic programming method for water resources systems planning and management under uncertainty. Li & Huang (2008) proposed an inexact two-stage stochastic nonlinear programming model for supporting decisions of water resources allocation within a multi-reservoir system. Wang & Huang (2011) developed an interactive two-stage stochastic fuzzy programming model for water allocation management, where the method can not only tackle dual uncertainties presented as fuzzy boundary intervals, but also permit in-depth analyses of various policy scenarios. Huang et al. (2012) developed an integrated optimization method for supporting agriculture water management and planning in Tarim River Basin, Northwest China, where the method couples ITSP and quadratic programming. In general, ITSP is effective for problems where an analysis of policy scenarios is desired and the related data are random/interval format in nature. However, in the previous study, the minimum cost or maximum net benefit are usually considered as the objective in a general ITSP model, which could lead to the problems of low system stability and unbalanced allocation risk. Most of the models generated by the ITSP methods for water resources management take the system benefit as the objective without considering the risk

aversion, which should also be incorporated in the proposed inexact stochastic programming approach.

Incorporating risk measures in the objective functions within other optimization methods is a fairly recent research topic. An alternative risk measure, namely conditional value-at-risk (CVaR), proposed by Rockafellar & Uryasev (2000), is a widely accepted risk measure (Ahmed 2004; Schultz & Tiedemann 2006; Fábián 2008). The CVaR model is a new risk measurement method based on probability distributions of random variables, and has been widely used for portfolio selection (Kall & Mayer 2005; Klein Haneveld & Van der Vlerk 2006; Schultz & Neise 2007; Liu et al. 2009). Previously, the application of CVaR in the water resources management field has been relatively limited. For example, Piantadosi et al. (2008) developed a stochastic dynamic programming model with CVaR for supporting urban storm water management. Shao et al. (2011) proposed a stochastic dynamic programming model with CVaR constraints for supporting water resources management under uncertainty. Nevertheless, most of the models take the system risk as the constraints, and no previous studies were focused on development of risk-aversion inexact two-stage stochastic programming (RITSP) method through integrating IPP, TSP, and CVaR into a general framework for water resources allocation management with considering the risk aversion in the system objective.

Therefore, the aim of this study is to develop a RITSP method for water resources allocation management under uncertainty. It is the first attempt where IPP, TSP, and CVaR methods are integrated into a general framework of a maximum benefit objective in the water resources allocation problem under uncertainties presented as interval values and probabilities. A case study will demonstrate the performance of the RITSP method in water resources management systems planning under uncertainty. Furthermore, it will be shown how it can be used to generate water allocation policies under a given risk level, as well as to determine which designs can most efficiently lead to the optimized system objectives.

METHODOLOGY

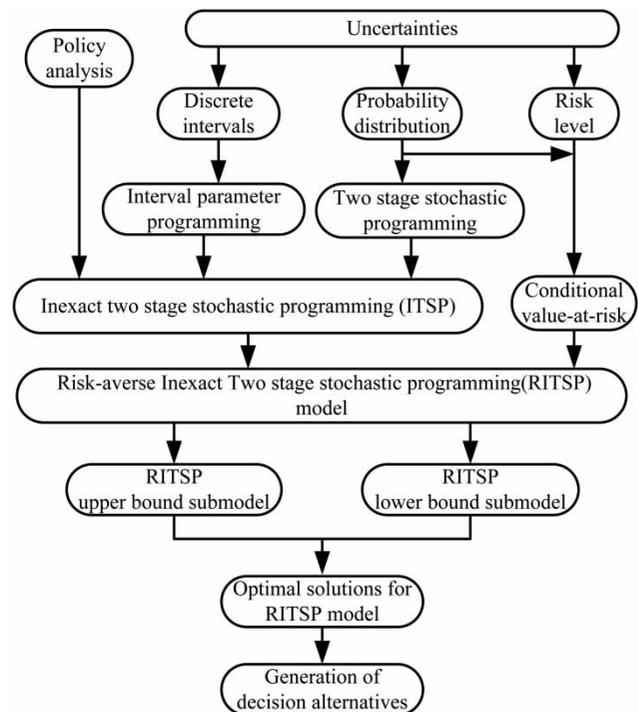An RITSP model was based on IPP, CVaR model, and TSP. Figure 1 presents the general framework of the

Figure 1 │ Framework of the RITSP model.

RITSP method, which is based on IPP, TSP, and CVaR techniques. Each technique has a unique contribution in enhancing the RITSP's capacities for tackling the uncertainties and system risk. For example, the probability distributions and policy implications were handled through TSP; the uncertainties presented as discrete intervals were reflected through IPP; the system risk was addressed by CVaR. The modeling framework would offer feasible and reliable solutions under different scenarios of allocation targets, which are helpful for decision makers (Maqsood et al. 2005).

Two-stage stochastic programming

Consider a typical water resources management system in a region, where a water resources manager is responsible for allocation of limited water to multiple competing users during a planning horizon. The water manager needs to promise each user an allocation target in the management process, which can help the water users make their generation plans. If the promised water is delivered, it will result in net benefit to the local economy and drive the

regional industry development; however, if the promised water is not delivered, the benefit will be reduced, due to the curtailed demand and the imposed penalty. Since the amount of available water is random, this water allocation problem can be formulated as a two-stage stochastic programming with the objective of maximizing the expected value of economic activity in the region.

The general form of TSP problems read: $z(x, \omega) = cx - Q(x, \omega)$, and a TSP model can be formulated as follows (Birge & Louveaux 1997):

$$f = \max cx - E_{\omega \in \Omega}[Q(x, \omega)] \tag{1a}$$

subject to

$$ax \leq b \tag{1b}$$

$$x \geq 0 \tag{1c}$$

where f is the system benefit, x is the first-stage decision of water allocation made before the random variable ω is observed ($\omega \in \Omega$), and c is the benefit coefficients of first-stage variable x in the objective function; a is the technical coefficients, b is right-hand side coefficients, and $Q(x, \omega)$ is the optimal value of the following non-linear programming:

$$\min q(y, \omega) \tag{2a}$$

subject to

$$D(\omega)y \geq h(\omega) + T(\omega)x \tag{2b}$$

$$y \geq 0 \tag{2c}$$

where y is the second-stage adaptive decision, which depends on the realization of the random variable. $q(x, \omega)$ denotes the second-stage cost function, while $\{D(\omega), h(\omega), T(\omega) | \omega \in \Omega\}$ are random model parameters with reasonable dimensions, which are functions of the random variable ω. By letting random variables ω take discrete values with probability levels p_h ($h = 1, 2, \ldots, v$ and $\sum p_h = 1$), the expected value of the second-stage

optimization problem can be expressed as:

$$E_{\omega \in \Omega}[Q(x, \omega)] = \sum_{h=1}^{v} p_h Q(x, \omega_h) \tag{3}$$

For each realization of random variable ω_h, a second-stage decision is made, which is denoted by y_h. The second-stage optimization problem can be rewritten as:

$$\min q(y_h, \omega_h) \tag{4a}$$

subject to

$$D(\omega_h)y_h \geq h(\omega_h) + T(\omega_h)x \tag{4b}$$

$$y_h \geq 0 \tag{4c}$$

Thus, Model (1) can be equivalently formulated as a linear programming model (Ahmed *et al*. 2004):

$$f = \max cx - \sum_{h=1}^{v} p_h q(y_h, \omega_h) \tag{5a}$$

subject to

$$ax \leq b \tag{5b}$$

$$D(\omega_h)y_h \geq h(\omega_h) + T(\omega_h)x \tag{5c}$$

$$x \geq 0 \tag{5d}$$

$$y_h \geq 0 \tag{5e}$$

Risk-averse two-stage stochastic programming

In the TSP, the first-stage decisions are deterministic and the second-stage decisions are allowed to depend on the elementary events, i.e., $y_h = y(\omega_h)$. Basically, the second-stage decisions represent the operational decisions, which change depending on the realized values of the random data. The objective function $Q(x, \omega)$ of the second-stage problem, also

known as the recourse (benefit) function, is a random variable and therefore, the total profit function $z(x, \omega)$ is a random variable. Determining the optimal decision vector x leads to the problem of comparing random profit variables $z(x, \omega)$. Comparing random variables is one of the main interests of decision theory in the presence of uncertainty. While comparing random variables, it is crucial to consider the effect of variability, which leads to the concept of risk. The preference relations among random variables can be specified using a risk measure. One of the main approaches in the practice of decision making under risk uses mean-risk models (Ogryczak & Ruszczyński 2002). In these models, one minimizes the mean-risk function, which involves a specified risk measure $\rho : z \to R$, where ρ is a functional and z is a linear space of F-measurable functions on the probability space (Ω, F, P):

$$\max\{E(z(x, \omega)) - \lambda \rho(z(x, \omega))\} \qquad (6)$$

In this approach, λ is a nonnegative trade-off coefficient representing the exchange rate of mean benefit for risk, and also refers to it as a risk coefficient, which is specified by decision makers according to their risk preferences. Usually, when typical dispersion statistics, such as variance, are used as risk measures, the mean-risk approach may lead to inferior solutions. In order to remedy this drawback, models with alternative asymmetric risk measures, such as downside risk measures, have been proposed (Ogryczak & Ruszczynski 2002), and conditional value-at-risk (*CVaR*) measure which is based on the value-at-risk (*VaR*) was widely applied in many areas to downside risk measures among the popular risk-aversion methods.

VaR is a measure computed as the maximum profit value (e.g., z) such that the probability of the profit being lower than or equal to this value (e.g., l) is lower than or equal to $1 - \alpha$:

$$VaR = \max\{l | p(z \le l) \le 1 - \alpha\} \qquad (7)$$

CVaR at level α, in a simple way, is defined as follows (Rockafellar & Uryasev 2000, 2002):

$$CVaR(z) = E(z | z \le VaR(z)) \qquad (8)$$

VaR has the additional difficulty, for stochastic problems, that it requires the use of binary variables for its modeling. Instead, computation of *CVaR* does not require the use of binary variables and it can be modeled by the simple use of linear constraints. The concept of *CVaR* is illustrated in Figure 2. *CVaR(z)* is the conditional expected value not exceeding the value under the confidence level α. The CVaR at the confidence level α is given by:

$$CVaR_\alpha(z) = \inf_{\xi \in R}\left\{\xi - \frac{1}{1-\alpha}E\big([\xi - z]_+\big)\right\} \qquad (9)$$

where ξ is an auxiliary variable, which is the maximum value at the cumulative probability α.

