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DETERMINISTIC APPROACH TO WATERSHED MODELING

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Physically-based, deterministic models, are considered in this paper. Physically-based, in that the models have a theoretical structure based primarily on the laws of conservation of mass, energy, or momentum; deterministic in the sense that when initial and boundary conditions and inputs are specified, the output is known with certainty. This type of model attempts to describe the structure of a particular hydrologic process and is therefore helpful in predicting what will happen when some change occurs in the system.

GENERAL, PHYSICALLY-BASED WATERSHED MODEL

The general question being asked in research is: "Is it possible to develop a physically-based, distributed model of watershed behavior?" Such a model would consist of a set of linked partial-differential equations in up to three space variables and time. These equations, with boundary conditions defined by the surface and subsurface geometry of the basin, would operate upon distributed rainfall input data to estimate streamflow and groundwater flow from the basin as well as soil moisture levels. A conceptual, three-dimensional hydrologic model proposed by Freeze and Harlan is shown in Fig. 1.

Contribution from the Northern Plains Branch, Soil and Water Conservation Research Division, Agricultural Research Service, USDA, in cooperation with the Colorado Agricultural Experiment Station.

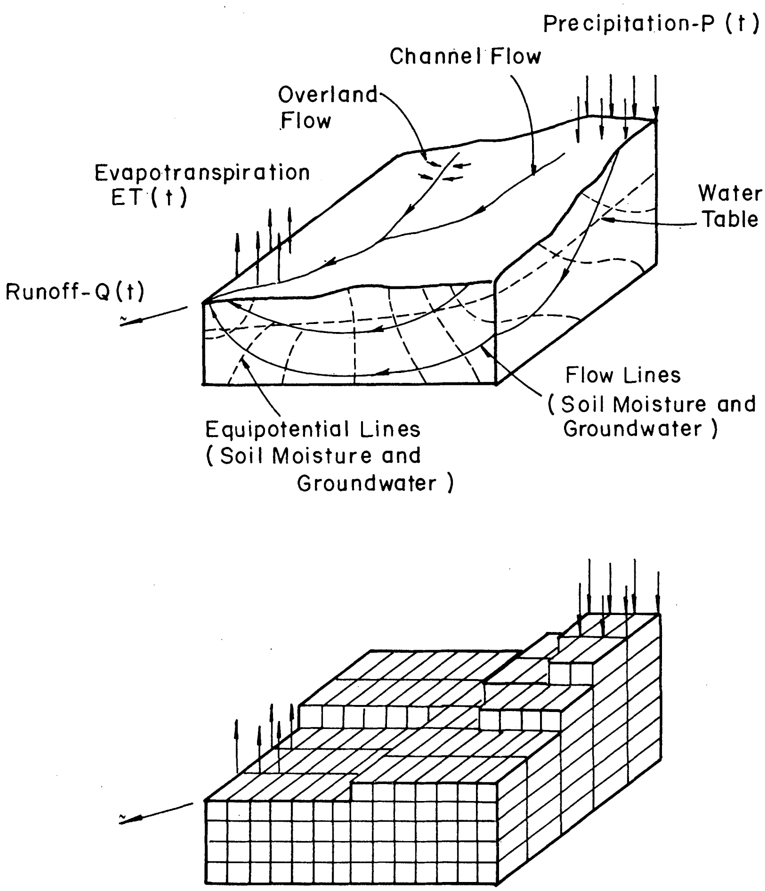

Fig. 1.
Schematic diagram of (A) hydrologic basin and (B) three-dimensional nodal model of hydrologic basin (from Freeze & Harlan 1969).

Although a general watershed model is desirable for engineering design, the model should be widely adaptable so that it can be used for different sized projects and for locations with varying amounts of available data. The general model is composed of a number of components, each of which may be represented in several ways. The general research question is not directly amenable to experimental testing because it is very difficult to prove (or disprove) the validity of a general model. For this reason it is good strategy to subdivide the general problem into first-order questions that can be tested. Some examples of these questions are:

(1) Can a deterministic model of infiltration be developed?
(2) Can a deterministic model of overland and open-channel flow be developed?
(3) Can an evapotranspiration model for conditions of less than potential ET be developed?

There are obviously several other important components in a general watershed model but the remainder of this paper will de devoted to a brief discussion of these three.

INFILTRATION

An infiltration model is perhaps the most critical component in a general watershed model because small errors in infiltration predictions may result in large errors in runoff predictions.

The physically-based approach toward an infiltration model has developed around the theory of single-phase flow in a porous medium. The equation for vertical, unsaturated flow in a porous medium can be readily derived from a mass balance in an element of soil along with Darcy's law relating soil moisture flux to the unsaturated hydraulic conductivity and the potential gradient.

