Chapter 2

Formation of Host Clouds of First Stars in the Early Universe

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It is widely accepted that various structures of the universe are formed through the gravitational instability of the density perturbations imprinted by the quantum fluctuations. First stars are also formed as a consequence of the evolved density perturbations. However, they are not formed from the stellar size perturbations such as $M \sim 1 - 10^2 M_\odot$. Recent theoretical investigations suggest the first stars are formed in more massive “parent” clouds, with $M \gtrsim 10^6 M_\odot$. The key physics of the formation of such small hosts is the formation of hydrogen molecules in them. Since the virial temperature of such clouds are less than $10^4 K$, they are not cooled by atomic cooling processes. Therefore, hydrogen molecules are required as the main coolant of primordial gas at $10^4 K$, because their rotation/vibration levels are excited even at such low temperature.

In this chapter, gravitational collapse of density perturbations in Cold Dark Matter (CDM) universe is reviewed. Second, the formation processes of hydrogen molecules and the cooling mechanisms in primordial gas are summarized. Finally, the condition of the formation of cold dense gas clouds that host the first stars is shown.

§ 1. Growth of density perturbations in CDM universe

In this section, the parameters of background CDM cosmology are outlined and the gravitational instability of density perturbations is discussed. Finally, the notion of “collapse/virialized epoch” is defined.

1.1. Cosmological parameters and CDM density perturbations

There are four basic parameters $h, \Omega, \Omega_\Lambda, \Omega_B$ and two parameters $\delta_H, n$ which characterize the expanding universe and the density perturbations. Here $h$ denotes the Hubble parameter normalized by 100 km/s/Mpc, $\Omega$ is the density parameter of CDM and baryons, $\Omega_\Lambda$ is the normalized density of the vacuum energy, and $\Omega_B$ denotes the normalized density of baryons. $\delta_H$ is the density fluctuation at horizon-crossing, and $n$ denotes the slope of the “initial” spectrum of density perturbation. Once we know these parameters, we can trace the growth of density fluctuations at a given scale.

First of all, the observed value of $h$ has been controversial until the early 90’s. However, after HST observation of Cepheid in VIRGO cluster, the accuracy of cosmic distance ladder has greatly improved. Moreover, a few new method of distance determination helped the convergence of the observed value of $h$. As a result,

$$ h \simeq 0.7 $$  

(1.1)

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is a widely accepted consensus at present with approximately 10 percent errors.

Consequently, it has been suggested that $\Omega_A$ should be of order unity, because the age of the universe should be longer than that of the globular clusters, if we adopt $h \sim 0.7$. However, the extreme difficulty to find the theoretical basis on the origin and the nature of $\Omega_A$, the presence of cosmological constant has been suspected. Recent observations of high-$z$ supernova and the higher multipole moments of CMB fluctuations have dramatically changed such a situation.\(^3\)-\(^7\) Observations of high-$z$ supernova\(^8\),\(^9\) revealed that the expansion of universe is accelerating ($q_0 < 0$), and $0.8\Omega - 0.6\Omega_A \simeq -0.2$\(^8\) is suggested. On the other hand, the CMB observations by the projects MAXIMA-1,\(^3\),\(^4\) Boomerang\(^5\) and DASI\(^6\),\(^7\) strongly suggest that the universe is flat, i.e. $\Omega + \Omega_A \sim 1$. Combining most recent results of these observations, we have most confident values:

$$\Omega \simeq 0.3,$$

$$\Omega_A \simeq 0.7.$$

In this paper we use Eqs. (1.1), (1.2) and (1.3), and for our canonical cosmological model. We should remark that the CMB observations will be dramatically upgraded in the next a few years, because the data from satellite MAP are started to be supplied. The satellite PLANK will also be launched in 2007, which will improve the data significantly. These next generation observations will constrain the cosmological parameters more severely. We also have to remark that the observation on the cluster of galaxies also suggest the low $\Omega$ universe, which is consistent with the above value.

Next, we move on to the power spectrum of the density fluctuations. The variance of the density fluctuations smoothed within a radius $r$ (denoted as $\bar{\delta}^2(r)$) is related to the power spectrum $P(k)$ by the following well known formula:

$$\langle \bar{\delta}^2(r) \rangle = \int d^3k \tilde{W}^2(kr)P(k). \quad (1.4)$$

Here $\tilde{W}^2(kr)$ denote the window function in $k$ space, that smooth out the smaller scale fluctuations than $r$:

$$\tilde{W}^2(kr) = 3 \left( \frac{\sin kr}{(kr)^3} - \frac{\cos kr}{(kr)^2} \right) = \frac{3}{kr} j_1(kr), \quad (1.5)$$

where $j_1$ is the spherical Bessel function. The power spectrum $P(k)$ in CDM cosmology is often expressed by the transfer function $T_{\text{CDM}}(k)$. $T_{\text{CDM}}(k)$ represents the change of the power spectrum due to the evolution before the cosmic recombination epoch. The relation between $P(k)$ and $T_{\text{CDM}}(k)$ is

$$P(k) = AT_{\text{CDM}}^2(k)k^n. \quad (1.6)$$

Here the power index $n$ denotes the “initial” spectrum of the density fluctuations, and $n = 1$ denotes the scale free spectrum. $A$ represents the amplitude of the density fluctuations.
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Fig. 1. Spectrum of CDM density fluctuation is plotted against smoothed mass scale. The vertical axis denotes 1-$\sigma$ density fluctuation smoothed over the mass scale $M$. The horizontal axis shows the amplitude of density fluctuation at $z = 0$, assuming the linear growth rate even after $\delta > 1$. The assumed cosmological parameters are $\Omega = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_B h^2 = 0.02$ and $h = 0.7$. We adopt the transfer function by BBKS, followed by the modification by Hu and Sugiyama. The normalization of the spectrum was given by the COBE 4th year data, using the formula shown in Bunn and White.

