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SNOWMELT – RUNOFF MODEL FOR STREAM FLOW FORECASTS

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Following the development of rainfall-runoff models the attention of the international hydrologic program is now increasingly focused on the snowmelt-runoff. The present simple model is based on taking into account the variability of the degree-day factor, recession coefficient and snow coverage. It can be adapted to heterogenous conditions of snow accumulation and temperature in mountainous basins.

The accumulation of snow during the winter months in mountainous basins and in northern regions provides an opportunity to forecast the resulting runoff. This paper deals with a procedure of day-to-day discharge computations which has been derived in two representative basins of Central Europe. The aim was to develop a snowmelt runoff model using only such input data as can be obtained or at least extrapolated in hydrological catchments of practical interest and not only in special cases of well-equipped experimental areas. The possibility of estimating the future trend of these data is also essential for forecasting purposes.

DETERMINING THE MELTWATER PRODUCTION

Of the numerous meteorological factors involved in the snowmelt, the air temperature is generally available and can be extrapolated to remote parts of a basin. A well-established method uses the number of degree-days obtained as the average of positive temperatures over a 24 hour period to calculate the resulting meltwater depth:

$$M = a \cdot T_d \qquad (1)$$

where

M is the daily snowmelt depth in cm
a is the degree-day factor in cm · °C^{-1} · d^{-1}
T_d is the number of degree-days in °C · d

Due to varying conditions in the snowpack and the fact that the air is not the only source of heat, the degree-day factor varies over a wide range. The snow density appears to be a useful index for estimating degree-day ratios in the absence of direct measurements (Martinec 1960):

$$a = 1.1 \cdot \varrho_r \qquad (2)$$

where

ϱ_r is the snow density relative to water
(specific gravity)

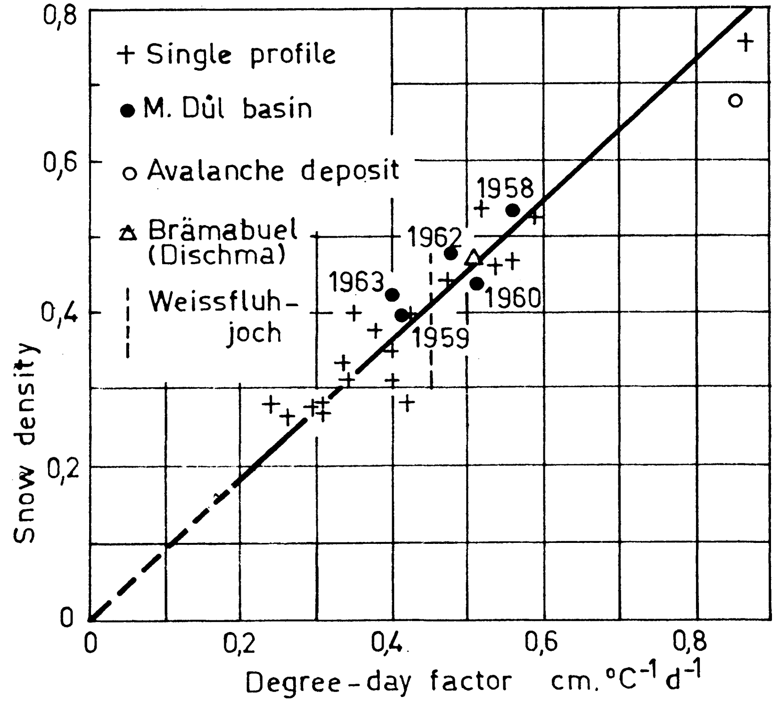

Fig. 1.
Relation between snow density and degree-day factor.

The experimental results in Fig. 1 represent conditions in Central Europe (46°–51° northern latitude). Lower values of *a* can be expected at higher northern latitudes since the effect of solar radiation, which is automatically included in the degree-day factor, would be generally reduced. Deviations must also be expected in different wind conditions.

The snow density seems to represent other properties of snow cover affecting the snowmelt: Its increase is generally accompanied by a decrease of the albedo (Corps of Engineers, 1956) which promotes the gain of heat from radiation. An old snow of high density frequently has an increased free water content and a low thermal quality. (The thermal quality of snow is the ratio of the amount of heat required to produce a given volume of water from snow to the amount of heat required to melt the same volume of water from pure ice at 0°C.) The increasing densities of snow in Fig. 1 indicate a growing proportion of slush, which reduces the thermal quality. On the other hand, the high density of ice is accompanied by a high thermal quality having a reverse effect on degree-day ratios.