Thus, Model (6) can be redefined as:

$$\max\{E(z(x, \omega)) - \lambda CVaR_\alpha(z(x, \omega))\} \qquad (10)$$

In addition, $CVaR_\alpha(z + a) = CVaR_\alpha(z) + a, a \in R$ (Birbil *et al.* 2009), therefore, $CVaR_\alpha(z(x, \omega)) = CVaR_\alpha(cx - Q(x, \omega)) = cx - CVaR_\alpha(Q(x, \omega))$, Model (10) can be reformed as the following linear programming problem:

$$\mathrm{Max} f = (1 - \lambda)cx$$
$$- \sum_{h=1}^{v} p_h q(y_h, \omega_h) + \lambda\left\{\xi - \frac{1}{1-\alpha}\sum_{h=1}^{v} p_h V_h\right\} \qquad (11a)$$
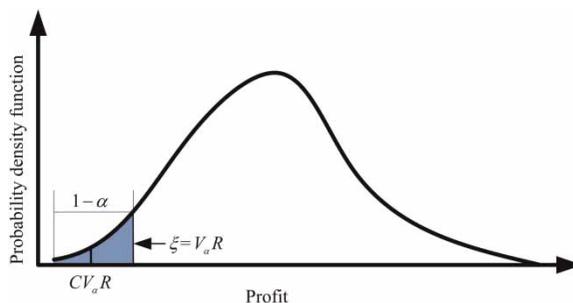
subject to

$$ax \le b \qquad (11b)$$

Figure 2 │ *VaR* and *CVaR* illustration.

$$D(\omega_h)y_h \geq h(\omega_h) + T(\omega_h)x, \quad h = 1, 2, \ldots, v \tag{11c}$$

$$V_h \geq \xi - cx + q(y_{h,}\omega_h), \quad h = 1, 2, \ldots, v \tag{11d}$$

$$V_h \geq 0, \quad h = 1, 2, \ldots, v \tag{11e}$$

$$x \geq 0 \tag{11f}$$

$$y_h \geq 0, \quad h = 1, 2, \ldots, v \tag{11g}$$

Risk-averse inexact two-stage stochastic programming

However, in water resources optimization problems, uncertainty presented as interval numbers is more straightforward than probability density functions (PDFs) due to the poor quality of information that can be obtained (Li et al. 2006). Thus, by introducing the interval parameter programming to quantify those uncertainties presented in terms of interval values, Model (11) can be transformed into the following RITSP model:

$$\text{Max} f^{\pm} = (1 - \lambda)c^{\pm}x^{\pm}$$
$$- \sum_{h=1}^{v} p_h q(y_h^{\pm}, \omega_h^{\pm}) + \lambda\left\{\xi^{\pm} - \frac{1}{1 - \alpha}\sum_{h=1}^{v} p_h V_h^{\pm}\right\} \tag{12a}$$

subject to

$$a^{\pm}x^{\pm} \leq b^{\pm} \tag{12b}$$

$$D(\omega_h^{\pm})y_h^{\pm} \geq h(\omega_h^{\pm}) + T(\omega_h^{\pm})x^{\pm}, \quad h = 1, 2, \ldots, v \tag{12c}$$

$$V_h^{\pm} \geq \xi^{\pm} - c^{\pm}x^{\pm} + q(y_h^{\pm}, \omega_h^{\pm}), \quad h = 1, 2, \ldots, v \tag{12d}$$

$$V_h^{\pm} \geq 0, \quad h = 1, 2, \ldots, v \tag{12e}$$

$$x^{\pm} \geq 0 \tag{12f}$$

$$y_h^{\pm} \geq 0, \quad h = 1, 2, \ldots, v \tag{12g}$$

where $a^{\pm}, c^{\pm}, x^{\pm}, b^{\pm}, y_h^{\pm}, \xi^{\pm}, \omega_h^{\pm}, V_h^{\pm} \in \{R^{\pm}\}$, and $\{R^{\pm}\}$ denotes a set of interval parameters and/or variables; superscript '±' means interval-valued feature; the '−' and '+' superscripts represent lower and upper bounds of an interval parameter/variable, respectively.

Solution of the RITSP model

Model (12) can be transformed into two deterministic sub-models that correspond to the lower and upper bounds of desired objective function value. This transformation process is based on an interactive algorithm, which is different from the best/worst case analysis (Huang et al. 1992). The objective function value corresponding to f^+ is desired first because the objective is to maximize net system benefit. The sub-model to find f^+ can be first formulated as follows (assume that $c^{\pm} \geq 0$, $A^{\pm} \geq 0$, and $b^{\pm} \geq 0$):

$$\text{Max} f^+ = (1 - \lambda)c^+x$$
$$- \sum_{h=1}^{v} p_h q(y_h^-, \omega_h^-) + \lambda\left\{\xi^+ - \frac{1}{1 - \alpha}\sum_{h=1}^{v} p_h V_h^-\right\} \tag{13a}$$

subject to

$$x = x^- + \mu(x^+ - x^-) \tag{13b}$$

$$0 \leq \mu \leq 1 \tag{13c}$$

$$a^-x \leq b^+ \tag{13d}$$

$$D(\omega_h^-)y_h^- \geq h(\omega_h^-) + T(\omega_h^-)x, \quad h = 1, 2, \ldots, v \tag{13e}$$

$$V_h^- \geq \xi^+ - c^+x + q(y_h^-, \omega_h^-), \quad h = 1, 2, \ldots, v \tag{13f}$$

$$V_h^- \geq 0, \quad h = 1, 2, \ldots, v \tag{13g}$$

$$y_h^- \geq 0, \quad h = 1, 2, \ldots, v \tag{13h}$$

where μ and y_h^- are decision variables. The optimal f_{opt}^+, μ_{opt} and $y_{h\,opt}^-$ would be obtained through solving the Submodel

(13), and $x_{opt} = x^- + \mu_{opt}(x^+ - x^-)$ is the optimized first-stage variable, which may correspond to the optimized upper-bound objective function value. Based on the above solutions, the second submodel for f^- can be formulated as follows:

$$\text{Max} f^- = (1 - \lambda)c^- x_{opt} - \sum_{h=1}^{v} p_h q(y_h^+, \omega_h^+)$$
$$+ \lambda \left\{ \xi^- - \frac{1}{1-\alpha} \sum_{h=1}^{v} p_h V_h^+ \right\} \tag{14a}$$

subject to

$$a^+ x_{opt} \leq b^- \tag{14b}$$

$$D(\omega_h^+) y_h^+ \geq h(\omega_h^+) + T(\omega_h^+) x_{opt}, \quad h = 1, 2, \ldots, v \tag{14c}$$

$$V_h^+ \geq \xi^- - c^- x_{opt} + q(y_h^+, \omega_h^+), \quad h = 1, 2, \ldots, v \tag{14d}$$

$$V_h^+ \geq V_h^- \geq 0, \quad h = 1, 2, \ldots, v \tag{14e}$$

$$x_{opt} \geq y_h^+ \geq y_h^- \geq 0, \quad h = 1, 2, \ldots, v \tag{14f}$$

Solutions of y_{opt}^+ can be obtained through Submodel (14). Through integrating solutions of Submodels (13) and (14), interval solution for Model (12) can be obtained as follows:

$$f_{hopt}^{\pm} = \left[f_{opt}^-, f_{opt}^+ \right] \tag{15a}$$

$$x_{opt} = x^- + \mu_{opt}(x^+ - x^-) \tag{15b}$$

$$y_{hopt}^{\pm} = \left[y_{hopt}^-, y_{hopt}^+ \right] \tag{15c}$$

CASE STUDY

A case study of regional water management is then provided for demonstrating applicability of the developed method. In the water resources system, a water manager is responsible for allocating the limited water resources to support the regional municipality, industrial and agricultural

development. In the study region, agriculture and industry are the dominant activities and the agricultural irrigation and industrial consumption accounts for more than 75% of the total water demands to promote the development of the regional economy; 20–25% of the total water consumption is used for drinking, cleaning, and other municipal purposes. The main elements of the problem involve the water resource availabilities, the water demands to satisfy current and potential future needs, and the water transportation systems. With less water available, the management of water resources is becoming more complex in order to satisfy all users, and existing public institutions lack the capacity and structure to properly deal with the situation. Since local economic development relies heavily on the availability of the water supply, the adaptive strategies to water shortage crises are of high importance to local government. Moreover, in water resources systems, the manager wants to obtain different options for water supply, and select the option or combination of options that provide the necessary amount of water in the most cost-effective manner while taking into account technical and social criteria. From an economic point of view, all users need to know how much water they can expect to obtain during the planning horizon in order to establish and make plans for rational production. However, a variety of complexities exist in the study problem. On the one hand, the hydrologic cycle is basically dependent upon the geology and climate, which determine the physical characteristics of the basin, the natural environment, and the variability of the mass and energy exchanges occurring within the river basin. On the other hand, human activities impact on the natural resources, mainly through land and water use, and produce changes in the dynamics of the natural environment and the hydrologic cycle, which may amplify the variability of those exchanges, affecting the hydrologic balance and the use of the natural resources (Victoria *et al.* 2005; Li *et al.* 2011). These complexities could become further compounded by not only interactions among many uncertain system components but also their economic implications caused by improper policies.

