A mathematical approach to find long-term strategies for the implementation of resource-orientated sanitation

I. Kaufmann Alves

ABSTRACT

In the present discussion of sustainability centralised water infrastructures are exposed to new challenges, which may cause a conceptual alteration in urban water management. If technologies for closing urban water and nutrient cycles are to at least partially replace existing systems, then intensive reconstruction work becomes essential. The paper presents the development and implementation of a mathematical approach to minimise environmental impact and economic costs on the way to more source-controlled future states in urban water management. To find an optimal transformation strategy, a simultaneous project scheduling and network flow problem was defined as a bi-criteria mixed-integer program. An optimal solution is found by minimising two objective functions concurrently – the economic costs and ‘ecologic costs’ for the period of consideration. This paper discusses the influence of the weighting of these two costs on optimal transformation strategies for a real catchment in Germany. The results show that the approach can very well support decision makers when showing all impacts of transformation processes in detail. All in all, the developed model can be seen as a first step in strategy-finding for transformations in existing urban water systems.

Key words | decision support, mathematical optimisation, mixed-integer linear program, resource-orientated sanitation, transformation strategies

NOMENCLATURE

\( A \) Set of arcs
\( A \) Flow type ‘flow contributing area’
\( b_f \) Factor for dimensioning of arc \( j \)
\( b^K_i \) Demand of a flow \( K \) in a node \( i \) (e.g. wastewater components, rainwater runoff)
\( B_{ipt} \in \{0,1\} \) Variable: begin of construction of arc \( j \) in mode \( p \) at time \( t \)
\( b_{kp} \) Investment costs of arc \( j \) in mode \( p \)
\( bud_t \) Budget at time \( t \)
\( COD \) Chemical oxygen demand
\( d_{ip} \) Construction time of arc \( j \) in mode \( p \)
\( \mathcal{F} \) Set of flow types \( A, R, T, FR, FT \)
\( f^K_j \) (Fictious) flow dependent costs for flow \( K \) on arc \( j \)
\( FR \) Flow type ‘pollution load in stormwater runoff’, here for parameter COD
\( FT \) Flow type ‘pollution load in dry weather flow’, here for parameter COD
\( g^A \) Annually mean rainfall
\( g^T \) Annually mean domestic wastewater flow in present state
\( H_i \) Predecessor arcs
\( i \) Number of node
\( j \) Number of arc
\( K \) Flow types in mathematical model
\( K_i(1) \) Parts of economic costs, \( i \in \{1,...,4\} \)
\( K_i(2) \) Parts of ecologic costs, \( i \in \{1,...,5\} \)
\( i^K_{jp} \) Lower bound of flow \( K \) for arc \( j \) in mode \( p \)
\( LB_{ipt} \in \{0,1\} \) Variable: compliance with minimum flow velocity
\( M_j \) Set of modes for an arc \( j \)
\( mk^K_i \) Flow dependent costs for flow \( K \) on arc \( j \)

doi: 10.2166/wst.2013.691
**INTRODUCTION**

**Background**

In industrialised countries complex water infrastructure systems have developed over decades and centuries. In the present discussion of sustainability they are exposed to new challenges. Especially, problems resulting from demographic and climate change can cause a conceptual alteration in urban water management. If resource-oriented sanitation techniques are to replace or complement existing centralised systems, then extensive reconstruction work becomes necessary. This system change is superposed by a high demand for rehabilitation in existing water supply and sewer networks. Currently, changes or rehabilitation measures are carried out rather as a result of locally limited and ‘spontaneous’ decisions.

The methodical and optimal procedure of an extensive or complete conversion is a challenging question. Such a retrofitting can only be realised successively over a long period and requires new strategies for transition. At all stages of a transition reliable water supply and disposal of