If we consider the volume of unit cross-sectional area and thickness dz, shown in Fig. 2, we can write the continuity equation for an incompressible fluid as

$$V = V + \frac{\delta V}{\delta z} dz + \frac{\delta \Theta}{\delta t} dz \qquad (1)$$

Where V is the macroscopic volumetric flux of liquid water and Θ is the volumetric water content.

From Eq. (1) we have

$$\frac{\delta \Theta}{\delta t} = \frac{-\delta V}{\delta z} \qquad (2)$$

Darcy's law for unsaturated flow states

$$V_z = -K_\Theta \frac{\delta \psi}{\delta z}$$

where K_Θ is the hydraulic conductivity and ψ is the soil water potential,

$$\psi = h + z$$

where h is the soil water (matric) pressure head and z is the height above an

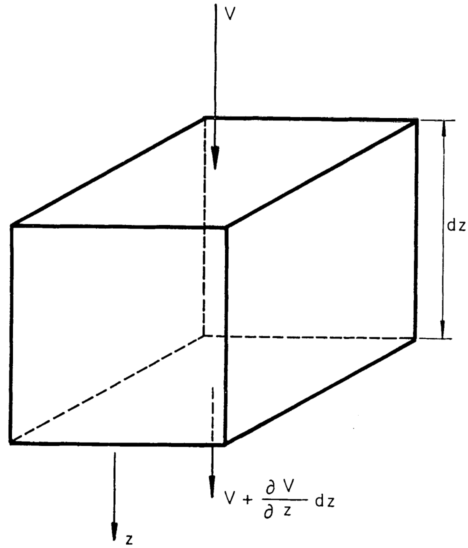

Fig. 2.
Elemental soil volume.

arbitrary reference level. By substituting the expression for V into Eq. (2) we obtain

$$\frac{\delta \Theta}{\delta t} = \frac{\delta}{\delta z}\left(K_{\Theta}\frac{\delta \psi}{\delta z}\right) = \frac{\delta}{\delta z}\left[K_{\Theta}\left(\frac{\delta h}{\delta z} + 1\right)\right] \tag{3}$$

Eq. (3) cannot be solved analytically except for some special cases; it can, however, be solved by finite-difference techniques. The solution gives Θ as a function of z and t as shown in Fig. 3. The mass infiltration during the time interval $t - t_o$ is given by

$$F = \int_{o}^{\infty} \Theta(z,t)dz - \int_{o}^{\infty} \Theta(z,t_o)dz$$

The boundary conditions required are also illustrated in Fig. 3. The initial condition $\Theta(z,0)$ is arbitrary, but must be specified. During the time interval $t_s - t_o$ the soil surface is unsaturated and the infiltration rate is controlled by the rainfall rate. Thus we have

$$-K_{\Theta}\left(\frac{\delta h}{\delta z} + 1\right) = q \tag{4}$$

where q is the rainfall rate.

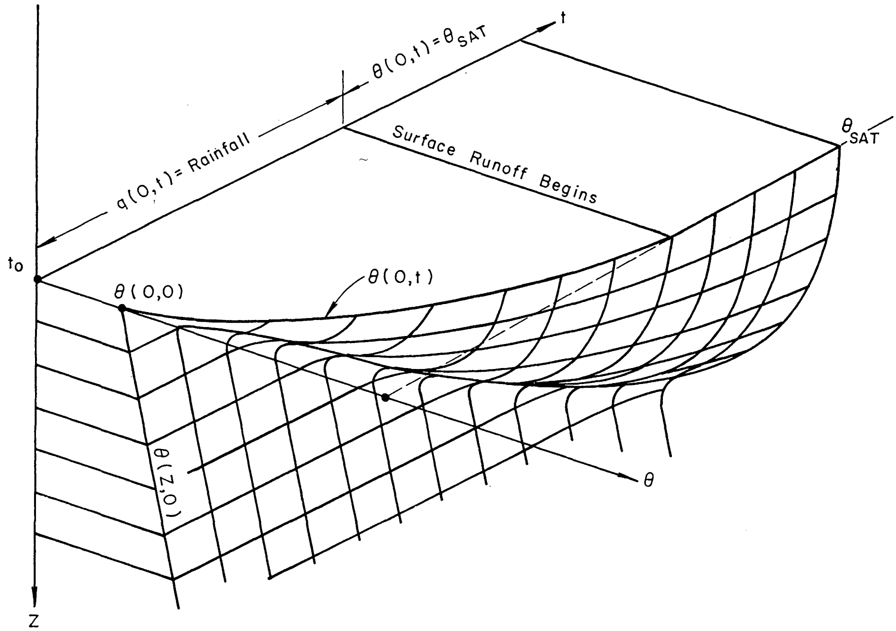

Fig. 3.
Solution surface of Richards' equation.