Several authors have shown the fitting formula of the transfer function for CDM universe. For example, the formula given by Bardeen et al. (hereafter BBKS) with some modifications is

$$T_{\text{CDM}} = \frac{\ln (1 + 2.34q)}{2.34q} \left( 1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4 \right)^{-1/4}, \quad (1.7)$$

with $q = k/(h\Gamma)$, and $\Gamma$ is given by the following equations:

$$\Gamma = \sqrt{a_0 \Omega h \left( \frac{2.728}{2.7} \right)^2}, \quad (1.8)$$

$$a_0 = a_1^{-\Omega_B/\Omega} a_2^{-\Omega_B/\Omega^3}, \quad (1.9)$$

$$a_1 = (46.9\Omega h^2)^{0.670} \left( 1 + (32.1\Omega h^2)^{-0.532} \right), \quad (1.10)$$

$$a_2 = (12.0\Omega h^2)^{0.421} \left( 1 + (45.0\Omega h^2)^{-0.582} \right). \quad (1.11)$$

Amplitude $A$ is fixed by the amplitude of the temperature fluctuation of cosmic microwave background. The horizon-crossing amplitude $\delta_H$ is related with $A$ by the
following equation:

\[ A = 2\pi^2 \delta_H^2 H_0^{-3-n}. \]  \hspace{1cm} (1.12)

\( \delta_H \) is determined by the CMB fluctuations, using the COBE 4th year data, the following fitting formula is obtained:

\[ \delta_H = 10^{-5} (2.422 - 1.166e^\Omega + 0.800e^{\Omega_A} + 3.780\Omega \\
- 2.267\Omega e^{\Omega_A} + 0.487\Omega^2 + 0.561\Omega_A \\
+ 3.392\Omega_A e^\Omega - 8.568\Omega\Omega_A + 1.080\Omega_A^2). \]  \hspace{1cm} (1.13)

1.2. Linear growth of density perturbations

The seeds of the density fluctuations grow as the universe expands. In the linear regime of the perturbation growth, the growth rate of the over density is known very well. In this section, we summarize the growth rate for 1) \( \Omega = 1, \Omega_A = 0 \), 2) \( \Omega = 0.3, \Omega_A = 0 \) and 3) \( \Omega = 0.3, \Omega_A = 0.7 \). The growth rate \( D(z) \) of density perturbation is defined as

\[
\frac{D(z)}{D(z_i)} = \frac{\bar{\delta}(r)}{\bar{\delta}_i(r)},
\]  \hspace{1cm} (1.14)

where \( z \) is the redshift that specify the stage of the evolution, and the suffix \( i \) denotes an arbitrary epoch during its evolution. The normalization for \( D(z) \) is chosen so that at high redshift the function is

\[
D(z) \rightarrow \frac{1}{1+z} \quad \text{for} \quad z \rightarrow \infty.
\]  \hspace{1cm} (1.15)

For Einstein-de Sitter universe (\( \Omega = 1, \Omega_A = 0 \)), \( D(z) \) is simply proportional to the scale factor, i.e.

\[
D(z) = \frac{1}{1+z}.
\]  \hspace{1cm} (1.16)

For general case (open or \( \Lambda \)-dominated universe), the expression is

\[
D(z) = E(z)G(z),
\]  \hspace{1cm} (1.17)

where

\[
E(z) = \left( \Omega (1+z)^3 + \Omega_A + (1-\Omega - \Omega_A)(1+z)^2 \right)^{1/2},
\]  \hspace{1cm} (1.18)

\[
G(z) = \frac{5\Omega}{2} \int_z^\infty \frac{1+z}{E(z)^3} dz.
\]  \hspace{1cm} (1.19)

The growth rates for three different cosmological parameters are plotted in Fig. 2. The parameters are 1) \( \Omega = 1, \Omega_A = 0 \), 2) \( \Omega = 0.3, \Omega_A = 0 \) and 3) \( \Omega = 0.3, \Omega_A = 0.7 \). The growth rate is the largest for Einstein-de Sitter case, and the smallest for open universe. For the most plausible case (flat-\( \Lambda \)), the growth rate is intermediate.
1.3. Non-linear growth of density perturbation and “collapse/virialized epoch”

The growth of density perturbation in non-linear regime cannot be described by analytical formula in general. However, it can be described by a simple formula for spherically symmetric density perturbations. The equation of motion of a mass shell $r(t)$ of the perturbed region is

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2},$$  \hspace{1cm} \text{(1.20)}

where $M$ is the mass enclosed within the radius $r$. If we are interested in the infalling solution, it is characterized by a positive parameter $C$, and is given as\(^{14}\)

$$r(\alpha) = \frac{GM}{C} \left( 1 - \cos \alpha \right),$$  \hspace{1cm} \text{(1.21)}

$$t(\alpha) = \frac{GM}{C^{3/2}} \left( \alpha - \sin \alpha \right),$$  \hspace{1cm} \text{(1.22)}

where $\alpha$ is the parameter “development angle”. In this solution, there are two typical epoch, $t_{ta}$ and $t_c$. $t_{ta}$ denotes the time when the radius turn-around from the expansion to the collapse phase. From Eq. (1.21), the development angle at turn-around is directory obtained as

$$\alpha_{ta} = \pi.$$  \hspace{1cm} \text{(1.23)}
Corresponding time and radius are

\[ r_{ta} = \frac{2GM}{C}, \]
\[ t_{ta} = \frac{\pi GM}{C^{3/2}}. \] (1.24)

We should remark that the ratio of the average density within the shell \( r(t) \) and the background density is determined independently of the initial condition for \( \Omega = 1, \Omega_A = 0 \) universe at \( t = t_{ta} \):

\[ \frac{\bar{\rho}(M; t_{ta})}{\rho_{\text{univ}}(t_{ta})} = 1 + \bar{\delta}(M, t_{ta}) = \frac{9\pi^2}{16}. \] (1.25)

If we extrapolate the evolution of linear density perturbation to the nonlinear regime, the linear extrapolated overdensity at the turn-around is

\[ \bar{\delta}_{\text{linear}}(M; t_{ta}) = \frac{3}{20}(6\pi)^{2/3} \simeq 1.06 \] (1.26)

for Einstein-de Sitter universe.

Similarly, “the collapse epoch” is defined when the mass shell \( r(t) \) is zero. At this time, the development angle \( r \) and \( t \) are

\[ \alpha_c = 2\pi, \] (1.27)
\[ r_c = 0, \] (1.28)
\[ t_c = \frac{2\pi GM}{C^{3/2}} (= 2t_{ta}). \] (1.29)

It is worth noting that the density of the system diverges at \( t = t_c \). In reality, physical density does not become infinite, instead, the system will be violently relaxed to a virialized system. Thus, the “collapse” epoch is sometimes expressed as “virialized” epoch.

Linear extrapolated density perturbation for Einstein-de Sitter case is

\[ \bar{\delta}_{\text{linear}}(M; t_c) = \frac{3}{20}(12\pi)^{2/3} \simeq 1.69. \] (1.30)

Thus, a density perturbation collapses to a bound object when the amplitude of density contrast obtained by linear theory exceeds 1.69 for Einstein-de Sitter case. This critical linear density contrast is always written as \( \delta_c \), and it weakly depends on cosmological parameters. However, since we are interested in the formation of first stars at \( z \gtrsim 10 \) for \( M \lesssim 10^8M_\odot \), differences due to the cosmological parameters are small. Therefore, the “turn-around” and “collapse” epochs of the “parent” systems are well characterized by the linearly extrapolated critical densities 1.06 and 1.69.