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>MV</td>
<td>Set of arcs for which flow velocity is tested</td>
</tr>
<tr>
<td>nd_{IST}</td>
<td>Remaining useful life span for elements at time t = 0</td>
</tr>
<tr>
<td>nd_{jp}</td>
<td>Useful lifespan of arc j in mode p</td>
</tr>
<tr>
<td>nds_{jp}</td>
<td>Useful lifespan of arc j in mode p after rehabilitation</td>
</tr>
<tr>
<td>NI</td>
<td>Set of arcs not existing in present state</td>
</tr>
<tr>
<td>p</td>
<td>Mode</td>
</tr>
<tr>
<td>r_{ipi}</td>
<td>Necessary quantity of resources i for arc j in mode p</td>
</tr>
<tr>
<td>R</td>
<td>Flow type ‘design rainfall runoff’</td>
</tr>
<tr>
<td>ra_{it}</td>
<td>Available resources of resource type i at time t</td>
</tr>
<tr>
<td>res</td>
<td>Resources</td>
</tr>
<tr>
<td>$\mathcal{K}$</td>
<td>Set of resource types</td>
</tr>
<tr>
<td>RT</td>
<td>Sum of flow types ‘design rainfall runoff’ and ‘dry weather flow’</td>
</tr>
<tr>
<td>$S_{ipi} \in {0,1}$</td>
<td>Variable: rehabilitation of arc j in mode p at time t</td>
</tr>
<tr>
<td>sk_{ip}</td>
<td>Costs for rehabilitation and reinvestment respectively of arc j in mode p</td>
</tr>
<tr>
<td>spk_{i}</td>
<td>Higher sewer flushing costs for arc j</td>
</tr>
<tr>
<td>T</td>
<td>Flow type ‘dry weather flow’</td>
</tr>
<tr>
<td>Ti</td>
<td>Successor arcs</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time of consideration</td>
</tr>
<tr>
<td>$\tau^{(p)}$</td>
<td>Timespan, in which an arc j in mode p exists</td>
</tr>
<tr>
<td>$\tau^{(p)}_{B}$</td>
<td>Timespan, in which an arcs j in mode p can be built</td>
</tr>
<tr>
<td>$\tau_{B}$</td>
<td>Time for transformation</td>
</tr>
<tr>
<td>u_{ip}^{K}</td>
<td>Upper bound of flow K for arc j in mode p</td>
</tr>
<tr>
<td>$\nu_{ip}^{RT}$</td>
<td>Maximum flow RT in arcs j in mode p representing transport elements</td>
</tr>
<tr>
<td>uk_{ip}</td>
<td>Operational costs of arc j in mode p</td>
</tr>
<tr>
<td>$\mathcal{V}$</td>
<td>Set of nodes</td>
</tr>
<tr>
<td>$X_{R}^{K}$</td>
<td>Variable: flow K on arc j at time t</td>
</tr>
<tr>
<td>$X_{R}^{A}$</td>
<td>Variable: flow A on arc j at time t</td>
</tr>
<tr>
<td>$X_{R}^{R}$</td>
<td>Variable: flow R on arc j at time t</td>
</tr>
<tr>
<td>$X_{R}^{FR}$</td>
<td>Variable: flow FR on arc j at time t</td>
</tr>
<tr>
<td>$X_{R}^{FT}$</td>
<td>Variable: flow FT on arc j at time t</td>
</tr>
<tr>
<td>$X_{gew}$</td>
<td>Flow R discharged to receiving waters</td>
</tr>
<tr>
<td>$X_{gew}$</td>
<td>Flow T discharged to receiving waters</td>
</tr>
<tr>
<td>$X_{gew}$</td>
<td>Flow FR discharged to receiving waters</td>
</tr>
<tr>
<td>$X_{gew}$</td>
<td>Flow FT discharged to receiving waters</td>
</tr>
<tr>
<td>$X_{rs}$</td>
<td>Actual values of resources protection components</td>
</tr>
<tr>
<td>$X_{wh}$</td>
<td>Actual values of water balance components</td>
</tr>
<tr>
<td>$Y_{ipi} \in {0,1}$</td>
<td>Activation variable: activation of arc j in mode p at time t</td>
</tr>
<tr>
<td>Z</td>
<td>Interest rate</td>
</tr>
<tr>
<td>ZF</td>
<td>Objective function</td>
</tr>
<tr>
<td>$z_{f}$</td>
<td>Factor for calculation time-dependent operational costs on arc j</td>
</tr>
<tr>
<td>$Z_{rs}$</td>
<td>Goal values for resources protection components</td>
</tr>
<tr>
<td>$Z_{wh}$</td>
<td>Goal values for natural water balance components</td>
</tr>
<tr>
<td>$Z_{gew}$</td>
<td>Set of arcs containing receiving waters</td>
</tr>
<tr>
<td>$Z_{r}$</td>
<td>Set of arcs containing resources protection values</td>
</tr>
<tr>
<td>$Z_{w}$</td>
<td>Set of arcs containing natural water balance values</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Time step</td>
</tr>
<tr>
<td>$\gamma_{i}(1)$</td>
<td>Weights for portions of economic costs, i $\in$ {1,...,4}</td>
</tr>
<tr>
<td>$\gamma_{i}(2)$</td>
<td>Weights for portions of ecologic costs, i $\in$ {1,...,5}</td>
</tr>
<tr>
<td>$\lambda_{j}^{K}$</td>
<td>Flow diversion factor of flow K for arc j</td>
</tr>
</tbody>
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The methodical and optimal procedure of an extensive or complete conversion is a challenging question. Such a retrofitting can only be realised successively over a long period and requires new strategies for transition. At all stages of a transition reliable water supply and disposal of
wastewater have to be guaranteed. To ensure that every step of reconstruction ecologically and economically benefits the future, an optimised strategy should be investigated. As these two aspects – financial efforts and impact on the environment – have to be minimised, the development of a mathematical optimisation model seems obvious.