Thus, the manager needs to create a plan to effectively allocate the uncertain supply of water to the three users in order to maximize the overall system benefit while simultaneously considering the uncertainties in the system. In addition, based on the regional water management policies, an allowable flow

level to each user must be regulated. If the promised water amount is delivered, the net benefit will be generated. However, if the promised water amount is not delivered, either the water must be obtained from higher price alternatives or the supply must be decreased by reducing the scale of production to fill the so-called deviation, causing economic losses (Li *et al*. 2006, 2007). Moreover, the existence of multiple uncertainties associated with the water resources system will aggravate the risk of system impairment and failure. Therefore, it is desirable that the risk control should be considered in the water allocation planning program. The problem under consideration of the risk of water resources system transforms into how to effectively allocate water to various sectors in order to achieve a maximum benefit assuming a given risk level under uncertainties. To solve such a problem, the proposed RITSP is considered to be a suitable approach for dealing with the study problem:

$$\text{Max} f^{\pm} = \sum_{i=1}^{3}(1-\lambda)NB_i^{\pm}W_i^{\pm} - \sum_{i=1}^{3}\sum_{h=1}^{7} p_h C_i^{\pm} D_{ih}^{\pm}$$

$$+ \lambda\left\{\xi_\alpha^{\pm} - \frac{1}{1-\alpha}\sum_{h=1}^{7} p_h V_h^{\pm}\right\} \quad (16a)$$

subject to

  [constraints of water availability]

$$\sum_{i=1}^{3}\left(W_i^{\pm} - D_{ih}^{\pm}\right) \leq q_h^{\pm}, \; \forall h \quad (16b)$$

  [constraints of extreme allocation amounts]

$$W_{i\,\max}^{\pm} \geq W_i^{\pm} \geq D_{ih}^{\pm}, \quad \forall i, t, h, \quad (16c)$$

$$W_i^{\pm} - D_{ih}^{\pm} \leq W_{i\,\min}^{\pm}, \quad \forall i, t, h, \quad (16d)$$

$$V_h^{\pm} \geq \xi^{\pm} - \sum_{i=1}^{3} NB_i^{\pm}W_i^{\pm} + \sum_{i=1}^{3} C_i^{\pm} D_{ih}^{\pm}, \; \forall h \quad (16e)$$

  [nonnegative constraints]

$$V_h^{\pm} \geq 0, \quad \forall h \quad (16f)$$

$$D_{ih}^{\pm} \geq 0, \quad \forall i, h \quad (16g)$$

where $f^{\pm}$ is the net system benefit over the planning horizon (\$); i is the index of water users, where $i = 1$ for municipality,

$i = 2$ for industrial production, and $i = 3$ for agricultural sector; h is the index of scenarios where $h = 1, 2,\dots, 7$; $W_i^{\pm}$ is the allocation target of water that is promised to user i; $D_{ih}^{\pm}$ is the amount of water deficit by which the water allocation target $W_i^{\pm}$ is not met in scenario h; $NB_i^{\pm}$ is the net benefit of user i per unit of water allocated; $C_i^{\pm}$ is the reduction of net benefit to user i per unit of water not delivered; λ is a nonnegative trade-off coefficient representing the exchange rate of mean benefit for risk; $\xi_\alpha^{\pm}$ is an auxiliary variable, which is the maximum benefit at the cumulative probability α; α is the confidence level; $V_h^{\pm}$ is a positive auxiliary variable under scenario h; p_h is probability of occurrence for scenario h; $q_h^{\pm}$ is the available water resources in scenario h; $W_{i\,\max}^{\pm}$ is the minimum allowable allocation amount for user i.

For Model (16), if $W_i^{\pm}$ are considered as uncertain inputs, the existing methods for solving inexact linear programming problems cannot be used directly. In this study, an optimized set of target values will be identified by having μ_i in Model (17) be decision variables. This optimized set will correspond to the highest possible system benefit under the uncertain water allocation targets. Accordingly, let $W_i = W_i^{-} + \mu_i \Delta W_i$, where $\Delta W_i = W_i^{+} - W_i^{-}$, $\mu_i \in [0, 1]$. μ_i are decision variables that are used for identifying an optimized set of target values $W_i^{\pm}$ in order to support the related policy analyses (Huang & Loucks 2000). For example, when $W_i^{\pm}$ approach their upper bounds (i.e., when $u_i = 1$), a relatively high benefit would be obtained if the water demands are satisfied; however, a high penalty may have to be paid when the promised water is not delivered. Conversely, when $W_i^{\pm}$ reach their lower bounds (i.e., when $u_i = 0$), we may have a lower cost and a higher risk of violating the promised targets. Therefore, by introducing decision variables u_i, and according to Huang & Loucks (2000), the model can be transformed into two deterministic submodels based on an interactive algorithm. Since the objective is to maximize the net system benefit, the submodel corresponding to upper-bound objective function value (f^{+}) is first desired. Thus, we have:

$$\text{Max} f^{+} = \sum_{i=1}^{3}(1-\lambda)NB_i^{+}\left(W_i^{-} + \mu_i \Delta W_i\right) - \sum_{i=1}^{3}\sum_{h=1}^{7} p_h C_i^{-} D_{ih}^{-}$$

$$+ \lambda\left\{\xi_\alpha^{+} - \frac{1}{1-\alpha}\sum_{h=1}^{7} p_h V_h^{-}\right\} \quad (17a)$$

subject to

$$\Delta W_i = W_i^+ - W_i^- \tag{17b}$$

$$0 \leq \mu_i \leq 1 \tag{17c}$$

$$\sum_{i=1}^{3} \left(W_i^- + \mu_i \Delta W_i - D_{ih}^- \right) \leq q_h^+, \quad \forall h \tag{17d}$$

$$W_{i\,\max}^+ \geq W_i^- + \mu_i \Delta W_i \geq D_{ih}^-, \quad \forall i, t, h, \tag{17e}$$

$$\left(W_i^- + \mu_i \Delta W_i \right) - D_{ih}^- \leq W_{i\,\min}^+, \quad \forall i, t, h, \tag{17f}$$

$$V_h^- \geq \xi^+ - \sum_{i=1}^{3} NB_i^+ \left(W_i^- + \mu_i \Delta W_i \right) + \sum_{i=1}^{3} C_i^- D_{ih}^-, \quad \forall h \tag{17g}$$

$$V_h^- \geq 0, \quad \forall h \tag{17h}$$

$$D_{ih}^- \geq 0, \quad \forall i, h \tag{17i}$$

where f_{opt}^+, $D_{ih\,\mathrm{opt}}^-$, and $u_{i\mathrm{opt}}$ are solutions of the Submodels (17). Solution for f^+ provides the extreme upper bound of system benefit under uncertain inputs. Then, the optimized water allocation targets would be $W_{i\,\mathrm{opt}} = W_i^- + \Delta W_{kt} u_{i\,\mathrm{opt}}$. Consequently, the submodel corresponding to the lower bound of the objective function value (i.e., f^-) is:

$$\mathrm{Max}\, f^- = \sum_{i=1}^{3} (1-\lambda) NB_i^- W_{i\,\mathrm{opt}} - \sum_{i=1}^{3} \sum_{h=1}^{7} p_h C_i^+ D_{ih}^+$$
$$+ \lambda \left\{ \xi_\alpha^- - \frac{1}{1-\alpha} \sum_{h=1}^{7} p_h V_h^+ \right\} \tag{18a}$$

subject to

$$W_{i\,\mathrm{opt}} = W_i^- + \Delta W_{kt} u_{i\,\mathrm{opt}} \tag{18b}$$

$$\sum_{i=1}^{3} \left(W_{i\,\mathrm{opt}} - D_{ih}^+ \right) \leq q_h^-, \quad \forall h \tag{18c}$$

$$W_{i\,\max}^- \geq W_{i\,\mathrm{opt}} \geq D_{ih}^+, \quad \forall i, t, h, \tag{18d}$$

$$W_{i\,\mathrm{opt}} - D_{ih}^+ \leq W_{i\,\min}^-, \quad \forall i, t, h, \tag{18e}$$

$$V_h^+ \geq \xi^- - \sum_{i=1}^{3} NB_i^- W_{i\,\mathrm{opt}} + \sum_{i=1}^{3} C_i^+ D_{ih}^+, \quad \forall h \tag{18f}$$

$$V_h^+ \geq 0, \quad \forall h \tag{18g}$$

$$D_{ih}^+ \geq 0, \quad \forall i, h \tag{18h}$$

where f_{opt}^- and $D_{it\,\mathrm{opt}}^+$ are solutions of the Submodels (18). Thus, the solutions for Model (16) under the optimized targets can be obtained through incorporating the solutions of the two submodels.

Table 1 provides the water target demands and the related economic data. The data were obtained from a number of representative cases for water resources management (Loucks *et al*. 1981; Huang & Loucks 2000; Li *et al*. 2006, 2007). Since uncertainties exist in the system components, water allocation targets and economic data are expressed as intervals format. Let W_i^+ be the quantity of water that is promised to each user i. If this water is delivered, the resulting net benefit to the local economy per unit of water allocated is estimated to be $NB_i^\pm$. However, if the promised water is not delivered, either water must be obtained from alternative and more expensive sources, or demand must be curtailed by reduced production and/or increased recycling within the industrial concern, or by reduced irrigation in the agricultural sector. This results in a reduction of net benefit to user i of $C_i^\pm$ per unit of water not delivered $(C_i^\pm > NB_i^\pm)$. In addition, in the water resources system, the total amount of water available has

Table 1 | Water target demands and the related economic data

	User		
	Municipal	**Industrial**	**Agricultural**
Water allocation target, $W_i^\pm (10^6\,\mathrm{m}^3)$	[2.20, 4.00]	[3.00, 5.50]	[3.50, 6.50]
Minimum allowable allocation, $W_{i\,\min}^\pm (10^6\,\mathrm{m}^3)$	[1.00, 1.50]	[0.50, 1.00]	[0.60, 1.00]
Net benefit when water demand is satisfied, $NB_i^\pm (\$/\mathrm{m}^3)$	[90, 100]	[45, 55]	[25, 35]
Penalty when water is not delivered, $C_i^\pm (\$/\mathrm{m}^3)$	[125, 135]	[70, 80]	[45, 55]

characteristics of random and increasing or decreasing trend changes. Theoretically, there are other ways to generate the random variables. One is a survey from different experts, based on an assumption that there were not enough data available. A large group of experts are required to estimate the value of a certain parameter. Then the value of the parameter can be obtained via analyzing the estimations through sample statistic inductive methods. The other way is exemplified by the probability cumulative distribution function, which is based on that there are enough data available. According to the local policy of hypothetical cases, seven discrete water inflow values (i.e., very-low, low, low-medium, medium, medium-high, high, and very-high) are selected as the range of intervals. In addition, division of the targets into a number of predefined values associated with probabilities (8, 12, 16, 25, 15, 14, and 10%) can meet the requirement of the RITSP. Table 2 shows the water inflow levels and the associated probabilities of occurrence. From previous studies (Conejo *et al.* 2008; Pousinho *et al.* 2011), the value of α is commonly set between 0.90

and 0.99. In addition, the value of λ can be chosen as any real number. After a number of test runs, it was found that, if λ value is over 0.6, the solutions of optimal water allocation targets and water shortage amounts are the same as the one obtained under $\lambda = 0.6$. Therefore, in order to reflect the variation trend of allocation policies by changing the value of λ, the λ value is set between 0 and 1.

Uncertainties exist in many of the system components (provided as intervals for water allocation targets and economic data, as well as distribution information for the total water availability). The problems under consideration include: (1) how to suitably allocate water flows to achieve a maximized system benefit; (2) how to identify desired water allocation policies under different risk levels; and (3) how to seek cost-effective water resources management strategies under complex uncertainties. The developed RITSP is considered to be a suitable approach for dealing with these problems.

RESULT ANALYSIS AND DISCUSSION

Results have been obtained through solving the RITSP model. The solutions for the objective function value and most of the nonzero decision variables were interval numbers. Generally, solutions presented as intervals demonstrate that the related decisions should be sensitive to the uncertain modeling inputs (Li *et al.* 2006).

