For example, if the body must rotate down to bottom dead center, the dimension c, should be one half the maximum spring deflection.

4 Write the equilibrium equation for the body at some angle α, including moment components due to all parts of the yielding means. Cancel out the cosine factor as in Equation [3], and substitute for the spring force F the expression of Equation [5] or [9]. Rearrange and collect terms into the form of Equation [6] to determine X, Y, and Z. Compute the distance d from Equation [7], and r from Equation [8].

5 Check the geometry of the system to insure that there is sufficient clearance when the body is in its uppermost position and that the maximum deflection of the spring is not exceeded when the body is in its lowest position. If necessary, alter the dimension c slightly and revise the computations of step 4 accordingly.

6 In the design of balancing systems for commercial products, it will normally be necessary to provide adjustment nuts at the end of the tie rod (or cylindrical case) and also a slot on the line AD, as shown in Fig. 2. The purpose of these features, other than to accommodate changes in load, is to permit small final adjustments to be made in the dimensions r and d at the time of assembly, to correct for the small variations in free length and modulus customarily encountered in commercially manufactured springs. These adjustments, the magnitude of which will depend upon the spring tolerances specified, may be made either empirically or by correcting the computations of the distances r and d made in step 4.

Discussion

M. F. Spotts.[2] This paper should be of worth-while service to designers confronted with counterbalancing problems. By an ingenious arrangement of links, the author has shown how a pivotally mounted weight can be kept in static equilibrium regardless of the angular position of the weight. In other words, a spring with a straight-line stress-strain relationship can be made to produce a variable moment equal and opposite to that of the weight as it rotates on its pivot. Practical difficulties may arise at times in providing sufficient clearance for the spring and tie rod. These members may become rather large as compared to the size of the remainder of the mechanism.

Springs with nonlinear load-deflection characteristics are frequently needed in mechanical equipment. Volute springs and blocks of rubber can be used where a spring of stiffness increasing with deflection is required. The Belleville spring can be proportioned to give a wide variety of stress-strain curves. One of the most useful is that one where a considerable range of deflection can be obtained at a constant load. A linear leaf spring can be made to give a nonlinear curve by mounting the ends in suitably designed shackles.

Another case where a linear spring is made to give a nonlinear load-deflection curve consists of the torsion-bar spring frequently applied to vehicles. Such a spring with a Hickman type suspension is shown in Fig. 4 of this discussion. The spring runs parallel to the frame of the vehicle; the far end of the spring is fixed to the frame and the near end is carried in a bearing attached to the frame. The near end of the spring is connected by splines to the torque arm OA. Shackle AB connects the end of the torque arm to a projection of the axle extending to B. If the pin at A should be removed and the spring be completely unloaded, the torque arm would assume position OA_o at angle α with the horizontal. Point B is offset inwardly by amount a from the full radius r.

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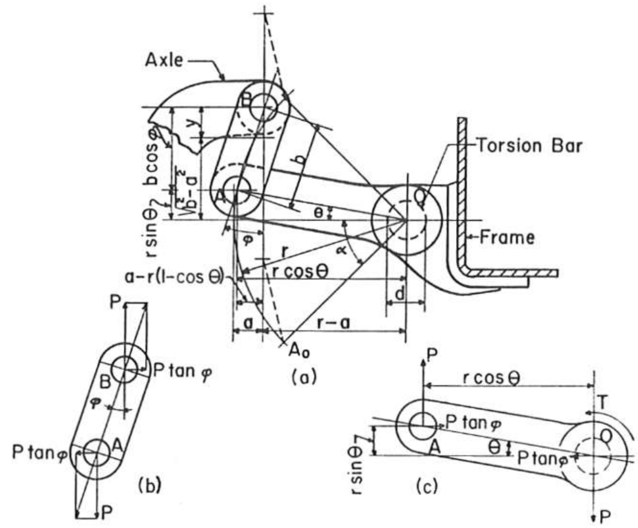

Fig. 4 Hickman Type Mount of Torsion-Bar Spring

Let P be the vertical load carried by the spring. Let angle θ represent the inclination of the torque arm with the horizontal, and angle φ represent the inclination of the shackle with the vertical. A relationship between angles θ and φ, from Fig. 1(a), is seen to be

$$\sin \varphi = \frac{a - r(1 - \cos \theta)}{b} \quad [10]$$

The shackle, as a two-force member, must have a resultant load lying along AB. It will therefore be subjected to the loads shown in Fig. 4(b). Fig. 4(c) shows these loads transferred to the end of the torque arm at A. The torque about point O is equal to

$$T = Pr \cos \theta + Pr \sin \theta \tan \varphi \quad [11]$$

The total angle of twist sustained by the spring is $\alpha + \theta$. By elementary theory for the torsion of round bars

$$T = \frac{JG(\alpha + \theta)}{l} \quad [12]$$

where J = polar moment of inertia for the cross section
G = modulus of elasticity in shear for the material
l = effective length of the spring

Equations [11] and [12] can be combined to give

$$Pr\,l\,(\cos \theta + \sin \theta \tan \varphi) = JG(\alpha + \theta) \quad [13]$$

Figure 4(a) shows that the axle travel y is equal to

$$y = r \sin \theta + b \cos \varphi - \sqrt{b^2 - a^2} \quad [14]$$

Equations [10], [13], and [14] permit the load-deflection curve of the spring to be plotted. Suitable values for θ are chosen, and the corresponding values of φ are found by Equation [10]. Load P and axle travel y are then found by Equations [13] and [14].