State-of-the-art and aim of the study

The use of linear optimisation models (linear programs – LPs) in urban water management history goes back almost 50 years. Since the 1960s operations research methods have been used for planning alternatives in regional wastewater systems (listed for example in de Melo & Câmara (1994)). In recent applications in wastewater disposal, LPs are used to find cost-efficient combinations of wastewater systems under pollution load constraints (Refsgaard 2001), to evaluate the sustainability of decentralised sanitation and reuse (DESAR) concepts (van der Vleuten-Balkema 2003) or to analyse the potential of wastewater reuse under physical and economic constraints (Chu et al. 2004). Furthermore, decision support approaches for the selection of sustainable urban drainage systems (SUDS) or DESAR concepts have been investigated in a number of studies including assessment methods for comparing different options of sanitation (e.g. Icke et al. 1999; Niederer et al. 2007; Thévenot 2008; Pearson et al. 2010). Only a few decision support approaches exist, which comprise technical option selection for integrated urban water management (Mitchel & Diaper 2006; Makropoulos et al. 2008). Most published decision support approaches are used to choose an appropriate future system solution. Research on a complete transformation of water infrastructure systems seems to be rather rare. Most studies (e.g. Kluge & Libbe 2006; Duffy & Jeffries 2011; Schiller 2010) focus on organisational and financial aspects, whereas the technical part is not investigated profoundly.

Therefore this paper will present the implementation of a mathematical tool to schedule time- and resource-restricted activities in such work- environments. A mathematical tool to find an optimised strategy for the realisation of resource-orientated drainage and sanitation concepts in existing urban areas. Two components of mathematical optimisation are of interest for the mentioned question – project scheduling problems (PSP) and network-flow problems (NFP). The aim of project planning is to schedule time- and resource-restricted activities in such a way that an objective function dependent on starting times of the activities is optimised. In NFP a flow function is implemented on a network of nodes and arcs. A flow-dependent objective function has to be optimised regarding flow conservation and capacity constraints. A simultaneous-constrained consideration of both problems has not been worked on so far. However, for a realistic modelling of long-ranging retrofitting and construction works in urban water management systems, this would be necessary. The presented approach will enable the evaluation and minimisation of time-dependent impacts not only on financial costs but also on the operability of systems and on the environment.

METHODOLOGY

Development and formulation of the model

The mathematical model is formulated as a bi-criteria mixed-integer program with two objective functions (financial effort and environmental impact). For the first time a simultaneous project scheduling and network flow problem, ProNet, has been defined. Details about the model structure, a complex arc-node-network, are given in Kaufmann et al. (2007). Within this network, nodes and arcs have to be valid for PSP and NFP simultaneously. Nodes in the activity network of the PSP represent construction activities (implementation of new devices, rehabilitations and adaptations). In the flow network of the NFP nodes represent at the one hand elements of urban settlements (surface and wastewater types) and on the other hand elements of water infrastructure (e.g. network nodes, storage or treatment facilities or sinks and sources such as ground, receiving waters or air). The arcs connect the nodes in direction of ‘flows’. Arcs are weighted, i.e. they are associated with some values: costs (\(b_{ip}, u_{ip}\)) and lifespans or installation periods (\(d_{ip}, nd_{IST}, nd_{ip}, nd_{ipj}\)) for the activity network and capacities and diversion factors for the different flows in the flow network (\(X_{ip}^K, X_{ip}^{K^T}, X_{ip}^{FS}\)). The defined flow variables \(X_{ip}^K\) of the model are: maximum design rainfall (\(X_{ip}^{KFR}\)), mean dry weather flow (\(X_{ip}^{K}\)) with its components, runoff contribution area (\(X_{ip}^{K}\)) and pollution loads in dry weather flow (\(X_{ip}^{K^T}\)) and stormwater runoff (\(X_{ip}^{FS}\)). These flows are ‘led’ through the network, which is expanded as new elements are built and reduced if elements are shut down respectively.

In the PSP, specifications for starting times of activities, construction periods, useful life spans as well as budget and resource restrictions are formulated in linear (un)equations.