Table 3 shows the solutions of water allocation targets ($W_{i\,opt}$) under different α and λ levels during the planning horizon. Various α and λ levels correspond to different system confidence levels and different levels of trade-off

Table 2 | Stream flow distribution

Flow level	Probability	Stream flows $q_h^{\pm}$ (10^6 m^3)
Very-low (V-L)	0.08	[3.80, 5.20]
Low (L)	0.12	[5.50, 6.50]
Low-medium (L-M)	0.16	[6.90, 8.20]
Medium (M)	0.25	[8.50, 9.80]
Medium-high (M-H)	0.15	[10.0, 11.5]
High (H)	0.14	[11.5, 12.9]
Very-high (V-H)	0. 10	[13.2, 14.5]

Table 3 | Optimal targets of the RITSP model

α level	$W_{i\,opt}$	λ level										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.90	$W_{1\,opt}$	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	$W_{2\,opt}$	5.40	4.80	4.00	4.00	4.00	4.00	3.20	3.20	3.20	3.20	3.20
	$W_{3\,opt}$	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50
0.95	$W_{1\,opt}$	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	$W_{2\,opt}$	5.40	4.00	3.20	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	$W_{3\,opt}$	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50
0.99	$W_{1\,opt}$	4.00	4.00	4.00	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20
	$W_{2\,opt}$	5.40	4.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	$W_{3\,opt}$	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50

between profit and risk, thus would lead to varied water allocation targets. For example, when $\alpha = 0.90$ and 0.95, water allocation targets for the municipal sector would be $4.00 \times 10^6 \, \text{m}^3$ under different λ levels; however, when $\alpha = 0.99$, the water allocation targets for this user would be 4.00×10^6 ($\lambda = 0$, 0.1, and 0.2), and $3.20 \times 10^6 \, \text{m}^3$ (the value of λ is from 0.3 to 1.0). Generally, water resources would first be allocated to the municipal sector, followed by the industrial and agricultural sectors. For example, the optimized allocation target for the municipality over the planning horizon would be close to its maximum value under different α and λ levels. This is because the municipality could bring about the highest benefit when its demand is satisfied; thus, the manager would have to promise larger amounts to it to achieve a maximized system benefit. The optimized allocation target for the agricultural sector would reach its minimum value under demanding conditions since this user is associated with the lowest benefit. In comparison, the optimized water allocation target for industry would fluctuate within its minimum and maximum values as α

and λ levels are varied; the benefit from industry lies between the profits from the municipality and agriculture. Moreover, the water allocation targets would decrease with increment of the λ levels, especially in a high confidence level. For example, when $\alpha = 0.99$, the water allocation targets for industry would be $5.40 \times 10^6 \, \text{m}^3$ ($\lambda = 0$), $4.00 \times 10^6 \, \text{m}^3$ ($\lambda = 0.1$), and $3.00 \times 10^6 \, \text{m}^3$ (the value of λ is from 0.2 to 1.0).

Variations in $W_i^{\pm}$ could reflect different policies of water resources management under uncertainty. When the water allocation targets reach their lower bounds, the corresponding policy may result in less water shortage and lower economic penalty. Moreover, the upper bounds of $W_i^{\pm}$ would lead to a strategy with higher allocated targets, resulting in a higher system benefit and a higher risk of penalty when the water inflow is in a lower level. Therefore, different policies in predefining the promised water allocation are associated with different levels of economic benefit and system failure risk.

Tables 4 to 6 present the water deficit ($D_{ih}^{\pm}$) under different scenarios in the planning horizon. The solutions of $D_{ih}^{\pm}$

Table 4 │ Solutions of $D_{ih}^{\pm}$ from RITSP model under $\alpha = 0.90$

		$\alpha = 0.90$						
H	**I**	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
1	1	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]
	2	[4.40,4.90]	[3.80,4.30]	[3.00,3.50]	[3.00,3.50]	[3.00,3.50]	[3.00,3.50]	[2.20,2.70]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
2	1	0	0	0	0	0	0	0
	2	[3.90,4.50]	[3.30,3.90]	[2.50,3.10]	[2.50,3.10]	[2.50,3.10]	[2.50,3.10]	[1.70,2.30]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
3	1	0	0	0	0	0	0	0
	2	[2.20,3.10]	[1.60,2.50]	[0.80,1.70]	[0.80,1.70]	[0.80,1.70]	[0.80,1.70]	[0,0.90]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
4	1	0	0	0	0	0	0	0
	2	[0.60,1.50]	[0,0.90]	[0,0.10]	[0,0.10]	[0,0.10]	[0,0.10]	0
	3	[2.50,2.90]	[2.50,2.90]	[1.70,2.90]	[1.70,2.90]	[1.70,2.90]	[1.70,2.90]	[0.90,2.20]
5	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[1.40,2.90]	[0.80,2.30]	[0,1.50]	[0,1.50]	[0,1.50]	[0,1.50]	[0,0.70]
6	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[0,1.40]	[0,0.80]	0	0	0	0	0
7	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0

Note: When $\lambda > 0.6$, the solutions of $D_{ih}^{\pm}$ are the same as those obtained under $\lambda = 0.6$.

Table 5 │ Solutions of $D_{ih}^{\pm}$ from RITSP model under $\alpha = 0.95$

		$\alpha = 0.95$						
h	i	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
1	1	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]
	2	[4.40,4.90]	[3.00,3.50]	[2.20,2.70]	[2.00,2.50]	[2.00,2.50]	[2.00,2.50]	[2.00,2.50]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
2	1	0	0	0	0	0	0	0
	2	[3.90,4.50]	[2.50,3.10]	[1.70,2.30]	[1.50,2.10]	[1.50,2.10]	[1.50,2.10]	[1.50,2.10]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
3	1	0	0	0	0	0	0	0
	2	[2.20,3.10]	[0.80,1.70]	[0,0.90]	[0,0.70]	[0,0.70]	[0,0.70]	[0,0.70]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.30,2.90]	[2.30,2.90]	[2.30,2.90]	[2.30,2.90]
4	1	0	0	0	0	0	0	0
	2	[0.60,1.50]	[0,0.10]	0	0	0	0	0
	3	[2.50,2.90]	[1.70,2.90]	[0.90,2.20]	[0.70,2.00]	[0.70,2.00]	[0.70,2.00]	[0.70,2.00]
5	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[1.40,2.90]	[0,1.50]	[0,0.70]	[0,0.50]	[0,0.50]	[0,0.50]	[0,0.50]
6	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[0,1.40]	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0

Note: When $\lambda > 0.6$, the solutions of $D_{ih}^{\pm}$ are the same as those obtained under $\lambda = 0.6$.

under the given targets reflect the variations of system conditions caused by inputs of the uncertain parameters. Generally, the water shortage solutions of the three users and scenarios can be similarly interpreted based on the results. As the water flow level increases, the water allocation target would be satisfied, and the water shortage would decrease. For example, when $\alpha = 0.90$ and $\lambda = 0.1$, the industrial water shortages would be $[3.80, 4.30] \times 10^6 \, \text{m}^3$, $[3.30, 3.90] \times 10^6 \, \text{m}^3$, $[1.60, 2.50] \times 10^6 \, \text{m}^3$, and $[0, 0.90] \times 10^6 \, \text{m}^3$, when flow levels are very-low, low, low-medium, and medium, respectively; there would be no shortages under medium-high, high and very-high flow levels. In addition, a trade-off could be analyzed by assigning different λ values in the model constraints when α is fixed. From Tables 3–6, a number of decision variables such as the target values ($W_{i \, \text{opt}}$) and the upper and lower bounds of the shortage ($D_{ih}^{\pm}$) amount would vary with different λ values. As the value of λ increases, the water allocation target and shortage of the three users would decrease. For example, when the available quantity of water is at the

medium level of stream flow under the scenario of α with the value of 0.95, the amount of industrial water shortage would be $[0.60, 1.50] \times 10^6 \, \text{m}^3$ ($\lambda = 0$), $[0, 0.90] \times 10^6 \, \text{m}^3$ ($\lambda = 0.1$), $[0, 0.10] \times 10^6 \, \text{m}^3$ (the value of λ is from 0.2 to 0.9), and 0 ($\lambda = 1.0$), respectively; the water shortage of the municipal sector would be 0 under different λ values, and the agricultural shortage would decrease from $[2.50, 2.90] \times 10^6 \, \text{m}^3$ to $[0.90, 2.20] \times 10^6 \, \text{m}^3$ when λ changes from 0.1 to 1.0. Generally, as λ increases, the allocation target and shortage would decrease, leading to a decreased amount of water shortage. It indicated that when the risk level λ increases, water managers would choose a conservative water allocation scheme to avoid the risk. In contrast, a lower λ value would result in alternatives with lower risk aversion. Moreover, when the confidence level of α increases, the allocation target would decrease, leading to a reduced amount of water shortage and increased water allocation balance among users. For example, under the low inflow level, the municipal water shortage would be 0, and agricultural deficit would be $[2.50, 2.90] \times 10^6 \, \text{m}^3$ with

Table 6 | Solutions of $D_{ih}^{\pm}$ from RITSP model under different $\alpha = 0.99$

		$\alpha = 0.99$						
h	i	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
1	1	[0.80,1.30]	[0.80,1.30]	[0.80,1.30]	[0,0.50]	[0,0.50]	[0,0.50]	[0,0.50]
	2	[4.40,4.90]	[3.00,3.50]	[2.00,2.50]	[2.00,2.50]	[2.00,2.50]	[2.00,2.50]	[2.00,2.50]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
2	1	0	0	0	0	0	0	0
	2	[3.90,4.50]	[2.50,3.10]	[1.50,2.10]	[0.70,1.30]	[0.70,1.30]	[0.70,1.30]	[0.70,1.30]
	3	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]	[2.50,2.90]
3	1	0	0	0	0	0	0	0
	2	[2.20,3.10]	[0.80,1.70]	[0,0.70]	0	0	0	0
	3	[2.50,2.90]	[2.50,2.90]	[2.30,2.90]	[1.50,2.80]	[1.50,2.80]	[1.50,2.80]	[1.50,2.80]
4	1	0	0	0	0	0	0	0
	2	[0.60,1.50]	[0,0.10]	0	0	0	0	0
	3	[2.50,2.90]	[1.70,2.90]	[0.70,2.00]	[0,1.20]	[0,1.20]	[0,1.20]	[0,1.20]
5	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[1.40,2.90]	[0,1.50]	[0,0.50]	0	0	0	0
6	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	[0,1.40]	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0

Note: When $\lambda > 0.6$, the solutions of $D_{ih}^{\pm}$ are the same as those obtained under $\lambda = 0.6$.

the scenarios of α with the values of 0.90, 0.95, and 0.99; for the industrial sector with different λ values, the amount of water deficit would decrease from $[3.90, 4.50] \times 10^6$ to $[1.70, 2.30] \times 10^6 \, \text{m}^3$ when α is a fixed value of 0.90, and reduce from $[3.90, 4.50] \times 10^6$ to $[0.70, 1.30] \times 10^6 \, \text{m}^3$ when the value of α is 0.99. In such a case, the extreme risk would be lowered and the system feasibility would be enhanced. In contrast, a lower α value would result in a higher possibility of system loss in extreme conditions.