At the time t_s the surface is saturated and the boundary condition becomes

$$\Theta(0,t) = \Theta_{SAT} \tag{5}$$

This condition holds as long as free water remains on the surface. When surface runoff ceases, the flux through the surface is zero, or

$$-K_{\Theta}\left(\frac{\delta h}{\delta z} + 1\right) = 0 \tag{6}$$

The example shown in Fig. 3 is for the semi-infinite case ($0 \leq z < \infty$) so no lower boundary condition is required. Under a shallow water table condition, the one-dimensional equation may not be appropriate because saturated flow occurs predominantly in a lateral direction and the water table elevation would fluctuate. A water table at a fixed depth may serve as an artificial boundary condition and not have a profound impact on infiltration rates.

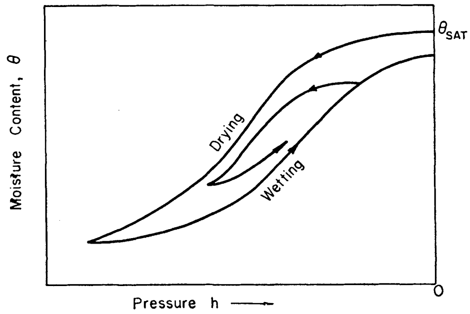

Fig. 4.
Typical moisture content-pressure relationship.

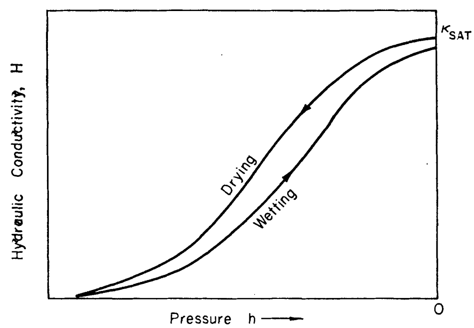

Fig. 5.
Typical hydraulic conductivity-pressure relationship.

To obtain solutions of Eq. (3) for real soils, the functional relationship between h and Θ and between K and Θ or K and h must be known: These relationships can be obtained for small disturbed samples in the laboratory. No method has yet been devised to measure these properties in the field. Even the laboratory procedures are difficult and time-consuming. Examples of $h-\Theta$ and $K-h$ curves are shown in Figs. 4 and 5.

Note that these are not single-valued functions but exhibit the characteristic of hysteresis; e.g., with constant Θ, h depends on whether or not the soil is wetting or drying and upon the past history.

The infiltration model specified by Eq. (3) is quite general in that soil properties can vary in the vertical direction so that various layered situations can be modeled successfully. However, Eq. (3) includes a number of simplifying assumptions that may invalidate it for some applications.

(1) The one-dimensional equation implies that the soil surface is a plane. On a rough, cultivated area, initial moisture flow may be strongly three-dimensional.

(2) Air movement is ignored. Strictly speaking, infiltration represents a two-phase flow problem; the simultaneous movement of water into the soil and the flow of air out of the soil. Air effects are probably insignificant when water tables are deep or where water on the surface is not continuous. Under ponding conditions with a shallow water table, infiltration rates computed by Eq. (3) would be too high.

(3) The soil remains a continuous porous medium irrespective of the moisture content and does not change in volume. Eq. (3) is not applicable to shrinking or swelling soils or to soils that crack upon drying. It also could not be applied to a newly plowed or cultivated field.

(4) The properties of the soil as defined by the soil curves are assumed to be time-invariant. In fact, soil properties near the surface can change as a result of raindrop impact, erosion, or freezing and thawing. These changes can have very important effects on runoff.

(5) It is also assumed that flow does not occur because of temperature or electrical gradients or in the vapor phase.

Numerical solutions to the one-dimensional equation of unsteady flow in unsaturated porous media have aided in understanding the process of infiltration. In the present state of development, this model may be useful in predicting the behavior of experimental plots and very small watersheds. Application to large watersheds with substantial spatial variations of rainfall or soil properties appears to require an excessive amount of input data and computational time.