§2. Main cooling processes in primordial gas

Before we proceed to the collapse of primordial gas clouds, we present a brief review of cooling processes in primordial gas clouds. Compared with the interstellar
medium which contains dusts and metals, primordial clouds have only light elements, and the hydrogen dominates the other species. Thus the cooling processes in primordial gas clouds are related to hydrogen atoms and hydrogen molecules.

2.1. Radiative recombination

The thermal energy loss associated with the recombination of proton with electron is caused by the photon emitted during the process. The recombined atom which is in an excited state in general, eventually decays to the lowest energy level by emitting photons. Accordingly, the energy loss per one recombination process is the difference between the energy of the electrons at a bound state of hydrogen atom and the kinetic energy of the free electron. The cross section is assessed by the assumption of case B recombination. In other words, we employ the recombination rate as the summation of the rates that the electron recombines to the levels except the lowest level. We use this recombination rate because the atom recombined directly to the lowest level emits the photon which is energetic enough to photoionize the nearby atom. Then the net energy loss associated with the recombination to the lowest state is negligible. The cross section of the recombination $\sigma_{fb}(n)$ is,

$$\sigma_{fb}(n) = \sigma_{bf}(n) \frac{m_e^2 c^2 v^2}{\nu_n^2 h^2 + m_e v^2/2} \frac{1}{n^2}, \quad (2.1)$$

where

$$\sigma_{bf}(n) = \frac{64\pi ng}{3\sqrt{3}} \alpha a_0 \left( \frac{h\nu_n}{h\nu_n + m_e v^2/2} \right)^3. \quad (2.2)$$

Here $\sigma_{bf}(n)$ is the photoionization cross section. $m_e, c, v, \nu_n, h, n, g, \alpha$ and $a_0$ denote electron mass, speed of light, the electron velocity, the frequency of an emitted photon in recombination to the $n$th level, Plank constant, the quantum number of the energy level, gaunt factor, fine structure constant and Bohr radius, respectively. Using the above expressions, we can calculate the recombination rate by averaging the electron velocity assuming Maxwellian distribution, as

$$\langle \sigma_{fb}(n)v \rangle = \int_0^\infty v f(v) \sigma_{fb}(n)dv = 3.262 \times 10^{-6} M(n, T), \quad (2.3)$$

where

$$f(v) = 4\pi \left( \frac{m_e}{2\pi kT} \right)^{3/2} v^2 \exp \left( \frac{-m_e v^2}{2kT} \right), \quad (2.4)$$

and

$$M(n, T) = \frac{e^{h\nu_n/kT}}{n^3 T^{3/2}} E_1(h\nu_n/kT). \quad (2.5)$$
Here $E_1(x)$ denotes the first exponential integral. Summing up the above expression for $n \geq 2$ we obtain the recombination rate. According to Spitzer,\cite{15} the recombination rate is written in the form as,

$$k_{\text{rec}} = 2.06 \times 10^{-13} \left( \frac{T}{10^4 \text{K}} \right)^{-1/2} \phi(1.58 \times 10^5 \text{K}/T),$$

(2.6)

where $\phi(x)$ is listed in Table I for different temperatures. In our calculations, we linearly interpolate this table to obtain the recombination rate. Next we can get the cooling rate due to the recombination. Assuming the case B recombination, the cooling rate is estimated as

$$A_{\text{rec}} = n_e n_p \sum_{n \geq 2} \frac{1}{2} m_e \int_0^{\infty} v^3 f(v) \sigma_{\text{fb}}(n) dv.$$  

(2.7)

In our calculations, we employ the Spitzer’s results again. The cooling rate is expressed as follows:

$$A_{\text{rec}} = 2.86 \times 10^{-27} n_e n_p T^{1/2} \chi(1.58 \times 10^5 \text{K}/T) \text{ erg s}^{-1} \text{cm}^{-3},$$

(2.8)

where $\chi(x)$ is the function listed in Table II for different temperatures.

<table>
<thead>
<tr>
<th>$T$ [K]</th>
<th>$\phi$</th>
<th>$T$ [10$^4$ K]</th>
<th>$\phi$</th>
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<td>1.06</td>
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<td>3.2</td>
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<td>100</td>
<td>0.36</td>
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<td>1000</td>
<td>0.074</td>
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<td>1.34</td>
<td>10000</td>
<td>0.011</td>
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<th>$\chi$</th>
<th>$T$ [K]</th>
<th>$\chi$</th>
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<td>1000</td>
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<td>64000</td>
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### 2.2. Collisional ionization

The collisional ionization of the hydrogen atom is also a cooling process. The energy loss in this process is not responsible to the radiation emitted away, but the ionization potential energy. In other words, the thermal energy of electrons is converted to the ionization energy in this process. The cross section of collisional ionization\cite{16} by an impact with a free electron is

$$\sigma_{\text{ion}} = a_1 q_1 \ln \frac{E/P_1}{E P_1} \left( 1 - b_1 \exp (-c_1 (E/P_1 - 1)) \right); \quad E \geq P_1,$$

(2.9)

where $q_1 = 1, a_1 = 4 \times 10^{-14} \text{cm}^2(\text{eV})^2, b_1 = 0.60, c_1 = 0.56, P_1 = 13.6 \text{eV}$ and $E$ is the energy of the electron. The rate coefficient is obtained by integrating $\sigma_{\text{ion}} v$ over...
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the Maxwellian distribution (Eq. (2.4)),

\[ \langle \sigma_{\text{ion}} v \rangle = \int_0^\infty v f(v) \sigma_{\text{ion}} dv \]

\[ = 2.68 \times 10^{-6} \frac{1}{(kT)^{3/2}} \]

\[ \times \left( \frac{1}{P_1/kT} E_1(P_1/kT) - \frac{b_1 \exp(c_1)}{P_1/kT + c_1} E_1(P_1/kT + c_1) \right) \text{cm}^3\text{s}^{-1}, \]

where \( E_1(x) \) again denotes the first exponential integral.