The ‘building variable’ \(B_{ipt}\) equals 1 when a device is implemented; variable \(S_{ipt}\) (rehabilitation or reinvestment of devices) has to become 1 when the useful lifespan is over. The indices mark the number of the respective arc (\(j\)), the mode in which an arc is activated (\(p\)) and the...
time \((t)\). (The mode represents different outcomes of characteristics of an arc, e.g. different plant sizes or different diameters of sewers.) With these variables, main constraints for the PSP are:

- **Compliance with resource restrictions**

\[
\sum_{j \in A} \sum_{p \in M_j} \sum_{k = \max\{0, t - d_{ip} + 1\}}^t r_{ip} B_{ipk} \leq r_{ai} \quad \forall i \in \mathcal{R}, \ t \in \mathcal{T}
\]

(1)

- **Compliance with budget restrictions**

\[
\sum_{j \in A} \sum_{p \in M_j} b_{ip} B_{ip} \leq bud_i \quad \forall t \in \mathcal{T}
\]

(2)

- **Construction of new devices, in a way they are finished in time for transformation \(T_B\)**

\[
B_{ip} = 0 \\
\forall j \in A, \ p \in M_j; d_{ip} > 0, \ t \in \mathcal{T}_B^{(j,p)} = \{T - d_{ip} + 1, \ldots, T\}
\]

(3)

The combination to ProNet is accomplished by the implementation of a variable \(Y_{ipt}\), which marks the date, onward from which a new element can be used after completion. The main constraints for this variable are:

- **Activation of arcs after installation period until time \(T\)** (end of period under consideration)

\[
Y_{ipt} = \sum_{r = 0}^{t - d_{ip}} B_{ipr} \quad \forall (j, p) \in \mathcal{N}, \ t \in \mathcal{T}_B^{(j,p)} = \{d_{ip}, \ldots, T\}
\]

(4)

- **Reinvestment/rehabilitation at the end of useful lifespan**

\[
\sum_{r = 0}^t \ Y_{ipt} \geq nd_{ip} \sum_{r = 0}^t S_{ipr} \quad \forall j \in A, \ p \in M_j, \ t \in \mathcal{T}
\]

(5)

\[
\sum_{r = 0}^t \ Y_{ipt} - nd_{ip} \leq \sum_{r = 0}^t nd_{ip} S_{ipr} \quad \forall j \in A, \ p \in M_j, \ t \in \mathcal{T}
\]

(6)

- **Flow conservation constraints for every node**

\[
\sum_{j \in A} X_{jt}^K - \sum_{j \in H_t} X_{jt}^K = b_t^K \quad \forall K \in \mathcal{F}, \ i \in \mathcal{V}, \ t \in \mathcal{T}
\]

(8)

- **Flow diversion constraints (e.g. infiltration rates, runoff coefficients, removal efficiencies)**

\[
X_{jt}^K \leq \lambda_j^k \left( b_t^K + \sum_{e \in H_t} X_{jet}^K \right) \quad \forall K \in \mathcal{F}, \ i \in \mathcal{V}, \ j \in T_t, \ t \in \mathcal{T}
\]

(9)

- **Activation of arcs in adequate mode \(p\)** (variable capacity boundaries)

\[
b_f \left( X_{jt}^R + X_{jt}^T \right) \leq \sum_{p \in M_j} u_t^{RT} Y_{ipt} \quad \forall j \in A, \ t \in \mathcal{T}
\]

(10)

\[
X_{jt}^K \leq \sum_{p \in M_j} u_t^{K} Y_{ipt} \quad \forall K \in \{T, A, RT\}, \ j \in A, \ t \in \mathcal{T}
\]

(11)

\[
X_{jt}^K \geq \sum_{p \in M_j} l_t^{K} Y_{ipt} \quad \forall K \in \mathcal{F} = \{R, T, A, FR, FT\}, \ j \in A, \ t \in \mathcal{T}
\]

(12)

An optimal solution represents a set of variables in every time step, which fulfils constraints and minimises objective functions. The objective functions economic costs \(K(1)\) and ecologic costs \(K(2)\) have to be minimised concurrently for the period of consideration. Ecologic costs as present values with an interest rate \(Z\) consist of:
• Investment and reinvestment costs

\[ K_1(1) := \sum_{t \in T} \left( \frac{1}{1 + Z_t^t} \left( \sum_{j \in A} \sum_{p \in M_t^j} b_{jp} B_{ipt} + s_{jp} S_{ipt} \right) \right) \]  

(13)

• (Time-dependent) operational costs

\[ K_2(1) := \sum_{t \in T} \left( \frac{1}{1 + Z_t^t} \left( \sum_{j \in A} \sum_{p \in M_t^j} u_{jp} Y_{ipt} + z_f u_{jp} \left( \sum_{r = 0}^t Y_{ipt} - nd_{jp} \sum_{r = 0}^t S_{ipt} \right) \right) \right) \]  