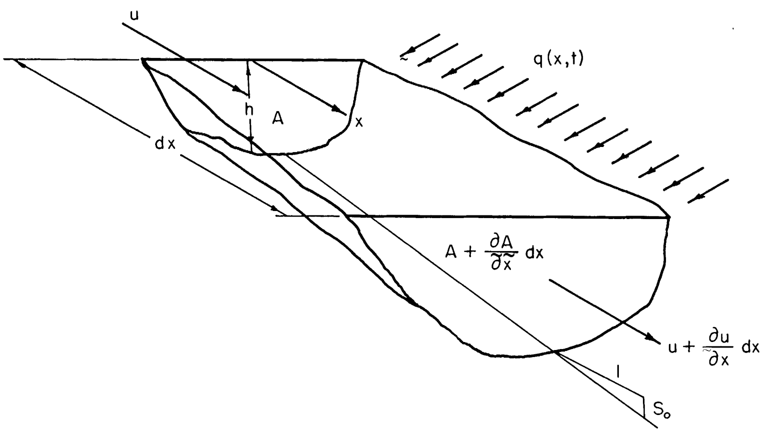

Fig. 6.
Definition sketch unsteady, nonuniform channel flow.

OVERLAND AND OPEN-CHANNEL FLOW

The most rapid response of a stream to rainfall on the watershed is that due to surface runoff. When rainfall rates are in excess of the infiltration rate, water accumulates on the surface and under the influence of gravity flows overland toward the stream channels. Both overland flow and open channel flow can be classified as unsteady, free-surface flow with lateral inflow (or outflow).

For a complex channel as shown in Fig. 6, the principle of continuity (conservation of mass) may be applied to obtain the equation:

$$\frac{\delta A}{\delta t} + \frac{\delta A u}{\delta x} = q\,(x,t) \tag{7}$$

where A is the cross-sectional area, u is the local velocity, $q(x,t)$ is the lateral inflow rate per unit length of channel and x and t are the space and time coordinates.

By applying the principle of conservation of momentum, the equation of motion is obtained

$$\frac{\delta u}{\delta t} + \frac{u \delta u}{\delta x} + \frac{g \delta h}{\delta x} = g(S_o - S_f) - q/_A\,(u - v) \tag{8}$$

where h is the local depth, g is the acceleration of gravity, S_o is the bed slope, S_f is the friction slope, and v is the x-component of the velocity of the lateral inflow (usually assumed to be zero). The friction slope S_f is given by either the Manning or Chézy formula. These are the fundamental equations of one-dimensional, open-channel flow. By far the largest part of books on open-channel hydraulics is concerned with the solution of these equations or of special cases of these equations.

Analytic solutions are not available for these equations so they must be solved numerically for all but some special cases. With the second and third generation digital computers now available and the advancing knowledge of numercial methods, unsteady flow computations can be readily carried out.

One of the advantages of physically-based models is that we can often find simplified approaches that give reliable results and, furthermore, we can develop physically-based objective criteria for determining when the simplified approach is permissible. As an example, consider the kinematic-wave approximation to Eqs. (7) and (8) for an overland flow problem. A definition sketch for overland flow is shown in Fig. 7. Eqs. (7) and (8) apply but h can be substituted for A in both equations. The initial conditions are

$$\begin{aligned} h(x,0) &= 0 \\ u(x,0) &= 0 \end{aligned} \tag{9}$$

the upstream boundary condition is

$$u(0,t) = 0 \tag{10}$$

and a possible downstream boundary condition is critical depth

$$u(L_o,t) = \sqrt{gh(L_o,t)} \tag{11}$$

The kinematic wave assumption utilizes Eq. (7) in its entirety but eliminates all of the derivative terms and the terms accounting for the momentum of the lateral inflow from Eq. (8). Thus Eq. (8) becomes

$$S_o = S_f \tag{12}$$

or using the Chézy formulation

$$u = C\sqrt{S_o h} \tag{13}$$

which is the relationship between u and h for uniform flow. This expression for u can be substituted into Eq. (7) and we obtain a single equation in one dependent variable, h,

$$\frac{\delta h}{\delta t} + \frac{\tilde{\delta}}{\delta x}(C\sqrt{S_o}h^{3/2}) = q \tag{14}$$

The upstream boundary condition is

$$h(0,t) = 0 \tag{15}$$

No downstream boundary condition is required.