2.3. **Bound-bound transition of hydrogen atom**

This is the most important cooling process around \( T \sim 10^4 \text{K} \). Bound-bound transition of hydrogen atoms emit the radiation whose energy amount to the energy difference between the two levels. We need the transition rate between the two levels and the level population to assess the cooling rate. The decay rate of the hydrogen atom at \( n \)th energy level to \( m \)th level is known as the Einstein’s \( A \) coefficient. \( A_{nm} \) (\( A \) coefficient from \( n \)th level to \( m \)th) is written as,

\[ A_{nm} = -\frac{8\pi^2 e^2 \nu_{nm}^2 f_{nm}}{m_e c^3}, \]

where \( \nu_{nm} \) and \( f_{nm} \) denote the frequency between the two levels and the emission oscillator strength, respectively. In Table III, \( A_{nm} \) and \( f_{nm} \) are listed.\(^{17}\) It is shown that the Lyman-\( \alpha \) line has a much larger transition rate than the other lines in this table. In fact, around \( T \sim 10^4 \text{K} \), Lyman-\( \alpha \) line cooling rate dominates the cooling rates by the other lines. The level population of a hydrogen atom is determined by the balance equation of the excitation rate and the de-excitation rate between the levels. In our calculations, the de-excitation rate is dominated by the radiative transition since the density of the cloud is sufficiently low. On the other hand, the excitation is controlled by the collision. As a result, the balance equations are

\[ \sum_{n>m} A_{nm} n_n + \sum_{n<m} C_{nm} n_n n_e = \sum_{n<m} A_{mn} n_m + \sum_{n>m} C_{mn} n_m n_e, \]

Table III. **A-coefficients of hydrogen atom.**

<table>
<thead>
<tr>
<th>( n ) - ( m )</th>
<th>( f_{nm} )</th>
<th>( A_{nm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>4.16 × 10^{-4}</td>
<td>4.70 × 10^8 s^{-1}</td>
</tr>
<tr>
<td>1 - 3</td>
<td>7.91 × 10^{-2}</td>
<td>5.57 × 10^7 s^{-1}</td>
</tr>
<tr>
<td>1 - 4</td>
<td>2.90 × 10^{-2}</td>
<td>1.28 × 10^7 s^{-1}</td>
</tr>
<tr>
<td>2 - 3</td>
<td>6.40 × 10^{-1}</td>
<td>4.41 × 10^6 s^{-1}</td>
</tr>
<tr>
<td>2 - 4</td>
<td>1.19 × 10^{-1}</td>
<td>8.42 × 10^5 s^{-1}</td>
</tr>
<tr>
<td>3 - 4</td>
<td>8.42 × 10^{-1}</td>
<td>8.99 × 10^6 s^{-1}</td>
</tr>
</tbody>
</table>

Table IV. **The parameters in the cross section of collisional excitation.**

<table>
<thead>
<tr>
<th>( n ) - ( m )</th>
<th>( A )</th>
<th>( \chi )</th>
</tr>
</thead>
<tbody>
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<td>1 - 2</td>
<td>24</td>
<td>0.28</td>
</tr>
<tr>
<td>1 - 3</td>
<td>22</td>
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<tr>
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</tr>
<tr>
<td>3 - 4</td>
<td>124</td>
<td>0.26</td>
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</table>
where $n_m$ and $C_{nm}$ denote the number density of hydrogen atoms at $n$th level and the collisional excitation rate from the $n$th level to the $m$th. The collisional excitation rate $C_{nm} = \langle \sigma_{nm} v \rangle^{18}$ is

$$C_{nm} = 10^{-8} \left( \frac{13.6 \text{ eV}}{h \nu_n - h \nu_m} \right)^{3/2} \left( \frac{\nu_m}{\nu_n} \right)^{3/2} \frac{1}{n} e^{-\beta} G(\beta) \text{ cm}^3\text{s}^{-1}, \quad (2.13)$$

where

$$\beta = (h \nu_n - h \nu_m)/kT \quad (2.14)$$

and

$$G(\beta) = \frac{A \sqrt{\beta(\beta + 1)}}{\beta + \chi}. \quad (2.15)$$

$A$ and $\chi$ are given in Table IV. The cooling rate is expressed as,

$$\Lambda = \sum_{n \geq 2} \sum_{m < n} A_{nm} n_m. \quad (2.16)$$

Here we must remark one important point below. Since we are interested in the evolution of the objects at pregalactic scale, they are highly optically thick at Lyman photon frequency, although we treat the cooling function as if the cloud is optically thin in Eq. (2.16). However, the energy density of maximally emitted Lyman-α photons from hydrogen atoms is lower than the ‘heat capacity’ of photons at $T \sim 10^4 \text{K}$. In other words, the space is not ‘filled’ with radiation although the system is highly optically thick.

The capacity [cm$^{-3}$] for the Lyman-α photons are

$$N_{\text{cap}} \approx \frac{4 \pi B_\nu(T) \Delta \nu}{c h \nu}, \quad (2.17)$$

where

$$B_\nu(T) = \frac{2h \nu^3 / c^2}{\exp(h \nu/kT) - 1}, \quad (2.18)$$

and $\Delta \nu$ denotes the Doppler broadening width of the line. The maximal number density of Lyman-α photons emitted from the gas particles is estimated as follows: once the cloud heated up to high temperature, say $T \gtrsim 10^5 \text{K}$, many Lyman photons are produced through the collisional excitation process of neutral atoms. However, the Lyman photons can escape from the cloud because it is highly ionized at this temperature. That is why the Lyman photons excited at high temperature are not trapped in the cloud. When the cloud cools down to the temperature $\sim 10^4 \text{K}$, the neutral fraction of hydrogen atoms increases. Consequently, the cloud becomes highly optically thick, and the Lyman photons are trapped in the cloud. On the other hand, the energy density of Lyman α photons trapped in the cloud is bounded by the thermal energy density of the gas particles at $T \sim 10^4 \text{K}$. Comparing this
Fig. 3. The ratio in Eq. (2.19) is presented for $n_{\text{H}} = 10^2 \text{ cm}^{-3}$. The ratio is smaller than 1 for $T > \sim 10^4 \text{ K}$ at which the Lyman $\alpha$ cooling is the most important process.

maximal number density of the Lyman $\alpha$ photons with the photon capacity of the space (Eq. (2.17)), we have

$$\frac{N_{\text{max}}}{N_{\text{cap}}} \approx \frac{3/2n_{\text{H}}kT}{\hbar \nu} \frac{ch\nu}{4\pi B_{\nu}(T)\Delta \nu}$$

$$= \frac{3/2n_{\text{H}}ckT}{4\pi B_{\nu}(T)\Delta \nu}.$$  \hspace{1cm} (2.19)

The ratio $N_{\text{max}}/N_{\text{cap}}$ is plotted as a function of temperature in Fig. 3. It is shown that the ratio is smaller than unity for $T > 10^4 \text{ K}$ at which the Lyman $\alpha$ cooling is the most important process.