The RITSP model can generate a great deal of water allocation strategies with different α and λ values under different inflow levels, in order to analyze the effects of α and λ on water allocation policies. Figures 3–5 present the optional water allocation schemes obtained through the RITSP model. Due to the highest benefit, water would be first allocated to the municipal sector under different α and λ values. For example, the water allocated to municipal sectors would reach the upper bound of the water allocation target (e.g., $4.00 \times 10^6 \, \text{m}^3$) under the scenarios of α with the values of 0.90 and 0.95; the water allocation would decrease

to $3.2 \times 10^6 \, \text{m}^3$ when the value of λ changes from 0.4 to 1.0 under the scenarios of α with the value of 0.99. In addition, under the lower level of water inflow, the amount of industrial allocation would be decreased, and the agricultural water allocation would increase, when the value of α increases from 0.90 to 0.99 under the same λ value. For example, under the low-medium water flow level, when λ is a fixed value of 0.6, the industrial water allocation would be $[2.30, 3.20] \times 10^6$, 3.00×10^6, and $[2.30, 3.00] \times 10^6 \, \text{m}^3$, and the amount of agricultural allocation would be $[0.60, 1.00] \times 10^6$, $[0.60, 1.20] \times 10^6$, and $[0.70, 2.00] \times 10^6 \, \text{m}^3$, under the condition of α increasing from 0.90 to 0.99. From Figures 3–5, the lower and upper bounds of the water allocation amount would vary with the change of α. This shows that the effect of the risk measure on the modeling outputs could be adjusted by changing the α value. Generally, a high α value would lead to a lower risk and enhanced system feasibility. The water allocated to the users with higher benefit would decrease, and the water supplied to the users with lower benefit would increase when
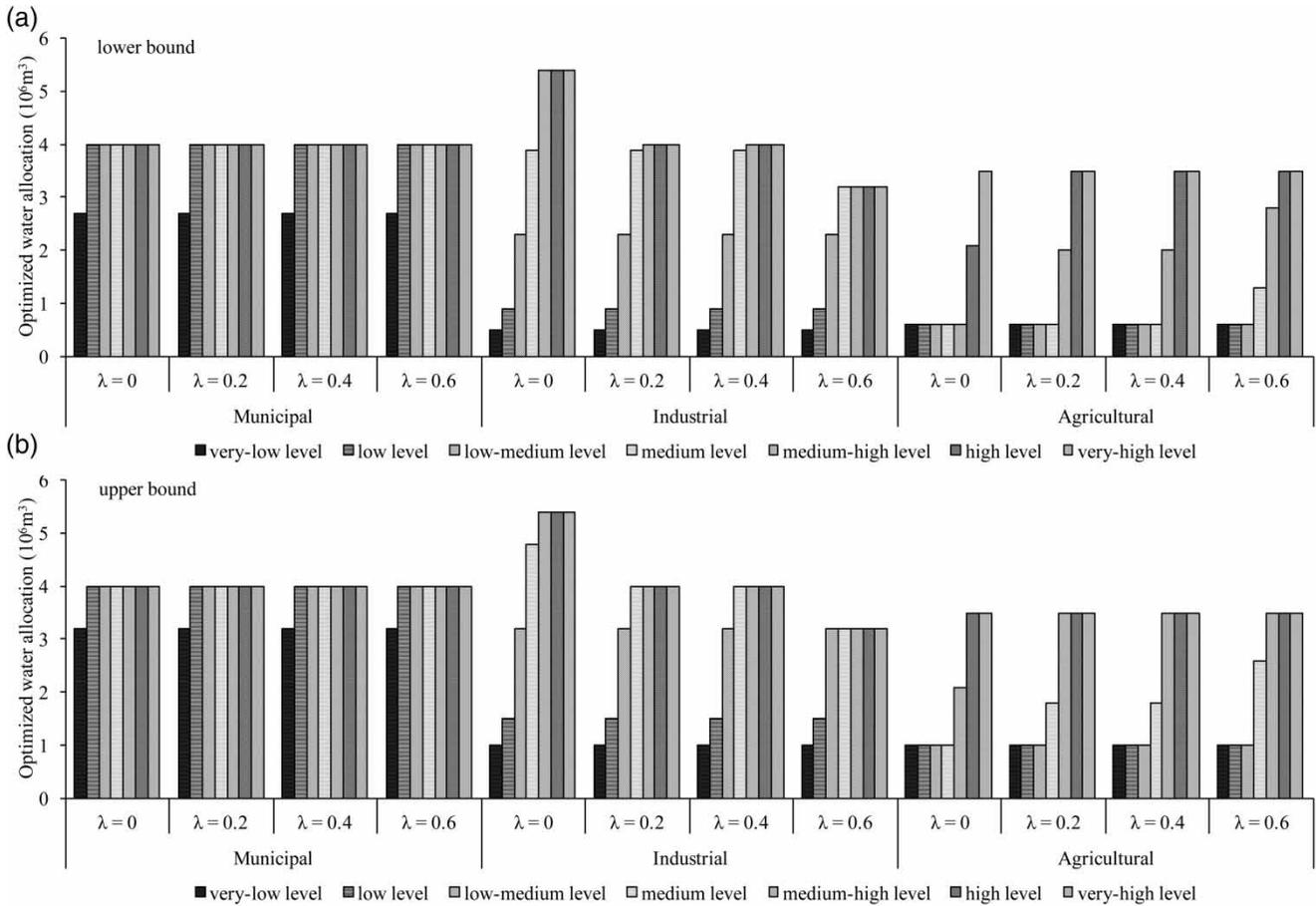
Figure 3 | Optimized water allocation schemes under $\alpha = 0.90$.

there is an α value increment with a fixed λ value, in order to reduce the risk of unbalance water allocation caused by the objective of maximum net benefit in water resources system planning and management.

Figures 6 and 7 show the varying trend of the RITSP model's objective, the system net benefit and recourse cost under different α and λ values. In general, the intervals of the model's objective would decrease as the value of λ increases when α is a fixed value. For example, when α is a fixed value of 0.90, the objective of the RITSP model would be \$ $[0.40, 0.64] \times 10^9$, \$ $[0.57, 0.96] \times 10^9$, \$ $[0.79, 1.62] \times 10^9$, \$ $[0.97, 1.62] \times 10^9$, \$ $[1.15, 1.95] \times 10^9$, \$ $[1.33, 2.28] \times 10^9$, \$ $[1.63, 2.62] \times 10^9$, \$ $[1.83, 2.95] \times 10^9$, \$ $[2.02, 3.28] \times 10^9$, \$ $[2.22, 3.62] \times 10^9$, and \$ $[2.42, 3.95] \times 10^9$ under the scenario of λ varying from 0 to 1.0, respectively (as shown in Figure 6(a)). In addition, first, the values of system net benefit would decrease and then would not

change as the value of λ increased. When α is a fixed value of 0.95, the net benefit would be \$ $[400.22, 640.89] \times 10^6$ ($\lambda = 0$), \$ $[427.87, 628.12] \times 10^6$ ($\lambda = 0.1$), \$ $[433.14, 613.28] \times 10^6$ ($\lambda = 0.2$), \$ $[434.30, 608.77] \times 10^6$ (the value of λ is from 0.3 to 1.0) respectively; the recourse cost would be \$ $[178.62, 290.28] \times 10^6$ ($\lambda = 0$), \$ $[114.39, 199.63] \times 10^6$ ($\lambda = 0.1$), \$ $[85.23, 158.37] \times 10^6$ ($\lambda = 0.2$), \$ $[78.74, 148.21] \times 10^6$ (the value of λ is from 0.3 to 1.0) respectively (as shown in Figure 7(b)). It indicated that increasing the value of λ would increase the relative importance of the risk term and also lead to a higher system risk, and the water managers would choose a conservative scheme with a lower system benefit. Moreover, as the value of α increases, the net benefit would decrease. For example, when λ is a fixed value of 0.6, the net benefit would be \$ $[433.14, 613.28] \times 10^6$, \$ $[434.30, 608.77] \times 10^6$, and \$ $[403.58, 557.12] \times 10^6$ under the scenarios of α with
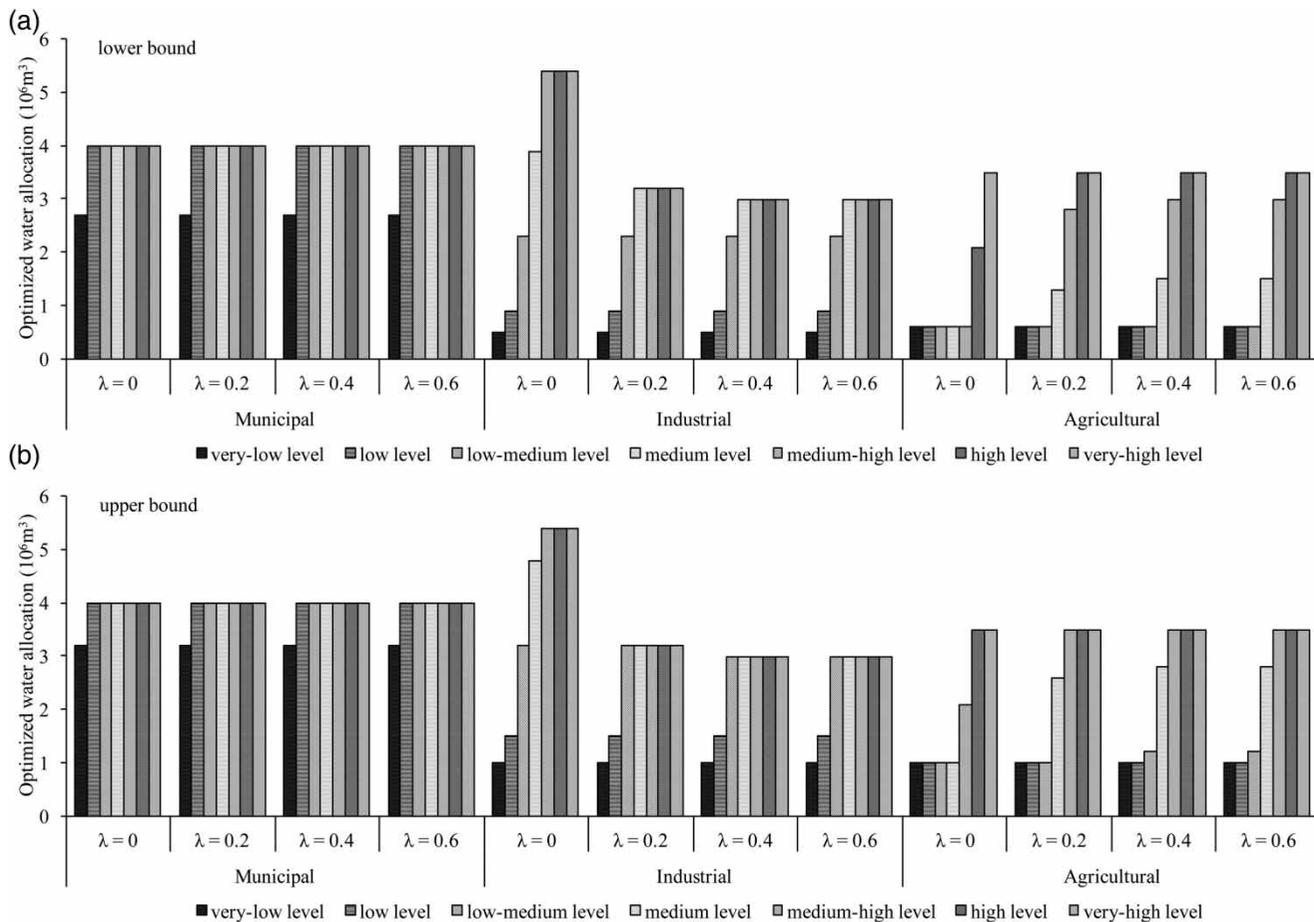
Figure 4 | Optimized water allocation schemes under $\alpha = 0.95$.