THE BASIN RESPONSE

Due to the retention, there is no correlation between the daily rates of snowmelt and runoff. The melt water produced on the first day of snowmelt does not completely leave the watershed but continues to flow off as a gradually receding discharge series characterized by the recession coefficient *k*:

$$k = \frac{R_{m + 1}}{R_m} \qquad (3)$$

where

R is the daily runoff depth and the index *m* refers to the sequence of days during the recession flow.

Disregarding losses, the melt water production on the first day of snowmelt, M_1, equals the total runoff R_t which is the sum of the receding discharge series:

$$M_1 \equiv R_t \equiv R_1 \frac{k^{\infty} - 1}{k - 1} \qquad (4)$$

Since

$k < 1$, Eq. (4) becomes

$$M_1 \equiv R_t \equiv R_1 \frac{1}{1 - k}$$

and

$$R_1 = M_1 (1 - k)$$

On the nth day of the snowmelt period the total runoff includes the recession flow from the previous day:

$$R_n = M_n (1 - k) + R_{n-1} \cdot k \qquad (5)$$

Recalling Eq. (1), a simple runoff concept (Martinec 1965) results:

$$R_n = c \cdot a \cdot T_{dn} (1 - k) + R_{n-1} \cdot k \qquad (6)$$

where

c is the runoff coefficient expressing the losses.

Thus one part of the daily melt water production appears in the hydrograph within 24 hours while the other part follows as the recession flow.

From

$$R_m = R_0 k^m = R_0 e^{-Km} \qquad (7)$$

where

R_0 is the initial daily runoff of a recession period

K is the coefficient of exhaustion [d^{-1}]

m is the number of days

it follows that

$$K = - \ln k \text{ and } k = e^{-K} \qquad (8)$$

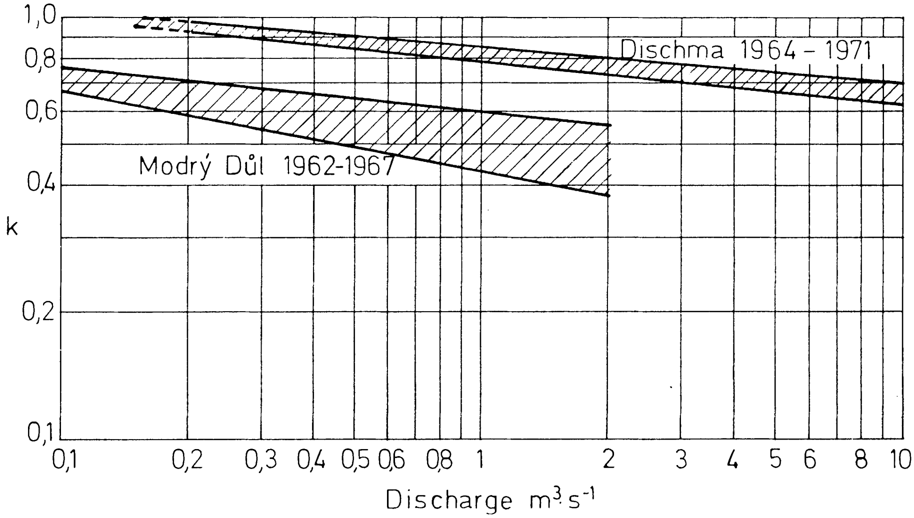

Fig. 2.

Range of k-values in relation to discharge in the basins Dischma (43,3 km^2) and Modry Dul (2,65 km^2).

Consequently, the concept of Eq. (6) is similar to a Swedish rainfall-runoff model (Bergström & Forsman 1973), which uses two different exhaustion coefficients, K_1 and K_2, simultaneously. Gradually changing values of *k*, in turn, have to be substituted into Eq. (6), which is equivalent to changing mixing ratios of K_1 and K_2. A relationship between k and the current intensity of runoff has been derived in two basins (Martinec 1970). At the same time, fluctuations from year to year have been observed, as shown in Fig. 2.

THE ROLE OF SNOW COVERAGE

The areal extent of snow cover in a basin can gradually decrease from 100 % to zero. This factor is often neglected in snowmelt-runoff models although it is likely to overshadow a possible refinement of Eq. (1). In basins with