2.4. Thermal bremsstrahlung emission

Radiation due to the acceleration of a charge in Coulomb field of another charge is called as bremsstrahlung or free-free emission. The energy emitted from the encounter of a proton with an electron is expressed as,

$$\frac{dE}{d\nu dV dt}(v, \nu) = \frac{16\pi e^6}{3\sqrt{3}c^3m_e^2v}n_p n_e g_{\text{ff}} \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1},$$  \hspace{1cm} (2.20)

where $v, \nu$ and $g_{\text{ff}}$ denote the velocity of primary electron, the frequency of the emitted radiation and the gaunt factor of free-free emission. Now averaging Eq. (2.20) over the velocity with Eq. (2.4) we obtain the emissivity of the thermal bremsstrahl-
H. Susa

lungs radiation. Performing the integral

\[
\frac{dE}{dVdtd\nu}(T,\nu) = \langle \frac{dE}{dVdtd\nu}(v,\nu) \rangle \\
= \int_{v_{\text{min}}}^{\infty} \frac{dE}{dVdtd\nu}(v,\nu)f(v)dv,
\]

(2.21)

where \( v_{\text{min}} \equiv \sqrt{2h\nu/m_e} \), we obtain

\[
\frac{dE}{dVdtd\nu} = \frac{32\pi e^6}{3m_e c^3} \left( \frac{2\pi}{3k m_e} \right)^{1/2} T^{-1/2} n_e n_p \exp \left( -h \nu / kT \right) \langle g_{\text{ff}} \rangle.
\]

(2.22)

The lower bound of the integral \( v_{\text{min}} \) is determined by the physical requirement that the incident kinetic energy must be above the radiated energy. The cooling rate is assessed by integrating Eq. (2.22) over the frequency. Performing the integral,

\[
A_{\text{ff}} = \int_0^{\infty} \frac{dE}{dVdtd\nu}(T,\nu)d\nu,
\]

(2.23)

we obtain

\[
A_{\text{ff}} = 1.426 \times 10^{-27} n_e n_p T^{1/2} \langle g_{\text{ff}} \rangle \text{ erg s}^{-1}\text{cm}^{-3},
\]

(2.24)

where \( \langle g_{\text{ff}} \rangle \) is the averaged Gaunt factor, which is fitted by the following empirical formula:

\[
\langle g_{\text{ff}} \rangle = 0.79464 + 0.1243 \log T \quad \text{for} \quad T < 3.2 \times 10^5 \text{ K}
\]

\[
= 2.13164 - 0.1240 \log T \quad \text{for} \quad T > 3.2 \times 10^5 \text{ K}.
\]

(2.25)

2.5. \( H_2 \) radiative cooling

The process of \( H_2 \) radiative cooling is very important for the primordial gas at rather low temperature \((T \lesssim 10^4 \text{ K})\). The hydrogen molecules have the energy levels corresponding to the vibrational transitions and the rotational transitions. The vibrational transitions are more important at high temperature \((10^3 \text{ K} \lesssim T \lesssim 10^4 \text{ K})\), and the others are more significant at low temperature \((T \lesssim 10^3 \text{ K})\). It is worth noting that there are a few qualitative differences between \( H_2 \) line cooling and that of hydrogen atoms. First of all, the ‘photon capacity’ is much smaller than the previous case, because the capacity is proportional to \( T^3 \) and the temperature is lower than \( 10^4 \text{ K} \), when the molecular line cooling is effective. Accordingly, we must treat the radiative transfer of molecular line photons carefully. Second, the Einstein’s A-coefficients are extremely smaller than the previous case, because the hydrogen molecules have no dipole moments. As a result, the absorption coefficients become much smaller. At the same time, the critical column density above which the cloud is optically thick become much larger. The small A-coefficients also affect the level population of the excited states. The collisional de-excitation process is effective for \( n \gtrsim 10^4 \text{ cm}^{-3} \) although the critical density for hydrogen atom is higher than \( 10^{15} \text{ cm}^{-3} \). When the cloud is optically thick, we have to solve the level populations
to estimate the emissivity and the radiative transfer equation for each line. This issue is discussed in Chapter 6 and Chapter 5.

In case the cloud is optically thin, the fitting formulae are already given by several authors. We also remark that the pregalactic clouds that we are going to discuss in this section, are always optically thin due to the low H$_2$ fraction ($y_{H_2} \lesssim 10^{-3}$) and the lower densities ($n_H \lesssim 10^6$ cm$^{-3}$) than those of the collapsing proto-stellar core.

First we show the formula given by Hollenbach and McKee.$^{20}$ According to their paper, the cooling function is expressed as

$$A_{H_2HM} = n_H L^H_{\nu r} + n_{H_2} L^H_{\nu r}$$

$$L^H_{\nu r} = \frac{L^H_{\nu r}(n \to \infty)}{L^H_{\nu r}(n \to \infty) + L^H_{\nu r}(n \to 0)} + \frac{L^H_{\nu r}(n \to \infty)}{L^H_{\nu r}(n \to \infty) + L^H_{\nu r}(n \to 0)},$$

where

$$L^H_{\nu r}(n \to \infty) = \frac{1}{n_{H,H_2}} \times \left( \frac{9.5 \times 10^{-22} T_3^{3.76}}{1 + 0.12 T_3^{2.1}} e^{-(0.13/T_3)^3} + 3 \times 10^{-24} e^{-(0.51/T_3)} \right),$$

$$L^H_{\nu r}(n \to \infty) = \frac{1}{n_{H,H_2}} \left( 6.7 \times 10^{-19} e^{-(5.86/T_3)} + 1.6 \times 10^{-18} e^{-(11.7/T_3)} \right),$$

$$L^H_{\nu r}(n \to 0) = 0.25 \left( 5 \gamma_2^H H_2 (E_2 - E_0) e^{-E_2/E_0} \right) + 0.75 \left( \frac{7}{3} \gamma_3^H H_2 (E_3 - E_1) e^{-E_3/E_1} \right),$$

$$L^H_{\nu r}(n \to 0) = \gamma_{10}^H H_2 e^{-E_{10}/kT} + \gamma_{20}^H H_2 e^{-E_{20}/kT},$$

with

$$\gamma_J^H = \left( \frac{1 \times 10^{-11} T_3^{1/2}}{1 + 60 T_3^{-4}} + 1 \times 10^{-12} T_3 \right) \left( 0.33 + 0.9 e^{-J/0.5} \right),$$

$$\gamma_J^H = (3.3 \times 10^{-12} + 6.6 \times 10^{-12} T_3) \left( 0.276 J^2 e^{-J/3T_3} \right),$$

$$\gamma_{10}^H = 1 \times 10^{-12} T_3^{1/2} \exp \left( -(1000/T) \right),$$

$$\gamma_{20}^H = 1.6 \times 10^{-12} T_3^{1/2} \exp \left( -(400/T)^2 \right),$$

$$\gamma_{10}^H = 1.4 \times 10^{-12} T_3^{1/2} \exp \left( -(12000/ (T + 1200))^2 \right),$$