the value of 0.90, 0.95, and 0.99, respectively (Figure 7). Thus, increasing the parameter λ and/or the parameter α implies a higher level of risk than the recourse cost and the total positioning profit, which together constitute the expected total benefit; change monotonically as a function of α.

Figure 8 illustrates how the optimal CVaR changes as the risk parameters α and λ increase through solving the RITSP model. Similar to the optimal objective of the RITSP model, CVaR also decreases as α increases by the definition of CVaR. When α increases the corresponding value-at-risk increases, and CVaR accounts for the risk of larger realizations. Thus, larger α values would lead to more conservative policies, which give more weight to worse scenarios. However, CVaR increases as λ increases. Due to the changing trade-off between the expectation and the CVaR criterion, larger λ values provide us with a lower

expected benefit and a higher CVaR value. Increasing λ leads to a more risk-averse policy with a lower system benefit and lower expected recourse costs in general. Thus, increasing the parameter λ and/or the parameter α implies a higher level of risk aversion, and water managers would choose a more risk-averse policy that would be a lower water allocation target for each user in order to avoid the risk of water shortage, and a well-balanced water allocation scheme to reduce the risk of conflicts over competition for water resources.

When the λ value is 0, the RITSP model would be an ITSP model for water resources system management under uncertainty. The detailed optimal water targets and water shortage from ITSP are presented in Tables 3–6. Differently from the RITSP model, the ITSP model aims to obtain the maximum benefit in the optimal process of water allocation, and it does not take the risk of model feasibility and
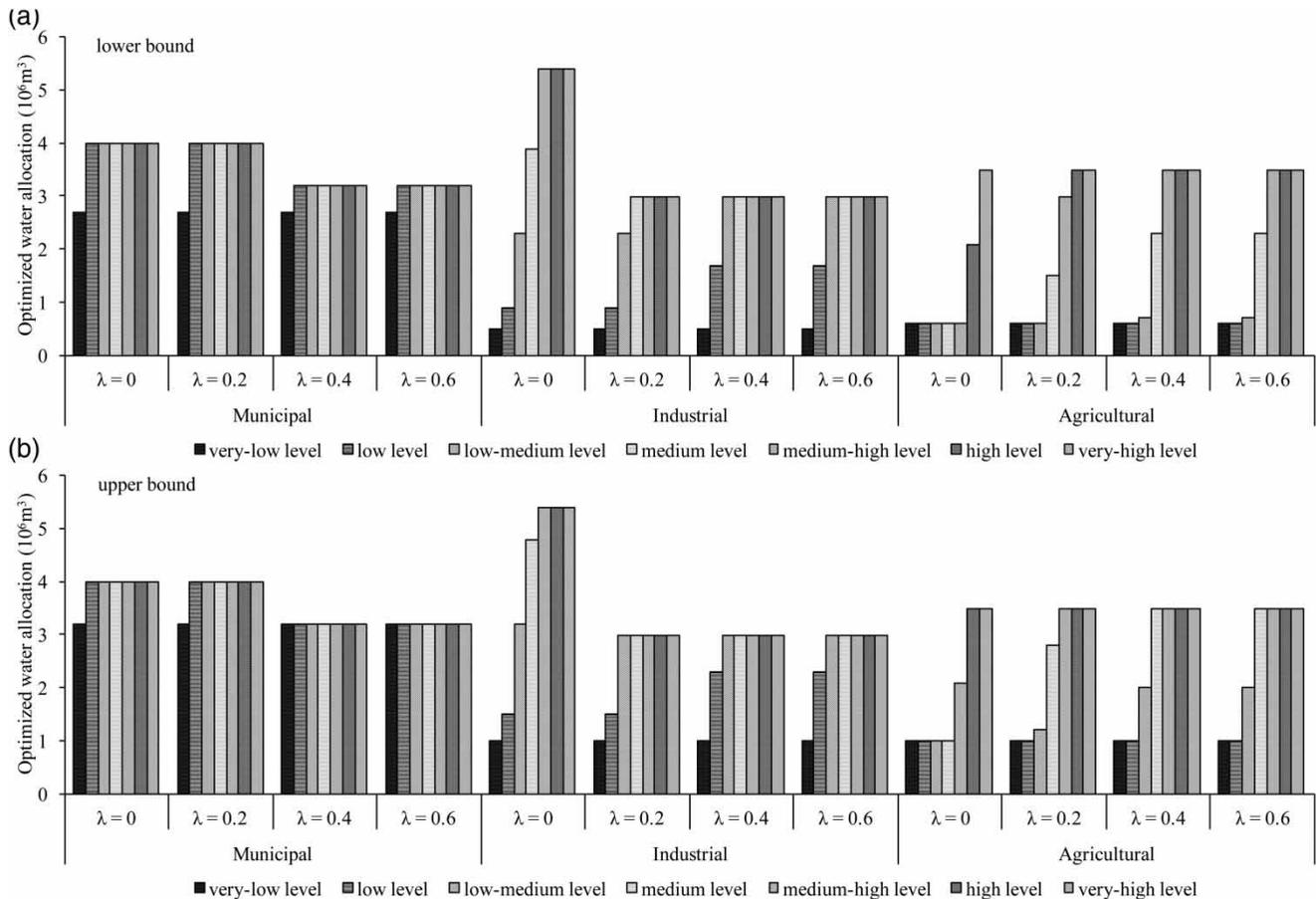
Figure 5 | Optimized water allocation schemes under $\alpha = 0.99$.

reliability into consideration. These limitations could lead to low system stability and unbalanced allocation patterns. For example, when λ value is equal to 0, the water allocation targets of the municipal and industrial sectors would first be satisfied and reach their upper bounds, due to a higher benefit, and the agricultural water allocation target would reach the lower bounds; especially, in the very-low inflow level, water shortage would first occur in the agricultural sector. Moreover, the net benefit of ITSP is higher than that of the RITSP model. This also implies that the system objective of the ITSP model is only to obtain a maximum benefit without regarding risk aversion. In addition, the width of interval net benefit in the RITSP model is narrower than that of the ITSP model. It is indicated that the system benefit relies on the water resources condition, and tends to fluctuate more intensively with the change of available water resources. Through integrating CVaR into the objective of a water resources system management model, managers could obtain a robust and riskless decision.

CONCLUSIONS

In this study, a RITSP model is developed for supporting regional water resources management problems under uncertainty. This method is based on an integration of IPP, CVaR model, and two-stage stochastic programming (TSP). It allows uncertainties presented as both probability distributions and interval values to be incorporated within a general optimization framework. Moreover, the risk-aversion method was incorporated into the objective function to reflect the preference of decision makers, such that the trade-off between system economy and extreme expected loss could be analyzed. Then, the developed
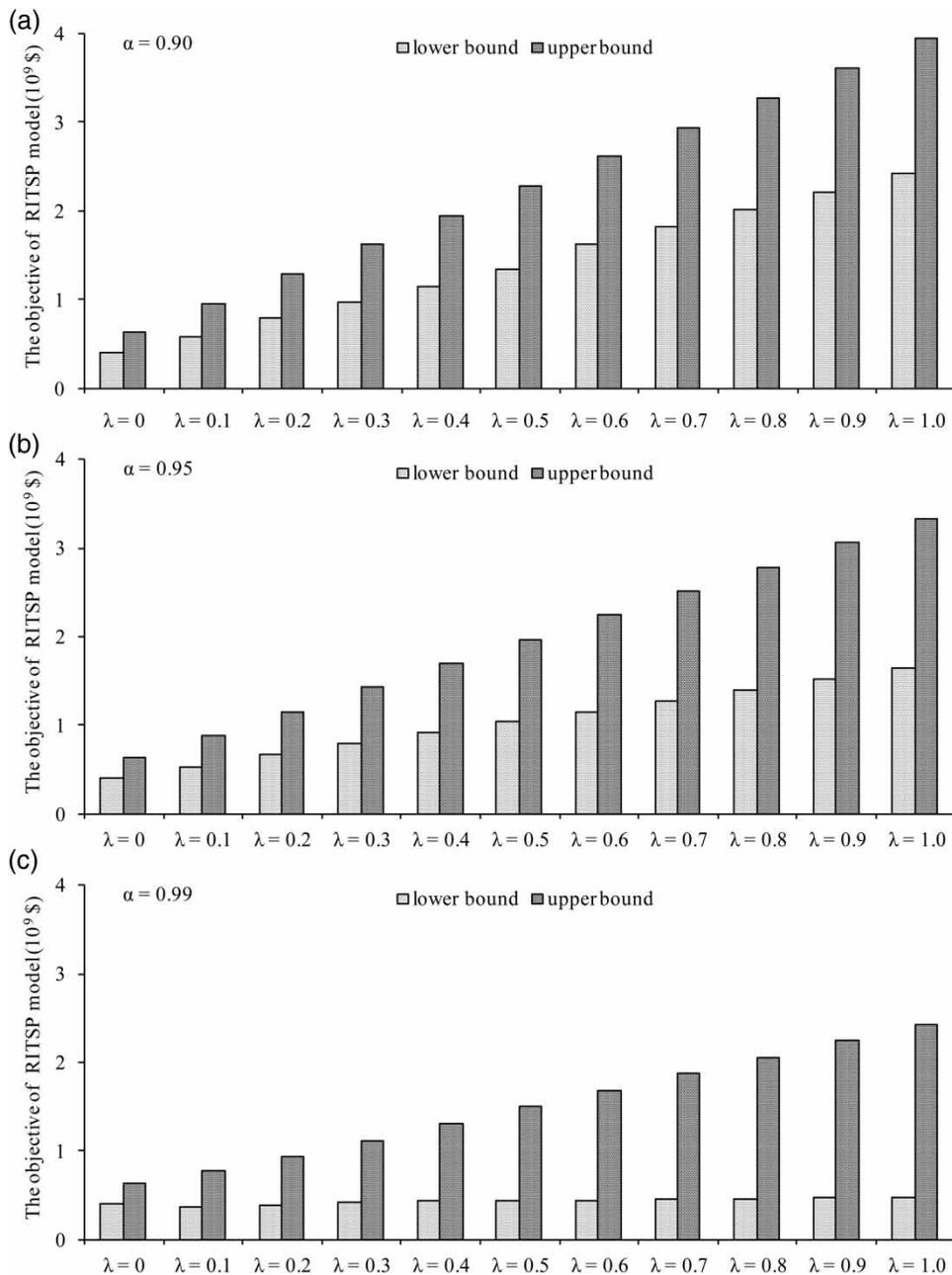
Figure 6 | The objectives of the RITSP model under different α and λ levels.