(14)

• Additional operational costs by reduction of minimal flow velocity

\[ K_3(1) := \sum_{t \in T} \left( \frac{1}{1 + Z_t^t} \left( \sum_{j \in MV} \sum_{p \in M_t^j} spk_{jp} L_{ipt} \right) \right) \]  

(15)

• Flow-dependent monetary costs (e.g. sewage tax)

\[ K_4(1) := \sum_{K \in F} \sum_{t \in T} \left( \frac{1}{1 + Z_t^t} \left( \sum_{j \in A} m_{jk} X_{ipt}^k \right) \right) \]  

(16)

and result in:

\[ K(1) := \sum_{i = 1}^4 \gamma_i(1) K_i(1) \]  

(17)

Ecologic costs \( K(2) \) are calculated as sum of:

• Costs for emissions in receiving waters

\[ K_3(2) := \sum_{g \in Z_{gew}} \sum_{t \in T} \left( X_{IT_{gew}} + X_{IT_{gew}} \right) \]  

(20)

• Costs for immissions

\[ K_4(2) := \sum_{g \in Z_{gew}} \sum_{t \in T} \left( X_{IT_{gew}} + X_{IT_{gew}} \right) \]  

(21)

• (Fictitious) flow-dependent costs

\[ K_5(2) := \sum_{K \in F} \sum_{t \in T} \sum_{j \in A} f_{Kj} X_{ipt}^k \]  

(22)

and result in:

\[ K(2) := \sum_{i = 1}^5 \gamma_i(2) K_i(2) \]  

(23)

The different ecologic cost portions can directly be accounted as differences or sums as formulated in Equations (18)–(21) or can be scaled to an interval \([0,1]\), as shown for two components in Figure 1, wherein 1 represents the highest detriment (can be chosen to characterise present state) and 0 being the matching of a target value (no detriment).

The solution of the LP represents an optimal strategy towards the future state, which could not be enhanced in both criteria (economic and ecologic costs). Generally, not only one solution of the optimisation problem exists but numerous reasonable Pareto-optimal solutions (see for example Ehrgott (2005)). Pareto solutions are those for which improvement in one objective can only occur with the worsening of the other objective. Therefore, a ‘compromise’ has to be found.

**Procedure of model application**

The implementation of the developed program follows the flowchart presented in Figure 2.

**Data analysis and preparation**

For the implementation of the mathematical model for real catchment areas the main work is data analysis and preparation. An important step is to subdivide the entire
catchment in subcatchments since the scale of single houses or lots would be too detailed. This step is dependent on numerous factors in the fields of subcatchment data, pollution data, topography parameters as well as sewer network and receiving water data. For these subcatchments then, appropriate activities for SUDS, DESAR and drainage of remaining flows can be prepared. These feasible, necessary or infeasible activities are arranged in a specific catalogue of activities. Here, for each catchment’s characteristics the expedient installation and operational costs for the measures – prepared for different possible sizes of the device depending on connected surfaces or dry weather flow components (cf. mode $p$) – are stored.

Beside the generation of this catalogue a simplification of existing sewer networks is advisable. For the resulting ‘computational network’, capacity and flow velocity constraints are tested. At the same time possible new transport elements such as foul sewers, vacuum pipes, stormwater sewers or surface drains have to be prepared with possible flow paths and discharge points. Within, all possible dimensions are provided with allocated capacities and costs.

In a detailed pre-processing routine the combination of parameters, activities, costs, connections and interrelations to the input formats for the mathematical model takes place.

**Mathematical modelling**

Subsequent to data preparation the model ProNet can be applied. Indeed, many of the implicated constraints or evaluations are dependent on the specific catchment. Goal values for a natural evaporation rate or infiltration rate and initial-state parameters are different for each application. Mean values for runoff coefficients or infiltration rates for surface types are dependent on precipitation characteristics and should be evaluated for the specific case if possible. Optimisation parameters have to be chosen and weighted.

To solve the multi-criteria problem two methods for constructing a single aggregate objective function are implemented in the optimisation tool: the weighted sum method and the $\varepsilon$-constraint method. At the first method, for the two objective functions, economic cost $K(1)$ and ecologic cost $K(2)$, scalar weights are specified and the sum of both has to be minimised. At the second method one objective function is limited to a specific value and the second objective function has to be minimised without exceeding the limit of the other objective. The problem itself is solved with the software ILOG CPLEX® 9.0 and 11.0 (ILOG 2005). The mathematical programming engine CPLEX® uses Branch-and-Cut-methods for solving mixed-integer linear programs.