$$\gamma_{20}^H = 0,$$

$$T_3 \equiv T/1000 K,$$

$$(E_2 - E_0)/k = 512 K,$$
Here, $L^{\text{H,H}2}$ and $\gamma^{\text{H,H}2}$ denote the expressions $L^\text{H}$, $L^\text{H}2$ and $\gamma^\text{H}$, $\gamma^\text{H}2$, respectively. $J$ represents the rotational level of hydrogen molecules. In the above expressions, $J = 2$ and $J = 3$ are used. The units of all $L$ and $\gamma$ are erg s$^{-1}$cm$^3$ and cm$^3$s$^{-1}$, respectively. In the above expression, $A_{\text{H}2\text{HM}}$ consists of two parts. The first term is the contribution from H-H$_2$ collisional excitation and the second is due to H$_2$-H$_2$ collision. We also find that $L_r^{\text{H},\text{H}2}$ and $L_v^{\text{H},\text{H}2}$ have two limiting values for $n \to \infty$ and $n \to 0$. The dependence of these limiting values on the density are different from each other. The former are proportional to $n^{-1}$ and the latter are independent of the density. The difference is due to the dependence of the fraction of the excited hydrogen molecules on the density. The level populations are determined by the balance between the de-excitation rates from higher levels to lower levels and the excitation rates. The excitation rates are always dominated by the collisional excitation processes, though the de-excitation rates are determined by both the collisional processes and the radiative decay. The collisional de-excitation rates are proportional to $n^2$ and the radiative decay rates are proportional to $n$. Accordingly, the de-excitation rates are dominated by the collisional process at high density, and the radiative decay dominates the de-excitation process for low density. Thus, the number density of excited molecules are proportional to $n^2$ at low density and to $n$ at high density. The emissivity ($n^2L$) is the product of the number density of the excited molecules and the radiative transition rates. Therefore, the limiting values $L(n \to \infty)$ and $L(n \to 0)$ in Eq. (2.27) have different dependence on density. The density at which the collisional excitation rates equal to the radiative decay rates is called the critical density and its value is $\sim 10^4$ cm$^{-3}$ for the lowest rotational transition. The fitting formulae in Eqs. (2.26) and (2.27) include these effects and also well fit around the critical density.

There also are another fitting formula given by Galli and Palla,$^{21}$ and they use updated data. The cooling function is expressed as,

$$A_{\text{H}2\text{GP}} = \frac{A_{\text{H}2\text{HM}}(n \to \infty)}{1 + n_{cr}/n_{H}} , \quad (2.42)$$

$$n_{cr} = n_{H} \frac{A_{\text{H}2\text{HM}}(n \to \infty)}{A_{\text{H}2\text{GP}}(n \to 0)} , \quad (2.43)$$

$$A_{\text{H}2\text{ GP}}(n \to 0) = \text{dex}(-103.0 + 97.59 \log T - 48.05 (\log T)^2$$
\[ + 10.80 (\log T)^3 - 0.9032 (\log T)^4) \]. \quad (2.44)$$

These two expressions are plotted in Fig. 4 for H-H$_2$ collisional excitation. Another function by Lepp and Shull$^{22}$ is also plotted for comparison.$^\ast$ Remark that these cooling functions are in good agreement with each other at high density (i.e. LTE case), however, for lower density, some inconsistencies are found. First of all the older compilation by Lepp and Shull overestimates the cooling rate at lower density.

$^\ast$ The fitting formula for H$_2$ cooling function in Lepp and Shull$^{22}$ contains a typographical error of which the authors should beware.
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2.6. $H_2$ dissociation cooling

Hydrogen molecules have lower potential energy than the state of two separated neutral hydrogens. Thus, the hydrogen molecules absorb the thermal energy of the particles which collide with hydrogen molecules during the dissociation processes. Then, the cooling rate is given as

$$
A_{\text{diss}} = 7.16 \times 10^{-12} \left( \frac{dn(H_2)}{dt} \right)_- \text{erg s}^{-1}\text{cm}^{-3},
$$

where $n(H_2)$ denotes the number density of hydrogen molecules. In the above equation, $(dn(H_2)/dt)_-$ is the dissociation rate of $H_2$. As will be discussed in the next section, collisional dissociation processes of hydrogen molecules have main three channels, which are $H_2$-$H^+$ collision, $H_2$-$H$ collision and $H_2$-$H_2$ collision. The first channel is efficient when the system is highly ionized. The rate coefficient of this channel is presented in Table V. The latter two channels are important when the temperature ($T \lesssim 3000$ K) significantly. On the other hand, for high temperature ($T \gtrsim 3000$ K) the latest function by Galli and Palla is three times larger than the other functions. The formula given by Hollenbach and Mckee is smaller than that of Galli and Palla by a factor of $2 \sim 3$ in wide temperature range.
system have low ionization degree. The reaction rates are expressed as,

\[
k_5 = k_{H_5}^{1+cr5/nH} k_{L_5}^{1/ncr5}, \quad \text{(H-H2 collision)} \quad (2.46) \\
k_7 = k_{H_7}^{1+cr7/nH} k_{L_7}^{1/ncr7}, \quad \text{(H2-H2 collision)} \quad (2.47)
\]

where

\[
n_{cr5} = \text{dex}(4.0 - 0.416 \log T/10^4 - 0.327(\log(T/10^4))^2) \, \text{cm}^{-3}, \quad (2.48) \\
n_{cr7} = \text{dex}(4.845 - 1.3 \log T/10^4 + 1.62(\log(T/10^4))^2) \, \text{cm}^{-3}, \quad (2.49) \\
k_{L5} = 1.12 \times 10^{-10} \exp(-7.035 \times 10^4/T) \, \text{cm}^{-3}, \quad (2.50) \\
k_{L7} = 1.18 \times 10^{-10} \exp(-6.95 \times 10^4/T) \, \text{cm}^{-3}, \quad (2.51) \\
k_{H5} = 1.20 \times 10^{-9} \exp(-5.24 \times 10^4/T) \, \text{cm}^{-3}, \quad (2.52) \\
k_{H7} = 1.30 \times 10^{-9} \exp(-5.33 \times 10^4/T) \, \text{cm}^{-3}. \quad (2.53)
\]