method has been confirmed through a case study of a water resources allocation problem involving three competing water users. A number of scenarios corresponding to different river inflow and risk levels was examined; the results of the case study suggest that the methodology is applicable to reflecting complexities of water resources management and can be used for providing bases for identifying desired water allocation plans with a maximized system, and reflecting the decision maker's attitude toward risk aversion.

The proposed method could help water resources managers identify desired management policies under various economic considerations. The study results suggested that the proposed approach was also applicable to many other environmental and energy management problems. The
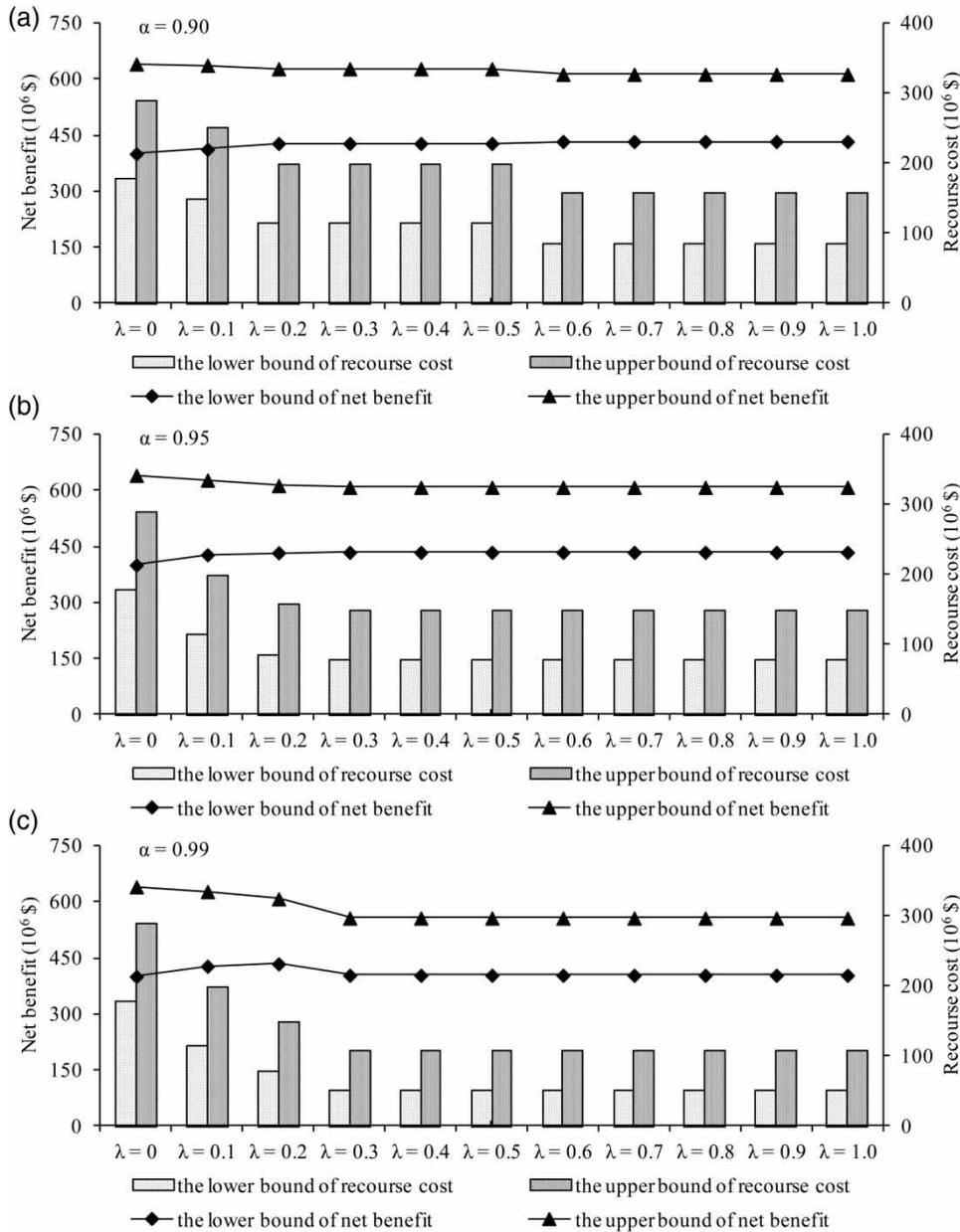
Figure 7 │ Net benefit and recourse cost under different α and λ levels.

risk-based framework could be used to assess the performance risk of unbalanced water resources allocation strategies in compliance with the economic and/or environmental management goals, and help managers identify desired water resources management policies under various environmental, economic, and system reliability considerations. It could also be coupled with other optimization methodologies to handle various types of uncertainties. However, compared with other approaches, there is still much space for improvement of the proposed model. For example, RITSP would have difficulties in dealing with the uncertainties in the model's right-hand side coefficients; the probability of random variable is estimated through statistical analysis, which would unavoidably bring errors to the
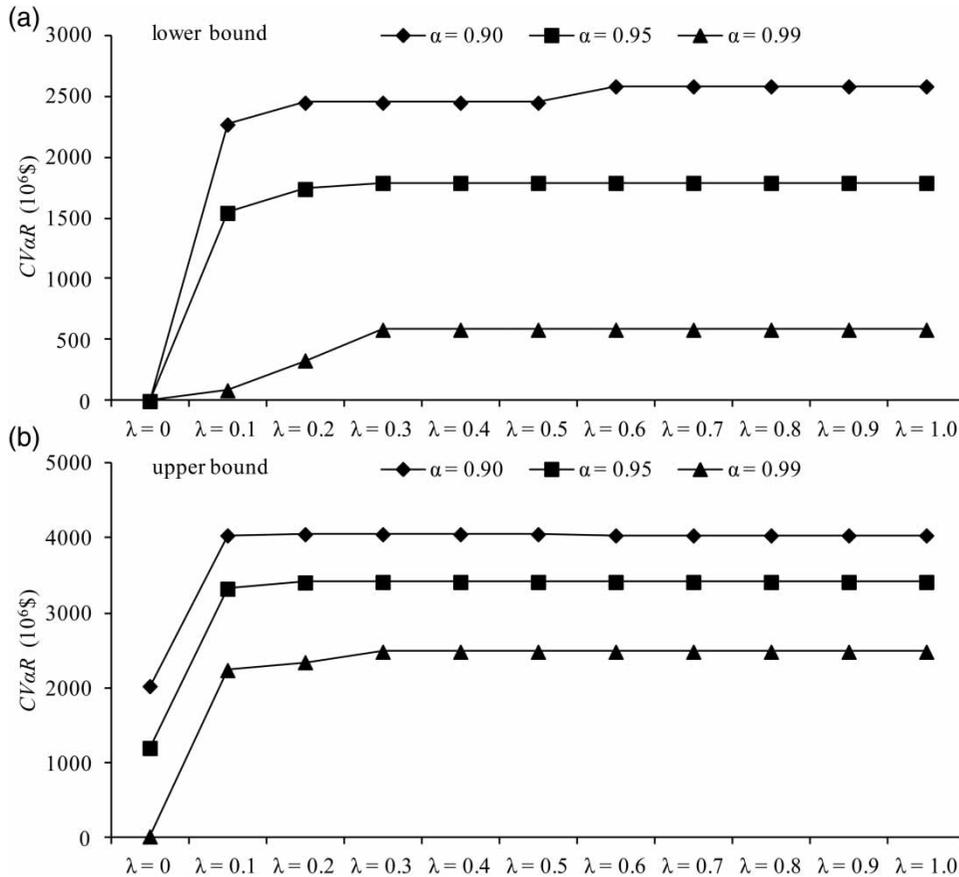
Figure 8 │ Optimal values of *CVaR* under different α and λ levels.

system; the selection of a suitable alternative among the obtained interval solutions under different α and λ values is of significant complexity and becomes an extra burden for water resources managers. It is also possible that fuzzy logic could be used instead of λ values to deal with uncertainties in many real-world optimization problems, due to the inherent ambiguity of the fuzzy subsets. Further studies are desired to mitigate these limitations.

REFERENCES

Ahmed, S. 2004 Mean-risk objectives in stochastic programming. Technical Report, Georgia Institute of Technology. E-print available at 2004, http://www.optimization-online.org.

Ahmed, S., Tawarmalani, M. & Sahinidis, N. V. 2004 A finite branch-and-bound algorithm for two-stage stochastic integer programs. *Math. Program. A* **100**, 355–377.