Eq. (14) retains all of the physical parameters present in Eqs. (7) and (8)

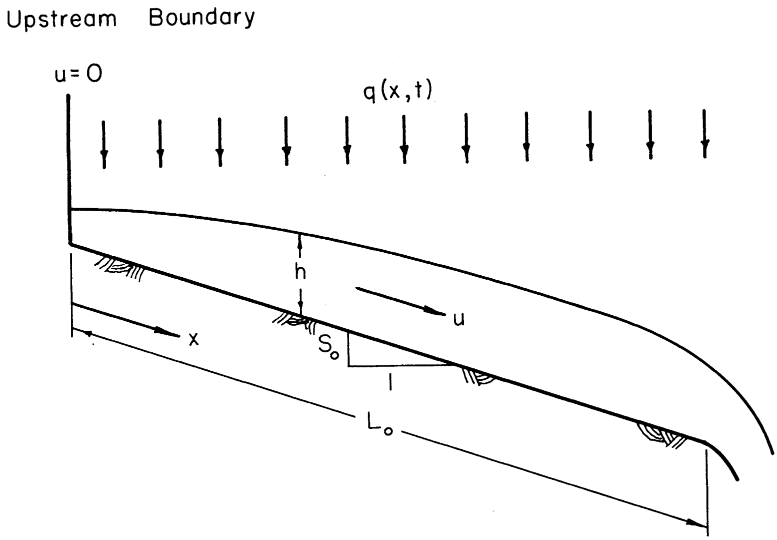

Fig. 7.
Definition sketch, overland flow.

but is much easier to solve although numerical methods are required for general cases.

It has been shown that the kinematic approximation is very good for overland flow when a dimensionless parameter, *k*, called the kinematic flow number, is greater than 20. The kinematic flow number is defined as

$$k = \frac{S_0 L_0}{H_0 F_0^2}$$

where S_0 is the bed slope, L_0 is the length of the flow plane, H_0 is the normal depth at the downstream boundary at the maximum steady-state flow rate, and F_0 is the Froude number at the downstream boundary at the same flow rate. For most hydrologically significant cases, *k* is on the order of 100 to several thousand. Therefore, it would be very difficult to distinguish between the solution to Eqs. (7) and (8) and that of Eq. (14).

Of the physically-based components of a watershed model, that describing unsteady flow is probably the most advanced and the most used in engineering practice. This is true for several reasons. The shallow-water equations were known as early as the nineteenth century and engineers have been solving special cases of these equations since then. With the advent of the digital computer, complete solutions became possible. Furthermore, the parameters for this model are quite straightforward with the possible exception of the friction coefficient. Even so, engineers have been selecting Manning's n for a long time and, while it is by no means completely satisfactory, it can be estimated quite accurately for some channels. This it not true for overland flow where the roughness elements penetrate the free surface and the flow may resemble flow through a porous medium. However, experimental evidence indicates that solutions to either the shallow-water equations or the kinematic-wave equations are adequate models of the phenomenon. More research needs to be done on the problem of estimating roughness coefficients for overland flow.

EVAPOTRANSPIRATION

In the time interval between precipitation events, storage of water in any watershed element is reduced by unsaturated and saturated lateral flows of water and by the process of evapotranspiration. Given some initial configuration of volumetric moisture content as a function of depth, the subsequent evapotranspiration will, to a large extent, govern the amount of ground water recharge and will also control the soil moisture distribution at the beginning of the next precipitation event. Through this influence on base flow and on surface runoff, evapotranspiration exerts a controlling effect on water yield.

Evapotranspiration rates from soil and vegetation depend upon the energy available to vaporize water, the unsaturated flow of water through soil to the surface and to plant roots, and on the type, density, and state of growth of plants.

It is possible to analyze the evapotranspiration process and to formulate a mathematical model that helps in understanding the process. Unfortunately these models are extremely complex and require so much input information that they are not useful as predictive tools. This complexity can be illustrated by considering a volume containing soil and the plant canopy as shown in Fig. 8. The evapotranspiration rate at any time is the net flux of water vapor through the sides and top of the above-ground portion of the volume element. Water movement is primarily in the liquid phase in the soil. In this form it moves through the soil to the surface and to the plant roots in response to tension gradients and is vaporized at the soil surface and at the surface of the plant leaves. The energy required to vaporize the water must come from radia-

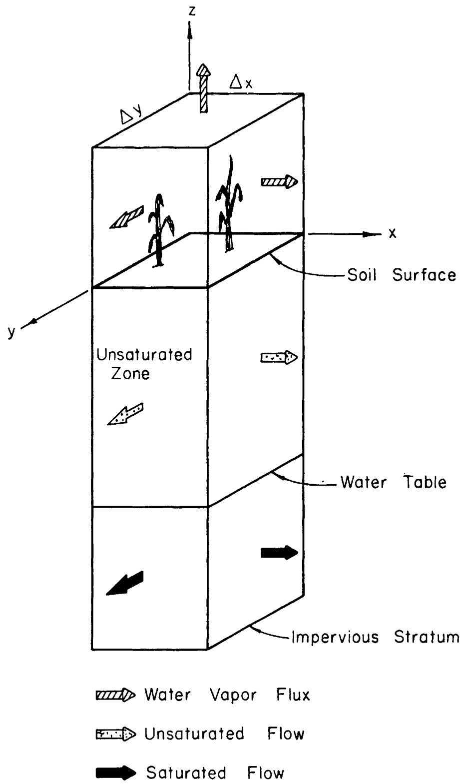

Fig. 8.
Soil-plant canopy volume.

tion, from the advection of sensible heat, or from heat stored in the soil. Water vapor is transported out of the canopy zone primarily by turbulent diffusion.