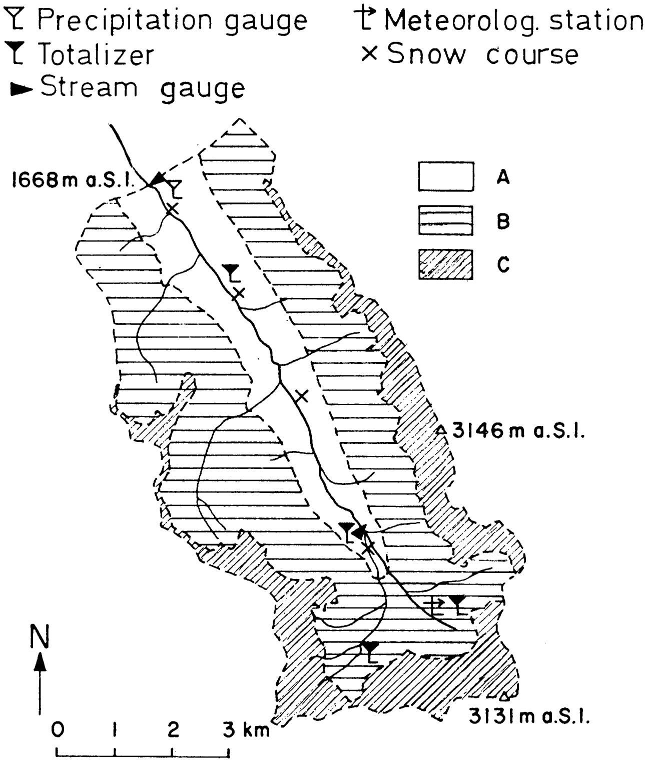

Fig. 3.
Situation and elevation zones of the Dischma basin.

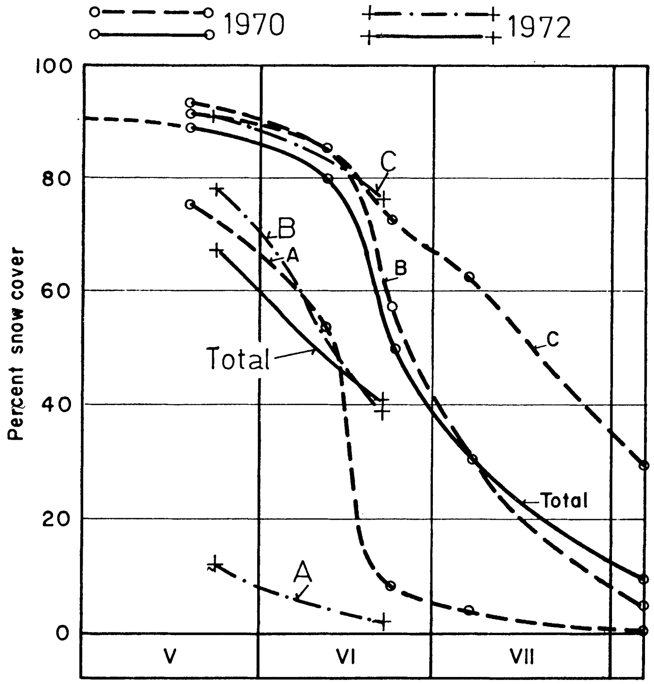

Fig. 4.
Seasonal decrease of snow coverage in partial areas of the Dischma basin.

varying conditions of snow ablation, separate curves of the decreasing snow coverage have to be used for partial areas. The representative basin Dischma in the Swiss Alps was divided, in view of the great range in elevation, into three zones, as shown in Fig. 3. The differing diminution of the snow coverage during the snowmelt season, evaluated from air photos, is demonstrated by Fig. 4. The difference between the curves of 1970 and 1972, respectively, is due partly to the greater accumulation of snow in 1970 (higher snow coverage in May), partly to exceptionally high temperatures in June 1970, resulting in a sharp decrease of the areal extent of snow cover.

COMPUTATION PROCEDURE

Taking into account the changing snow coverage and subdividing the basin into three partial areas with different snowmelt conditions, Eq (6) is rearranged as

$$R_n = \frac{c}{\underline{A}}(a_A T_{dA} S_A \underline{A}_A + a_B T_{dB} S_B \underline{A}_B + a_c T_{dC} S_C \underline{A}_C)(1-k) + R_{n=1} \cdot k \qquad (9)$$

where

R is the daily runoff depth in cm (to be converted, if preferable, to the average daily discharge in $m^3 \cdot s^{-1}$)

c is the runoff coefficient

$\underline{A}$ is the total area of the watershed in equal units as partial areas $\underline{A}_A$, $\underline{A}_B$, $\underline{A}_C$

a is the degree-day factor in $cm \cdot {}^{\circ}C \cdot d^{-1}$

T_d is the number of degree-days

S is the snow coverage (1.0 for a complete coverage)

k is the recession coefficient for a 1 day interval

n is an index expressing the sequence of days.

The number of degree-days is preferably determined for 24 hour periods starting with the approximate minimum (for example from 0600 hrs to 0600 hrs the next day). The resulting runoff refers to 24 hour periods shifted according to the time lag of the basin and starting again with the approximate daily minimum discharge. If rainfall occurs simultaneously, the daily amount is added to the snowmelt depth. The melting effect of rain can be generally neglected.

An example of snowmelt-runoff computation is given in Table 1 (disregarding losses):

Table 1.