**Interpretation**

In the solution of the mathematical optimisation the results for all elements of the mathematical network are documented for every calculated time step. For larger catchment areas with numerous subcatchments, this number of representations becomes too complex to review. Moreover, in different applications, different interpretations of the result are of interest: in some cases only objective functions’ values are important, in other cases specific points of the drainage system are of interest. Therefore in a substantial post-processing the desired representations of results are automatically generated.
Figure 2 | Flowchart for model application.
APPLICATION IN A CASE STUDY

Investigated catchment

The model has been applied to a suburb of Kaiserslautern in Germany, a rural catchment of about 3,000 inhabitants. Conditions of this town are representative for numerous rural structured communities in Germany. The entire catchment has a drainage area of about 90 ha and implies 35 ha of paved area. About 70% is drained by combined sewer systems whereas newer developed areas are drained by separate sewer systems. Two combined sewer overflow devices and one final sewer overflow tank are installed in the sewer system. Twenty hectares of the catchment is a business park, the effluent of which shows the characteristics of domestic wastewater. Dry weather flow amounts to 11.4 L/s and consists of 8.2 L/s foul sewage and 3.2 L/s infiltration water. The pollution of chemical oxygen demand (COD) in dry weather flow is 560 mg O₂/L. Wastewater is led to the central wastewater treatment plant (WWTP) of Kaiserslautern. For this study the catchment area was subdivided into six subcatchments according to numerous parameters such as land use, topography, subsurface conditions or population density.

Future state and boundary conditions

The approach presented here allows finding an optimised strategy to reach a favoured future state. The future state itself with the general decision for a sanitation concept is known. Moreover, goal values for a more natural water balance or a resource protection have to be defined. For this paper the chosen specifications for a possible sustainable future state are summarised in Table 1.

In this example different weights of \( K(1) \) and \( K(2) \) in objective function are examined by using the weighted sum method. Table 2 shows the six different investigated scenarios to reach the future state specified in Table 1. Among the two extreme cases S0 and S5, four meaningful combinations are calculated. For example, S0 characterises an optimal strategy for reaching the future state with minimal economic costs, regardless of environmental impact. In S5 reverse preferences in reaching the same future state are investigated: the strategy should have minimal ecologic impact while economic costs are non-relevant.

Another important definition is the criteria that should contribute to the objective function of ecologic costs \( K(2) \). So far, 11 different criteria in four fields are implemented in the model (cf. \( K_1(2) - K_4(2) \) in Equations (18)–(21)). Depending on specific conditions they have to be chosen in a balanced combination appropriate for the application case. For this example, the three fields of adapting a more natural water balance \( K_1(2) \), resources protection \( K_2(2) \) and emission \( K_3(2) \) were chosen and weighted equally with 1/3. Every criteria \( K_i(2) \) of \( K(2) \) is scaled to an interval from 0 to 1 (see Figure 1). The ecologic costs per time-step will then result as the weighted sum of the chosen criteria, which will sum up to a maximum of 1.

In this example a total period of consideration of 80 years is chosen. Within, the transformation to the favoured future state is calculated for a period of 50 years, and a period of 30 years afterwards is investigated to evaluate the ‘maintenance costs’ of the future state. The economic costs are calculated as total project costs with an interest rate of 3% relating to starting point of consideration \( t = 0 \). This interest rate is recommended for cost calculations in water management by LAWA (2005), where also neglecting of inflation is proposed. In addition, a budget of €2.5 million € per time step (\( \Delta T = 5 \) years) is given.

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<tr>
<th>Table 1</th>
<th>Specifications for favoured future state</th>
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</thead>
<tbody>
<tr>
<td>Overall specifications</td>
<td>Stormwater runoff and wastewater should not be mixed any more; near-nature and source-controlled management</td>
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<tr>
<td>Greywater</td>
<td>Completely decentralised treatment</td>
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<tr>
<td>Blackwater</td>
<td></td>
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<tr>
<td>Specific goal values</td>
<td>Evaporation rate</td>
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<tr>
<td></td>
<td>Infiltration rate</td>
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<td></td>
<td>Rate of rainwater utilisation</td>
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<tr>
<td></td>
<td>Rate of greywater recyclinga</td>
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<tr>
<td></td>
<td>Rate of fertiliser and energy recovery</td>
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<tr>
<td></td>
<td>Rate of direct reduction of use of potable water</td>
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<td></td>
<td>As a percentage of mean precipitation height</td>
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*aNot chosen for objective function as greywater should be treated centrally at WWTP
RESULTS AND DISCUSSION