2.7. \( H_2 \) formation heating

\( H_2 \) formation process is essentially the opposite to the \( H_2 \) dissociation process. The hydrogen molecules are in an excited state when they form. Hence, if the subsequent collisional de-excitation process is more efficient than the spontaneous
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decay process, the excitation energy is transported into the thermal energy of the surrounding particles by collision. If the spontaneous decay rate is greater than the collisional de-excitation rate, the energy is radiated away (H$_2$ line emission) and this process does not help to heat the gas cloud. We take into account this correction which should be adopted to $\Lambda_{\text{diss}}$. The heating rate$^{23}$ is expressed as

$$\Gamma_{\text{form}} = 7.16 \times 10^{-12} \left( \frac{dn(H_2)}{dt} \right) + \frac{1}{1 + n_{\text{cr}}/n_\text{H}} \text{erg s}^{-1}\text{cm}^{-3},$$  \hspace{1cm} (2.54)

where $n_\text{H}$ denotes the total number density of hydrogen nuclei, $(dn(H_2)/dt)_+$ is the formation rate of H$_2$, and $n_{\text{cr}}$ is the critical density which is defined as follows:

$$n_{\text{cr}} = \frac{10^6 \text{ cm}^{-3}}{\sqrt{Tn_\text{H}}} \left(1.6 \exp\left(-\frac{500}{T}\right)^2\right) n(\text{H}) + 1.4 \exp\left(-\frac{12000}{T + 1200}\right) n(\text{H}_2).$$  \hspace{1cm} (2.55)

In the above equation, $(1 + n_{\text{cr}}/n_\text{H})^{-1}$ is the correction term, due to the spontaneous decay of excited levels.

§3. H$_2$ in the early universe

In the interstellar clouds hosted in our Galaxy, H$_2$ is mainly formed on the surfaces of the dust particles, because hydrogen molecules are hardly be formed through direct collision of two hydrogen atoms due to the lack of dipole moment. Therefore, the absence of dust particles in primordial environment, indicates the lower formation efficiency of H$_2$ in primeval galaxies. However, hydrogen molecules are indispensable for the formation of first stars or their hosts. Thus, the importance of different channels of H$_2$ formation in the early universe is inferred. In this section, possible channels for the formation and dissociation of H$_2$ are presented, and relative importance is discussed. In addition, expected evolution of H$_2$ fraction in the expanding universe is reviewed, and major two paths for the formation of hydrogen molecules are discussed.

3.1. Possible formation channels

Possible channels for the formation of hydrogen molecules in zero-metal environments are shown. The reaction rates are listed in Table V.

a  H$^-$ process

$$\text{H} + e \rightarrow \text{H}^- + \gamma,$$
$$\text{H}^- + \text{H} \rightarrow \text{H}_2 + e,$$

b  H$_2^+$ process

$$\text{H} + \text{H}^+ \rightarrow \text{H}_2^+ + \gamma,$$
$$\text{H}_2^+ + \text{H} \rightarrow \text{H}_2 + \text{H}^+.$$

These two processes are preferred simply because dipole moment is necessary to form H$_2$ in two body reactions. Instead of direct collision of two neutral atoms, these
two channels use $H^- + H$ and $H_2^+ + H$. From this point of view one might expect
\[ H_2^+ + e \rightarrow H_2 + \gamma \]
or
\[ H_2^+ + H^- \rightarrow H_2 + H. \]
However, instead of the former reaction, collisional dissociation of $H_2^+$ ($H_2^+ + e \rightarrow 2H$) proceeds. Latter reaction proceeds actually, however, the fractions of $H^-$ and $H_2^+$ are always much smaller than the fraction of neutral hydrogen. Thus, the contribution of these processes are always unimportant.

In any case, two channels $a$ and $b$ are most important channels for the formation of $H_2$ in primordial gas. We also have to remark that these two reactions require electrons or protons as catalysts. Thus, degree of ionization is very important for the formation of $H_2$. In other words, the mechanism to provide the ionized material is required for the formation of $H_2$ in the early universe.

c  Three-body reactions

\[
3H \rightarrow H_2 + H, \\
2H + H_2 \rightarrow 2H_2.
\]
Significance of these three-body reactions are first pointed out by Palla, Salpeter and Stahler. They are important at high density ($n_H \gtrsim 10^8$ cm$^{-3}$) in the course of the collapsing prestellar core in primordial gas.

d  Direct collision between excited hydrogen atoms

\[ H(n = 1) + H(n = 2) \rightarrow H_2 + \gamma. \]
This channel is suggested by Latter and Black. This process is the collision between an excited neutral hydrogen, and a hydrogen at lowest energy level. The cross section of this reaction becomes much larger than that of the direct collision of hydrogen atoms in base state, owing to the dipole moment produced by the difference of the electron energy levels. The significance of this channel is lower than the channel $a$ and $b$. However it becomes important at high redshift ($z \gtrsim 1000$), because the presence of CMB photons destroy $H_2^+$ and $H^-$ at $z \gtrsim 250$ and channel $a$ and $b$ does not work. In addition, higher temperature of the universe help neutral hydrogen to be excited more efficiently.

3.2. Possible dissociation channels

In order to find the fraction of hydrogen molecules, we have to find the dissociation processes as well as the formation processes.

e  Dissociation by the impact with $H / H_2$

\[
H_2 + H \rightarrow 3H \quad \text{or} \\
2H_2 \rightarrow H_2 + 2H.
\]
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These processes are important at $T \gtrsim 2000 \text{ K}$ in the course of the collapses of prestellar cores of first stars. At lower temperature, the collision cannot provide enough energy for dissociation.

f Dissociation by the impact with $\text{H}^+$

\[
\text{H}_2 + \text{H}^+ \rightarrow \text{H}_2^+ + \text{H}, \\
\text{H}_2^+ + \text{e} \rightarrow 2\text{H}.
\]

This channel is most important when the gas cools from a hot ($T \gtrsim 8000 \text{ K}$) phase. The cross section is much larger than the previous reactions. Thus, a certain amount of ionized fraction due to the high temperature enables these dissociation process to be dominant.

g Dissociation by the impact with $\text{e}$

\[
\text{H}_2 + \text{e} \rightarrow 2\text{H} + \text{e}.
\]

This is also important at high temperature, however, this process is always dominated by the previous one ($\text{H}_2 + \text{H}^+$).

h Photo-dissociation

\[
\text{H}_2 + \gamma \rightarrow \text{H}_2^*, \\
\text{H}_2^* \rightarrow 2\text{H} + \gamma.
\]

This is the so-called Solomon process, which require two step reactions. Importance of this reaction will be discussed in Chapter 8.

This is a key process of the negative feedback by the first generation stars for further star formation.