Many of the component processes can be described mathematically, but a general model linking all of the components for realistic plant geometries would be extremely complex. It would include a soil moisture model, a model of plant behavior, and a model describing the turbulent transport of water vapor and heat. Input for such a model would include continuous radiation data, wind profiles and directions, air properties, and rainfall.

Because of the complexity of a general model, various simplifications have been made to develop physically-based models that apply under restricted conditions. One assumption that results in great simplification is that water supply to the plant and soil surface is not the limiting factor. Evapotranspiration then occurs at the potential rate and is governed primarily by the climatic conditions, although the albedo and the surface roughness characteristics also have an effect. Potential evapotranspiration can be calculated from (1) the energy budget, (2) the aerodynamic approach, and (3) the combined aerodynamic and energy budget approach. These models are discussed in several of the references cited at the end of this paper and will not be described herein.

Actual evapotranspiration is influenced by soil moisture content and the stage of plant growth. This relationship is a complex one and so far has resisted theoretical treatment. Empirical correlations of the ratio of actual evapotranspiration to potential ET have been used with some success for prediction.

At present, evapotranspiration models that can be used as components in watershed models consist of both physically-based and black box components. There are no indications that this situation will change appreciably in the near future.

NUMERICAL METHODS

Physically-based models of watershed components invariably involve ordinary or partial differential equations. Solution of these equations frequently requires the use of numerical methods. Such methods are required because the equations cannot be solved analytically or because the initial and boundary conditions cannot be described conveniently as a continuous function.

Fig. 9 illustrates the problems involved in numerical solutions of any differential equation used to describe some physical phenomenon. The box on top represents the "real world" system in all its natural complexity. This system might be visualized as a river reach, and the system behavior is defined by observed variables such as depth and velocity in space and time. The second box represents the partial differential equations that have been developed by physical reasoning as a representation of the real system. There are many assumptions and approximations involved in these equations so the true, continuous solutions to these equations would not coincide perfectly with reality. Finally, the third box represents the finite difference or discrete approximation to the differential equations. This discrete approximation to continuous functions also introduces errors so in general the solution of the finite-difference

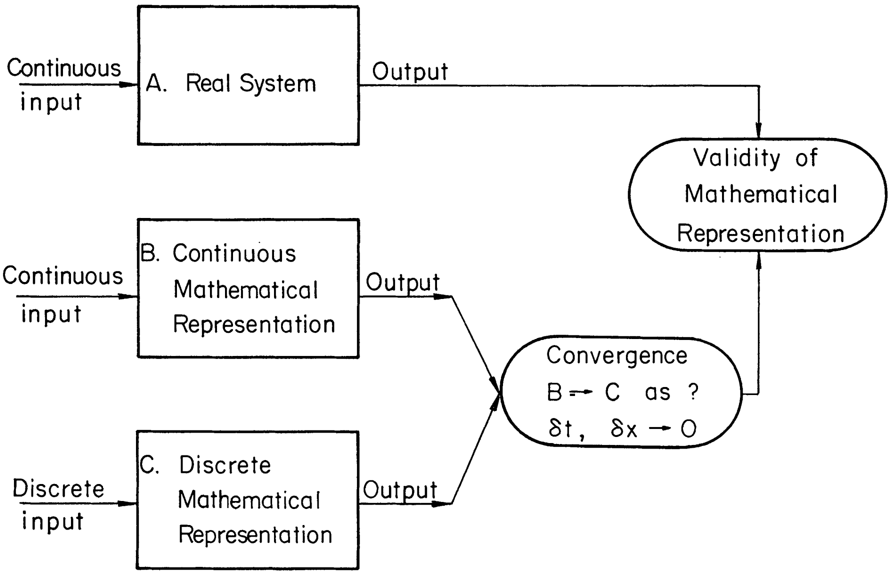

Fig. 9.
Logic of finite approximations.

equations will not coincide with the "true" solution to the differential equations.