Objective functions’ values

The economic and ecologic costs for the different transformation scenarios S0 to S5 are shown in Figure 3, where every scenario is characterised by a point in the $K(1)$–$K(2)$ graph. With increasing weight on ecologic costs $K(2)$ for the optimal strategy of transition, the economic costs will rise. The conditions of this specific case result in €32 million as minimum economic cost $K(1)$ to achieve and maintain the future state (scenario S0). This high value result is because numerous measures are required anyway to reach the future state. If only the ecologic costs $K(2)$ contribute to the objective function (scenario S5), a minimum of 7.2 results for these costs in the calculated 80 years. As sum of $K(2)$, a maximum of 16 (for 16 time steps in 80 years) could result, if no new elements were implemented. The environmental impact thus can be reduced to a great extent in optimal strategies.

A comparison of the strategies resulting in S0 and S2 shows, obviously, that with few additional financial efforts environmental impacts can be reduced efficiently. On the other hand it can be seen that higher weights than 1 for $K(2)$ (S3, S4 and S5) allow only a marginal reduction of ecologic costs with high financial effort. A more detailed comparison of scenario S2 and S5, differing by only 0.3 in ecologic costs but by 50% in economic costs, is done in the following.

Figure 4 allows a closer look at those aspects of the ecologic costs $K_i(2)$ that are chosen to contribute to the objective function. For the total period of consideration, in every axis of the radar chart the sum of costs for each criterion is shown for the both scenarios S2 and S5 in comparison to S0. That means for example that the ‘costs’ for adaptation to a near-nature infiltration rate in scenario S2 sum up to 0.6. Although the criteria $K_i(2)$ are weighted equally they are not minimised in the same way in the different scenarios. The difference in the sum of $K(2)$ in scenario S2 and S5 results in a lower impact on the water balance criteria in S5, lower emissions in S2 and nearly the same values for resources protection for both scenarios. So in sum, S3 produces lower ecologic costs when the impacts on criterion ‘rainwater utilisation’ are lower. The influence of this criterion is very high, as a high rate of rainwater should be used for substitution of potable water and these installations are more expensive than other natural rainwater management devices. In fact, the reduction of costs for this criterion can only be achieved by worsening other criteria, e.g. costs for emissions.

Table 2

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Economic costs $K(1)$</th>
<th>Ecologic costs $K(2)$</th>
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</thead>
<tbody>
<tr>
<td>S0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>S1</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>S2</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>S3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>S5</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

In Figure 5 it is shown that the results for $K_i(2)$ are dependent on the definition of ecologic costs, where criteria result from the evaluation of the distance from favoured goal values. That means the actual value of, for example, infiltration rate is compared to the more natural goal value of infiltrated percentage of rainwater. Figure 5 illustrates the minimisation of the three criteria for water balance. The values in the temporal sequence are shown for the discussed scenarios S2 and S5.

The corresponding costs for the implementation of devices in the field of SUDS, which will cause changes in the water balance, are shown as an area diagram in Figure 5. The costs are calculated at the beginning of implementation of devices. Due to installation periods and the used time step of 5 years, the effects can appear later. In S5 the three distances of water balance components are reduced more efficiently and right from the beginning. The early implementation of a bigger part of SUDS devices and the higher number of devices cause economic costs to be €7.5 million.
higher. If devices would have been implemented later, as done in S2, they would have caused lower costs $K(1)$ due to the interest rate. However, the distance from goal values then stays high for a longer period producing higher total costs $K(2)$. Even in S5, with highest weight on ecologic costs, the goal values are not reached exactly. On the one hand, the regeneration of a natural water balance in existing developments is difficult and on the other hand in the shown scenarios economic costs are restricted by a budget.

**Decision on a solution**

The choice of an appropriate solution out of the Pareto-optimal solutions is difficult. Recommendations for the
definition of optimisation criteria result from the respective objective of a system transformation (Kaufmann Alves 2012). So if new devices are to be implemented in a rather short time, then ecologic costs should have a high weight. This can be the case if, for example, receiving waters are very weak, the existing sewer system is surcharged or other regulations are not fulfilled. The discussion with local decision makers will lead to the definite choice of an optimal strategy for system transformation. In this application, S2 could be a good preference as, with rather moderate costs, a considerable reduction of environmental impact during the transformation process is possible. It has to be kept in mind that other criteria and weights $\lambda_i(2)$ for components of the ecologic costs $K_i(2)$ will have caused other strategies.

For decision-making processes many more result representations can be considered. At every point of interest, e.g. resulting pollution loads, changes in dry weather flow components connected to centralised networks or quantity and quality of WWTP inflow can be illustrated.