Birbil, S. I., Frenk, J., Kaynar, B. & Noyan, N. 2009 The VaR implementation handbook. In: *Risk Measures and Their Applications in Asset Management* (G. N. Gregoriou, ed.). McGraw-Hill, New York, pp. 311–337.

Birge, J. R. 1985 Decomposition and partitioning methods for multistage stochastic linear programs. *Oper. Res.* **33**, 989–1007.

Birge, J. R. & Louveaux, F. V. 1997 *Introduction to Stochastic Programming*. Springer, New York.

Cetinkaya, C. P., Fistikoglu, O., Fedra, K. & Harmancioglu, N. B. 2008 Optimization methods applied for sustainable management of water-scarce basins. *J. Hydroinformat*. **10**, 69–95.

Chang, N. B., Wen, C. G., Chen, Y. L. & Yong, Y. C. 1996 A grey fuzzy multiobjective programming approach for the optimal planning of a reservoir watershed, Part A: theoretical development. *Water Res*. **30**, 2329–2340.

Chaves, P. & Kojiri, T. 2007 Deriving reservoir operational strategies considering water quantity and quality objectives by stochastic fuzzy neural networks. *Adv. Water Resour*. **30**, 1329–1341.

Conejo, A. J., García-Bertrand, R., Carrión, M., Caballero, A. & Andrés, A. 2008 Optimal involvement in futures markets of a power producer. *IEEE Trans. Power Syst*. **23**, 701–711.

Fábián, C. I. 2008 Handling CVAR objectives and constraints in two-stage stochastic models. *Eur. J. Oper. Res*. **191** (3), 888–911.

Faye, R. M., Sawadogo, S., Lishoua, C. & Mora-Camino, F. 2003 Long-term fuzzy management of water resource systems. *Appl. Math. Comput*. **137**, 459–475.

Ferrero, R. W., Riviera, J. F. & Shahidehpour, S. M. 1998 A dynamic programming two-stage algorithm for long-term hydrothermal scheduling of multireservoir systems. *Trans. Power Syst*. **13** (4), 1534–1540.

Guo, P. & Huang, G. H. 2010 Inexact fuzzy-stochastic programming for water resources management under multiple uncertainties. *Environ. Model. Assess*. **15** (2), 111–124.

Guo, P., Huang, G. H. & Li, Y. P. 2010 An inexact fuzzy-chance-constrained two-stage mixed-integer linear programming approach for flood diversion planning under multiple uncertainties. *Adv. Water Resour*. **33** (1), 81–91.

Huang, G. H. 1996 IPWM: an interval-parameter water quality management model. *Eng. Optim*. **26**, 79–103.

Huang, G. H. 1998 A hybrid inexact-stochastic water management model. *Eur. J. Operat. Res*. **107**, 137–158.

Huang, G. H., Baetz, B. W. & Patry, G. G. 1992 An interval linear programming approach for municipal solid waste management planning under uncertainty. *Civil Eng. Environ. Syst*. **9**, 319–335.

Huang, Y., Chen, X., Li, Y. P., Willems, P. & Liu, T. 2010 Integrated modeling system for water resources management of Tarim River Basin. *Environ. Eng. Sci*. **27** (3), 255–269.

Huang, Y., Li, Y. P., Chen, X. & Ma, Y. G. 2012 Optimization of the irrigation water resources for agricultural sustainability in Tarim River Basin, China. *Agric. Water Manage*. **107**, 74–85.

Huang, G. H. & Loucks, D. P. 2000 An inexact two-stage stochastic programming model for water resources management under uncertainty. *Civil Eng. Environ. Syst*. **17**, 95–118.

Jairaj, P. G. & Vedula, S. 2000 Multi-reservoir system optimization using fuzzy mathematical programming. *Water Resour. Manage*. **14**, 457–472.

Kall, P. & Mayer, J. 2005 *Stochastic Linear Programming: Models, Theory, and Computation*. International Series in Operations Research and Management Science. Springer, New York.

Klein Haneveld, W. K. & Van der Vlerk, M. H. 2006 Integrated chance constraints: Reduced forms and an algorithm. *Comput. Manage. Sci*. **3**, 245–269.

Li, Y. P. & Huang, G. H. 2008 Interval-parameter two-stage stochastic nonlinear programming for water resources management under uncertainty. *Water Resour. Manage*. **22**, 681–698.

Li, Y. P., Huang, G. H. & Nie, S. L. 2006 An interval-parameter multi-stage stochastic programming model for water resources management under uncertainty. *Adv. Water Resour*. **29**, 776–789.

Li, Y. P., Huang, G. H. & Nie, S. L. 2007 Mixed interval-fuzzy two-stage integer programming and its application to flood-diversion planning. *Eng. Optim*. **39** (2), 163–183.

Li, Y. P., Huang, G. H., Nie, S. L. & Chen, X. 2011 A robust modeling approach for regional water management under multiple uncertainties. *Agr. Water Manage*. **98**, 1577–1588.

Li, Y. P., Huang, G. H., Wang, G. Q. & Huang, Y. F. 2009 FSWM: a hybrid fuzzy-stochastic water-management model for agricultural sustainability under uncertainty. *Agric. Water Manage*. **12** (96), 1807–1818.

Liu, Y., Cai, Y. P., Huang, G. H. & Dong, C. 2012 Interval-parameter chance-constrained fuzzy multi-objective programming for water pollution control with sustainable wetland management. *Procedia Environ. Sci*. **13**, 2316–2335.

Liu, C., Fan, Y. & Ordóñez, F. 2009 A two-stage stochastic programming model for transportation network protection. *Comput. Oper. Res*. **36**, 1582–1590.

Loucks, D. P., Stedinger, J. R. & Haith, D. A. 1981 *Water Resource Systems Planning and Analysis*. Prentice-Hall, Englewood Cliffs, NJ.

Lu, H. W., Huang, G. H. & He, L. 2010 Development of an interval-valued fuzzy linear-programming method based on infinite α-cuts for water resources management. *Environ. Model. Softw*. **25**, 354–361.

Luo, B., Maqsood, I., Yin, Y. Y., Huang, G. H. & Cohen, S. J. 2003 Adaptation to climate change through water trading under uncertainty – an inexact two-stage nonlinear programming approach. *J. Environ. Inf*. **2** (2), 58–68.

Lv, Y., Huang, G. H., Li, Y. P. & Sun, W. 2012 Managing water resources system in a mixed inexact environment using superiority and inferiority measures. *Stoch. Env. Res. Risk A* **26** (5), 681–693.

Maqsood, I., Huang, G. H. & Yeomans, J. S. 2005 An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty. *Eur. J. Oper. Res*. **167** (1), 208–225.

McIntyre, N., Wagener, T., Wheater, H. S. & Siyu, Z. 2003 Uncertainty and risk in water quality modelling and management. *J. Hydroinformat*. **5** (4), 259–274.

Ogryczak, W. & Ruszczyński, A. 2002 Dual stochastic dominance and related mean-risks models. *SIAM J. Optim*. **13** (2), 60–78.

Piantadosi, J., Metcalfe, A. V. & Howlett, P. G. 2008 Stochastic dynamic programming (SDP) with a conditional value-at-risk (CVaR) criterion for management of storm-water. *J. Hydrol*. **348** (3–4), 320–329.

Pousinho, H. M. I., Mendes, V. M. F. & Catalão, J. P. S. 2011 A risk-averse optimization model for trading wind energy in a market environment under uncertainty. *Energy* **36**, 4935–4942.

Rockafellar, R. & Uryasev, S. 2000 Optimization of conditional value at risk. *J. Risk* **2** (3), 21–41.

Rockafellar, R. & Uryasev, S. 2002 Conditional value-at-risk for general loss distributions. *J. Bank. Financ.* **26**, 1443–1471.

Russell, S. O. & Campbell, P. F. 1996 Reservoir operating rules with fuzzy programming. *ASCE J. Water Resour. Plan. Manage.* **122**, 165–170.

Schultz, R. & Neise, F. 2007 Algorithms for mean-risk stochastic integer programs in energy. *Rev. Invest. Oper.* **28**, 4–16.

Schultz, R. & Tiedemann, S. 2006 Conditional value-at-risk in stochastic programs with mixed-integer recourse. *Math. Program.* **105** (2), 365–386.

Seifi, A. & Hipel, K. W. 2001 Interior-point method for reservoir operation with stochastic inflows. *J. Water Resour. Plan. Manage.* **127** (1), 48–57.

Shao, L. G., Qin, X. S. & Xu, Y. 2011 A conditional value-at-risk based inexact water allocation model. *Water Resour. Manage.* **25**, 2125–2145.

Simonovic, S. 2009 A new method for spatial and temporal analysis of risk in water resources management. *J. Hydroinformat.* **11** (3–4), 320–329.

Slowinski, R., Urbaniak, A. & Weglarz, J. 1986 Probabilistic and fuzzy approaches to capacity expansion planning of a water supply system. In: *Systems Analysis Applied to Water and Related Land Resources* (L. Valadares Tavares & J. Evaristo da Silva, eds). Pergamon, Oxford, pp. 93–98.

Teegavarapu, R. & Elshorbagy, A. 2005 Fuzzy set based error measure for hydrologic model evaluation. *J. Hydroinformatic.* **7** (3), 199–208.

Tran, L. D., Schilizzi, S., Chalak, M. & Kingwell, R. 2011 Optimizing competitive uses of water for irrigation and fisheries. *Agric. Water Manage.* **101**, 42–51.

Victoria, F. B., Viegas Filho, J. S., Pereira, L. S., Teixeira, J. L. & Lanna, A. E. 2005 Multi-scale modelling for water resources planning and management in rural basins. *Agricult. Water Manage.* **77**, 4–20.

Wagner, J. M., Shamir, U. & Marks, D. H. 1994 Containing groundwater contamination: planning models using stochastic programming with recourse. *Eur. J. Oper. Res.* **7**, 1–26.

Wang, X. H. & Du, C. M. 2003 An internet based flood warning system. *J. Environ. Inf.* **2**, 48–56.

Wang, S. & Huang, G. H. 2011 Interactive two-stage stochastic fuzzy programming for water resources management. *J. Environ. Manage.* **92** (8), 1986–1995.

Xu, Y. & Qin, X. S. 2010 Rural effluent control under uncertainty: An inexact double-sided fuzzy chance-constrained model. *Adv. Water Resour.* **33** (9), 997–1014.

Zhang, X. H., Zhang, H. W., Chen, B., Guo, H. C., Chen, G. Q. & Zhao, B.A. 2009 An inexact-stochastic dual water supply programming model. *Commun. Nonlinear Sci.* **14**, 301–309.