Snowmelt-runoff computation in Dischma, 25 May 1973.

Zone	Area km^2 $\underline{A}$	Snow coverage S	T_d Degree-days °C · d	Degree-day factor $cm \cdot {}^{\circ}C \cdot d^{-1}$ a	Daily snowmelt depth cm water		Input into the basin $m^3 \cdot s^{-1}$
					over $\underline{A} \cdot S$	over $\underline{A}$	
A	8,9	0,10	7,40	0,45	3,33	0,33	0,34
B	24,5	0,75	4,37	0,45	1,96	1,47	4,17
C	9,9	0,90	0,97	0,45	0,43	0,39	0,45
Total	43,3						4,96

By recalling Eq. (5), the calculated input from snowmelt can be compared with the directly measured discharge:

$$Q_{25} = I_d\,(1 - k) + Q_{24} \cdot k \tag{10}$$

where

I_d is the input into the basin corresponding to the actual discharge.

By substituting:

Q_{25} (average discharge from 25 May, 1200, to 26 May, 1200) = 4,23 $m^3 \cdot s^{-1}$

Q_{24} (average discharge 24 hours earlier) = 4,12 $m^3 \cdot s^{-1}$

k = 0,7 (Fig. 2),

I_d = 4,5 $m^3 \cdot s^{-1}$ is obtained.

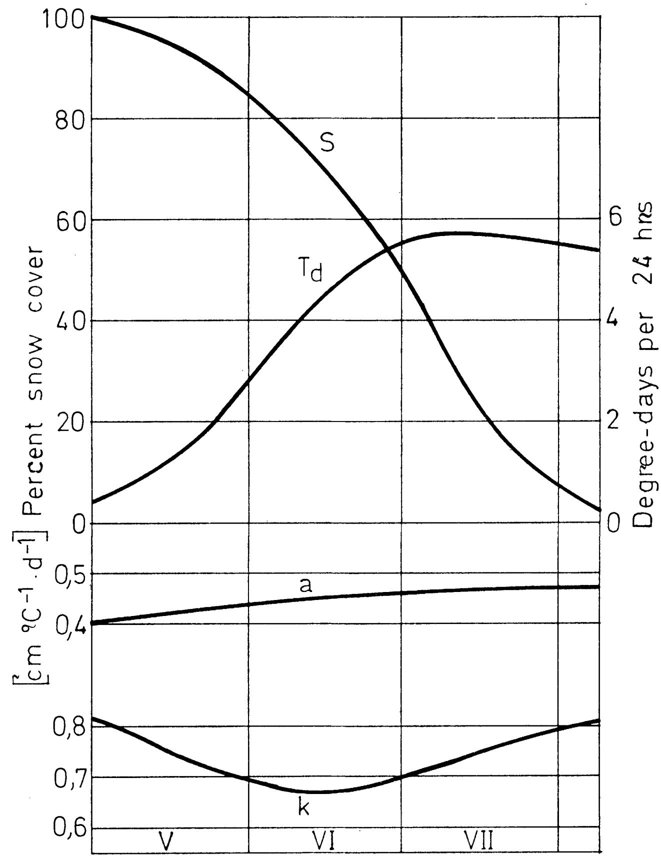

Fig. 5.

Seasonal changes of variables in snowmelt-runoff computation (hypothetical smoothed curves).

The temperature was measured at 2370 m a.s.l. (average elevation of the zone B) by an automatic meteorological station and extrapolated to the other zones by a lapse rate of 0,65 °C per 100 m.

Gradual changes of variables in day-to-day computations of snowmelt runoff by Eq. (9) are illustrated by a hypothetical example in Fig. 5.

In contrast to the above-mentioned HBV-2 rainfall-runoff model (Bergström & Forsman 1973), this computation procedure is sensitive to k values which depend on the discharge (Fig. 2). A computerized calculation of a snowmelt season should start with k corresponding to the previous day's flow and its value should be readjusted on each subsequent day according to the computed discharge of the previous day.

The accuracy of short-term forecasts depends in particular on the knowledge of the future trend of temperature. Seasonal forecasts of the runoff pattern in a snowmelt season can be carried out if the expected temperature conditions are given.

CONCLUSION

A successful application of runoff models depends on an appropriate assessment of variable factors involved. Representative basins help to narrow the range of estimates by providing the physical meaning of the respective values. By this approach, differences between various areas can be correlated with their specific characteristics. Coordinated presentation of data, as has been achieved for example in the Nordic countries (Nutsam 1969), and the mutual exchange of experience, improve the validity and exploitation of the gained knowledge. The acknowledged position of representative basins and snowmelt runoff models in the recently inaugurated international program in hydrology indicates that efforts in this field will be further increased in the future.

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