The most important result, if having decided for one strategy, will be a construction schedule, where construction and rehabilitation periods for the implemented devices are demonstrated in temporal and spatial sequence for every subcatchment. Figure 6 shows a small section for one subcatchment for scenario S2. In this solution, many devices of SUDS are implemented. The former combined sewer system is used to drain remaining stormwater runoff. Long installation periods for implementation of decentralised blackwater treatment are due to the fact that here all components, house installations as well as biogas plant and connections are included.

Discussion

To sum up these results, it can be seen that different optimal solutions are possible to reach one future state. The introduced approach can support local decision makers by making it possible to show all impacts in detail when calculating different scenarios.

The data analysis and preparation to formulate the mathematical network is very expensive. But for complex systems an optimal solution for transformation processes cannot be found manually if, besides the financial costs, aspects of the urban water and nutrient cycle have to be taken into account. Even the investigation of small settlements results in a difficult optimisation problem, as many interactions in water systems are modelled. Thus, simplifications are necessary. However, the developed optimisation tool produces feasible solutions for a succession of retrofitting measures, which meet functional and flow constraints. The objective functions represent the financial frame and quality of adaptation to target values in an adequate way. A problem is to specify the weights of the two introduced costs. Different dimensions and scales make it difficult to choose a predefined value. Also, diverse characteristics of ecological criteria complicate choice and weighting of cost portions $K_i(2)$. In the result, a lower value of $K_i(2)$ can imply on the one hand a better adaptation to goal values and a lower emission rate respectively. On the other hand, it can stand for a ‘faster’ adaptation with the same final values in a future state, as the sum of $K(2)$ has to be minimised. The definition of ecologic costs for transformation phases thus remains difficult. Nevertheless, new knowledge about time-dependent demonstration of environmental impact and effects on infrastructures is of great value.

Varied strategies for transformation (different weights, different optimisation criteria) do not differ much in choice of implemented devices, as with a given future state (e.g. complete decentralised treatment of blackwater) many retrofitting steps are necessary anyway. In fact, the dates of implementation during the transformation period and the plant sizes, especially for sustainable drainage elements, are the significant differences when choosing different optimisation criteria. With high weight on ecologic costs, effective measures are implemented at the beginning.
of the transformation period even though investment costs are high. Strategies that should result in low financial efforts (high weight on economic costs) initially show an implementation of measures with low expense.

The calculated results cannot be verified in reality at the moment, as so far an extensive transformation of larger settlements has not been undertaken. The trend of conclusions about transformations is in agreement with the literature: the further use of infrastructures – here sewers and WWTP – is considered as important in system transformations (e.g. Hiessl et al. 2003; Kluge & Libbe 2006; Hillenbrand 2009; Meinzinger 2010). Starting points of transformations are seen in short-term realisable measures with low technical and financial effort. Later areas with higher transformation effort could follow. Such strategies result in the model when a high weight is put on economic costs K(1).

At a transformation of urban water systems, a bigger part of retrofitting has to be done in private households. This could make it difficult to determine spatial and temporal distribution of measures in terms of optimal strategies. Communities and public policy-makers must ensure an aim-orientated way towards sustainable future systems (e.g. promotion and support programmes).

**CONCLUSIONS**

An extensive integration of new systems for drainage and sanitation in existing infrastructures requires, besides a suitable choice of techniques, a strategy for implementation. For development of such strategies mathematical optimisation methods can be an adequate instrument. The approach presented in this paper showed that:

- the overall aim of the developed model, to find and optimise a long-range transformation strategy, could be reached in different implementations,
- solutions resulting from calculations with the model are Pareto-optimal transformation strategies, at which weighting of introduced costs has a high influence,
- changes in optimisation criteria generally lead to an earlier or later implementation of respective measures and to different plant sizes,
- recommendations for the definition of optimisation criteria result from the respective objective of a system transformation,
- the interaction with local deciders is important and will lead to the definite choice of an optimal strategy for application.

All in all, the developed model can be seen as a first step in a strategy for finding transformations in existing urban water systems. It cannot replace engineering design studies but gives an important indication for strategies for extensive transformations. Until now, several important data have not been included in the model, e.g. variable boundary conditions. So far the work has focused on technical aspects (functioning, dimensioning and fulfilment of regulations); in future work different planning and policy levels should be included. Nevertheless, it could be shown that an extension of existing assessment methods in terms of a detailed temporal and spatial evaluation of the economic and ecological situation is essential. The exclusive planning and consideration of a favoured sustainable future state of urban water systems is inadequate for assessment of effects of extensive implementation of resource-orientated wastewater management.

**REFERENCES**


First received 2 January 2012; accepted in revised form 30 October 2012