If a perfect instrument for measuring discharge were used, the hydrograph shown as a solid line in Fig. 10 could be obtained. If an analytic solution to

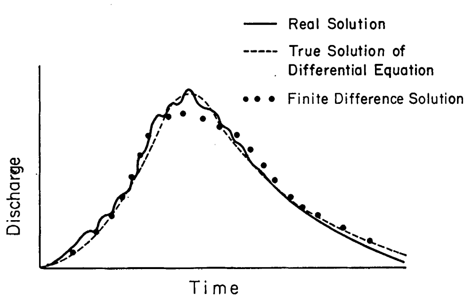

Fig. 10.
Comparison of solutions.

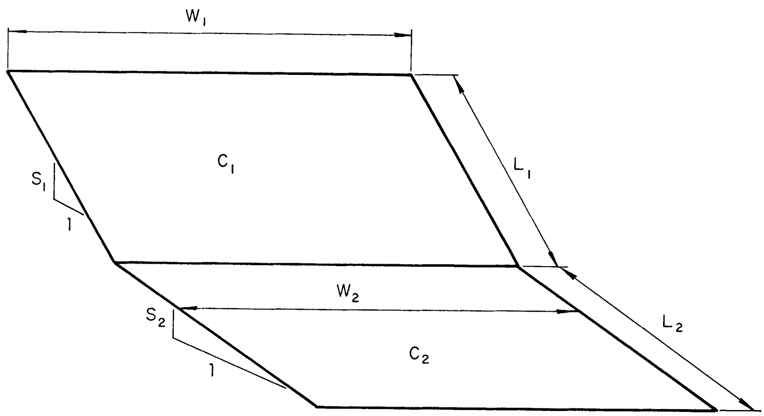

Fig. 11.
Two-plane cascade.

the differential equation were available, it would be the continuous solution to the mathematical model shown as a dotted line. The solution of the finite-difference equations is shown as a series of discrete points. Now having only the discrete solution, one must know something about the difference between this solution and the continuous "true" solution before coming to any conclusion as to how well the mathematical equations represent the physical system. This is the problem of *convergence*. Clearly for a desirable finite-difference scheme, the discrete solution should approach the true solution of the differential equation in the limit as the step size approaches zero.

The general problem of convergence involves two separable aspects: approximation and stability. It is beyond the scope of this paper to discuss these in detail but at a risk of oversimplification it can be said that the question of approximation deals with how well the finite-difference equations represent the continuous differential equations in the limit as the increments $\triangle x$ and $\triangle t$ approach zero. In an unstable scheme, small rounding errors are amplified by the finite-difference scheme until they completely dominate the solution. It is possible to have stable difference schemes that are poor approximations as well as unstable schemes that appear to be good approximations. In either case, the finite-difference solutions do not converge to the true solutions.

As an example of problems with numerical methods, consider the problem shown in Fig. 11. In this example a two-plane cascade receives a uniform lateral inflow. The upper plane is steeper than the lower plane; all other para-

Table 1.
Rectangular grid finite difference schemes.

Method	Finite difference equation	Order of approximation	Linear stability criterion
Single-step Lax-Wendroff	$h_j^{i+1} \equiv h_j^i - \Delta t \frac{k}{n}\left[\left(\frac{h_{j+1}^{iN} - h_{j-1}^{iN}}{2\Delta x}\right) - 1/2\left(q_{j+1}^i + q_{j-1}^i\right)\right] +$ $\frac{\Delta t^2\, Nk}{4n\Delta x}\left\{\left(h_{j+1}^{iN-1} + h_j^{iN-1}\right)\left[\frac{k}{n}\left(\frac{h_{j+1}^{iN} - h_j^{iN}}{\Delta x}\right) - 1/2\left(q_{j+1}^i + q_j^i\right)\right] -\right.$ $\left. - \left(h_j^{iN-1} + h_{j-1}^{iN-1}\right)\left[\frac{k}{n}\left(\frac{h_j^{iN} - h_{j-1}^{iN}}{\Delta x}\right) - 1/2\left(q_j^i + q_{j-1}^i\right)\right] + \frac{2\,n\Delta x}{Nk\Delta t}\left(q_j^{i+1} - q_j^i\right)\right\}$	$0(\Delta x)^2$	$\frac{\Delta t}{\Delta x} \leq \frac{n}{\sqrt{Nkh^{N-1}}}$
Upstream differencing	$h_j^{i+1} = h_j^i - \frac{Nk}{n}\frac{\Delta t}{\Delta x}\left(h_j^{iN} - h_{j-1}^{iN}\right) + q_j^i\,\Delta t$	$0(\Delta x)$	$\frac{\Delta t}{\Delta x} \leq \frac{n}{2.75kNh^{N-1}}$
Brakensiek's four point implicit	$\frac{h_j^{i+1} - h_j^i + h_{j-1}^{i+1} - h_j^i}{2\Delta t} + \frac{k}{n\Delta x}\left(h_j^{i+1N} - h_{j-1}^{i+1N}\right)$ $- 1/4\left(q_{j-1}^{i+1} + q_j^{i+1} + q_{j-1}^i + q_j^i\right) \equiv 0$	$0(\Delta x)$	Unconditionally stable