Fig. 5 gives the load-deflection curve for a suspension of the following values:

Spring diameter, d = 1.200 in.
Effective length, l = 120 in.
Polar moment of inertia, $J = \frac{\pi d^4}{32}$ = 0.2036 in.[4]
Torque-bar radius, r = 6 in.
Shackle length, b = 3 in.
Offset, a = 1 in.

Initial angle, α = 45 deg
Modulus of elasticity in shear, G = 11,400,000 psi

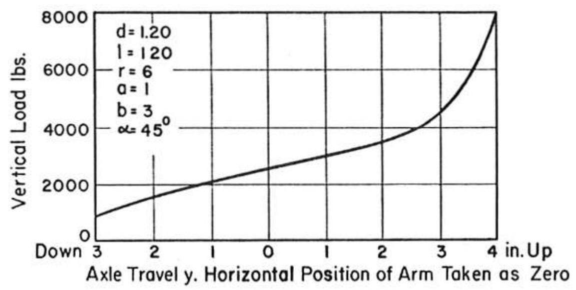

FIG. 5 LOAD-DEFLECTION CURVE FOR TORSION-BAR SPRING

The manner in which the stiffness of the suspension increases with increase of load is shown by the curve. This is a desirable property for vehicle mounts since a heavy overload will continue to be supported by the springs without bottoming of the frame. In addition, the lengths can be proportioned so that the riding qualities will remain practically unchanged. The natural frequency of vibration of a heavy load and stiff spring can be made equal to the natural frequency of the vehicle when the load is light and the spring is soft.

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F. HIRSCH.[3] Although they will be materially aided by Mr. Rouverol's paper, design engineers concerned with counterbalancing systems might find desirable a further explanation of the following:

(*a*) The dimensional limitations of d in Fig. 3 and the proper dimensional relation of d to b and c of Fig. 3.

(*b*) Indication of the origin of the summations X, Y, and Z occurring in Equations [6], [7], and [8].

(*c*) Provisions for vertical and horizontal movement in high-temperature piping installations.

(*d*) Provisions to be made for vibration isolation in piping installations.

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AUTHOR'S CLOSURE

The author would like to thank Professors Spotts and Hirsch for their comments. Professor Spotts' analysis of the Hickman type suspension adds some useful material to the paper. It might be well to point out that while the balancing method outlined in my own analysis could readily be altered to give a soft-spring type of suspension, for example, by making distance d slightly larger than the value given by Equation [7], there would seem to be little if any advantage in so doing. The characteristics of a good spring balance are not necessarily similar to those of a good vehicle suspension system, as the latter requires a variation in load-deflection relations of the type indicated in Professor Spotts Fig. 5.

Taking in order Professor Hirsch's suggestions regarding points in need of clarification:

(*a*) The only dimensional limitation on distance d of Fig. 3 is that it must satisfy Equation [7]; it corresponds to the similarly labeled dimension in Fig. 2. The dimensions b and c in Fig. 3 are shown as being equal, but need not necessarily be so. The dimension b would normally be selected so as to equal about one fourth of the required vertical travel of the load, and dimension c would be about half the safe spring deflection. The assignment of these two dimensions enables the distance d to be computed from Equation [7].

(*b*) With regard to the origin of the summations X, Y, and Z of Equation [6], they come directly from the equilibrium equation when terms containing Δ and r are grouped together, and the Δ or r, as the case may be, is factored out. Since both of these terms appear only in the first power, such collecting and factoring will always be possible.

(*c*) A device embodying the principles of Figs. 2 and 3 used as a constant-support pipe hanger would provide a constant lifting force despite vertical movement of the piping. Horizontal movement could be accommodated either by mounting the hanger so that its frame could swivel about a vertical axis, if the Fig. 2 type of mechanism is used, or, if there were adequate space above the pipe, simply by inserting a long vertical rod between the pipe and the hanger.

(*d*) Since the device is designed to provide constant lifting force regardless of vertical position of the load, no spring forces would be transmitted to the supporting frame by a vibrating load, provided the amplitude of vibration was less than the vertical travel afforded by the type of mechanism shown in Fig. 3. There would, however, be some small forces transmitted due to the inertia of the various parts of the yielding means. These could be kept relatively small by designing for lightness of the yielding means parts and by the strategic location of pivot points at or near centers of percussion.