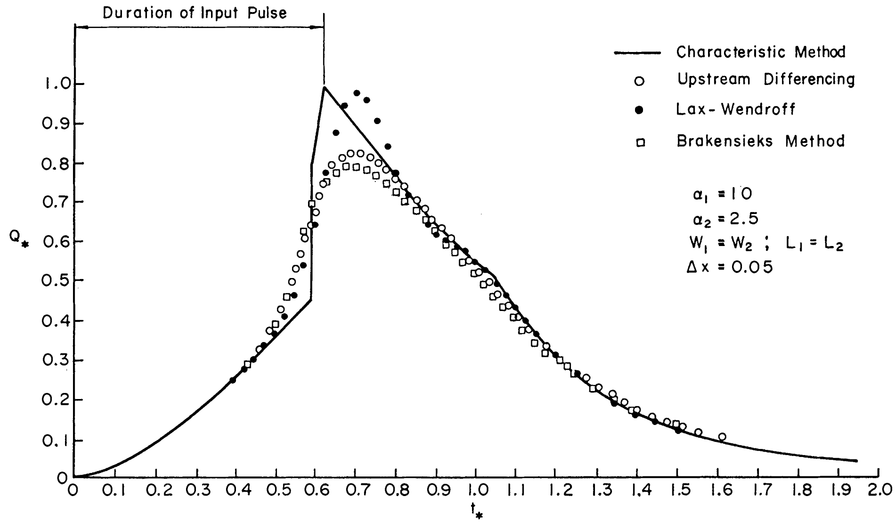

Fig. 12.
Comparison of finite-difference methods.

meters are the same. The kinematic equations were used to describe the unsteady flow. The hydrograph at the lower boundary was computed by four methods: (1) the single-step Lax-Wendroff; (2) the upstream differencing method; (3) Brakensiek's four-point implicit method (Brakensiek 1967), and (4) by numerical integration along the characteristics. The finite-difference formulations of the rectangular grid schemes are shown in Table 1, and the results are compared in Fig. 12.

This is a very severe test case in that the solution contains a discontinuity or kinematic shock as well as a very sharp peak. The characteristic solution is the most accurate. As one should expect, the second-order scheme gave the best results of the rectangular grid methods. The first-order schemes have errors of approximately 20 % at the peak. All rectangular schemes anticipate the shock and delay the peak. This is typical of such methods because the disturbance must follow the grid points.

The importance of this demonstration lies in the physical significance to be attached to parameters obtained by fitting computed hydrographs to observed data. Suppose that a real world situation exists where the kinematic approximation is very good and the hydrograph shown as the solid line had been ob-

served. Now using some objective criterion, such as least squares, the Chézy roughness parameter could be adjusted until an optimum, $\hat{C}$, was reached. This optimized parameter, $\hat{C}$, would then be related to the "true" parameter, C, in the following manner: $\hat{C} = C + \varepsilon$
where ε is an error term that depends on the difference scheme used as well as the $\triangle x$ and $\triangle t$ interval. It is quite likely that the error term will not have a mean of zero so that estimates will be biased and will differ from those obtained by other techniques. If one obtains optimized infiltration parameters as well as roughness parameters, it appears quite likely that the infiltration parameters will be strongly affected by the accuracy of the difference scheme used.

SUMMARY

In this paper, three psysically-based, deterministic components of a general watershed model have been briefly considered: infiltration, overland and open-channel flow, and evapotranspiration. The basic theory underlying each of these models is well developed. Progress in applying these models is quite different. Of the three, the overland and open-channel flow model is being used in many practical applications, and will replace the black box approach in many cases. Because of stringent data requirements or limitations imposed by some assumptions, the infiltration model cannot be used in some cases. It may be applied for computations of infiltration for plots and very small watersheds. Physically-based models for actual (as compared with potential evapotranspiration) are extremely complicated and do not offer an attractive alternative to combined theoretical-empirical methods.

Solution of the ordinary or partial differential equations describing physical processes on a watershed generally involves numerical methods. The problem of convergence of numerical methods and the possible effect of finite-difference solutions on optimized parameters must be considered.

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