3.3. Cosmic residual $\text{H}_2$

Before we proceed to the formation of cooled objects due to the $\text{H}_2$ line cooling, it is useful to assess the amount of hydrogen molecules formed in the expanding uniform universe. Generally speaking, the two body reaction such as formation of hydrogen molecules or proton-neutron conversion reactions, proceeds faster than the cosmological expansion at very early universe. The reason is like this: the reaction timescales change as $t_{2\text{body}} \propto (1 + z)^{-3}$, whereas the cosmological expansion time scale changes as $t_{\text{H}} \propto (1 + z)^{-2}$ (radiation dominant) or $t_{\text{H}} \propto (1 + z)^{-1.5}$ (matter dominant). In any case, if $z$ is large enough, 2-body reaction timescales become shorter than the cosmological expansion timescale. If another view is carried out, cosmological expansion proceeds faster than the 2-body reactions at lower redshift. This is the well known freeze-out mechanism of species in the universe.

The presence of a certain amount of electrons are crucial for the formation of hydrogen molecules, as shown in the previous subsection. Thus, the amount of residual electrons that failed to recombine at the cosmic recombination epoch
is important. It is assessed by equating the cosmic expansion timescale and the recombination timescale.\(^{38}\)

According to the most recent calculation,\(^{21}\) the residual fraction of electron is

\[
y_{e}^{\text{res}} = 3 \times 10^{-4}.
\]  

(3.1)

Provided these free electrons, hydrogen molecules are formed through the channels \(a\) and \(b\). However, at \(z \gtrsim 200\), both of the channels are blocked because the intermediate products \(\text{H}^-\) and \(\text{H}_2^+\) are photodissociated by the impact with CMB photons. For \(200 \gtrsim z \gtrsim 100\), only \(\text{H}^-\) is destroyed, and the channel starts working. For \(z \lesssim 100\), both of the channels are open, and \(\text{H}_2\) molecules are formed efficiently. Finally, \(\text{H}_2\) fraction almost freezes out at \(z \lesssim 70\). Several authors have calculated such an evolution of \(\text{H}_2\) abundance.\(^{21),25}\) For example, Galli and Palla\(^{21}\) calculated the evolution numerically, and they found the residual fraction of hydrogen molecules for canonical cosmological parameters is

\[
y_{\text{H}_2}^{\text{res}} = \begin{cases} 
1.1 \times 10^{-6}, & z < 100, \\
1 \times 10^{-7}, & 100 < z < 250, \\
1 \times 10^{-7} \left( \frac{1+z}{250} \right)^{-14}, & z > 250.
\end{cases}
\]  

(3.2)

(3.3)

(3.4)

Fig. 5. Comparison among the free-fall time, Hubble expansion time and cooling time with cosmologically residual \(\text{H}_2\). Three solid lines labeled as \(1\sigma, 2\sigma\) and \(3\sigma\) denote the loci where the CDM density fluctuations are expected to “collapse” for \(\Omega = 0.3, \Omega_\Lambda = 0.7\). Shaded region is the prohibited domain in which the Compton heating maintain \(T > T_{\text{vir}}\).
It is important to note that this fraction is too small to cool the pregalactic gas clouds within the free-fall time or Hubble expansion time (Fig. 5). As shown in Fig. 5, along the line of expected collapsing clouds for CDM cosmology (lines labeled $1\sigma, 2\sigma$ and $3\sigma$), no clouds $T_{\text{vir}} < 10^4$ K can be cooled, due to the absence of enough H$_2$. For $T_{\text{vir}} > 10^4$ K, the clouds are cooled by H bound-bound transitions. This result infer that the presence of some other paths to form hydrogen molecules in the collapsing nonlinear clouds is required for the formation of smaller cooled gas clouds with $T_{\text{vir}} < 10^4$ K.

3.4. Major two distinct H$_2$ formation paths in primordial gas

As discussed in the previous sections, free electrons or protons are necessary to form hydrogen molecules. Consequently, the formation processes of hydrogen molecules in the prestellar cores of first stars or their host subgalactic clouds are characterized by the mechanism of electron supply. From this point of view, there are major two possible path for the formation of hydrogen molecules.

The first one is the path which profits the residual electrons in the universe. As discussed in the previous section, significant amount of electrons failed to recombine with protons at the cosmic recombination epoch. Using these electrons as catalysts in the gravitationally contracting gas clouds, hydrogen molecules could be formed to some extent.

For example, in the numerical calculations based upon the 1-zone models (Fig. 6),$^{24,39-41}$ the initial phase of the collapse of a subgalactic cloud is adiabatic, and a dense and high temperature region is formed ($T \sim 10^3$ K, $n_H \sim 10^2 - 10^3$ cm$^{-3}$). In the course of its collapse, hydrogen molecules are formed using the cosmic residual electrons as catalysts. The formed hydrogen molecules behave as coolants, and the gas cloud starts to cool by the ro-vibrational transitions of H$_2$. As a result, the density of the clouds are getting higher and higher.

When the density exceeds the critical density of hydrogen molecules ($n_H \sim 10^4$ cm$^{-3}$), the level population of H$_2$ is determined by the balance between collisional excitation and collisional de-excitation. Thus, the dependence of cooling
rate on density is somewhat changed, the evolution in \( n_\mathrm{H} - T \) plane is changed above critical density. After this point, the three body reactions become dominant at \( n_\mathrm{H} \sim 10^8 \text{ cm}^{-3} \), and the clouds are turned into fully molecular clouds. As a result, the system becomes optically thick for \( \text{H}_2 \) lines. Further evolution will be discussed in Chapter 6.

Another path for the formation of \( \text{H}_2 \) is found in the collapsing massive clouds (\( T_{\mathrm{vir}} \gtrsim 10^4 \text{ K} \)) or photo-ionized gas. In these cases, electrons are provided by the collisionally ionized postshock regions of the collapsed massive clouds, or photo-ionized media. This process will also be discussed in detail in Chapter 3.

\section*{§4. Cooled mass of collapsed objects}

Now we are ready to proceed to the issue of the formation epoch and the mass of first cooled objects that host the first stars in CDM universe. There are three methods to attack this problem. We review these methods and results. In this section, we use \( \Omega = 0.3, \Omega_\Lambda = 0.7, \Omega_B h^2 = 0.02, h = 0.7 \) as canonical cosmological parameters.

\subsection*{4.1. Cooling mass by simplified model}

First approach is taken in Tegmark et al.,\textsuperscript{42}) which employs simple one zone collapse dynamics and evaluates the \( \text{H}_2 \) cooling. The assumed density perturbation is spherical “top-hat”. It is well known that the density of the system evolves as (see also §1.3)

\[
\frac{\rho}{\rho_{\text{univ}}} = \frac{9}{2} \left( \frac{\alpha - \sin \alpha}{1 - \cos \alpha} \right)^2, \quad (4.1)
\]

where \( \rho_{\text{univ}} \) denotes the averaged density of the background universe. The development angle \( \alpha \) is related to \( z \) by the following equation:

\[
\frac{1 + z_{\text{vir}}}{1 + z} = \left( \frac{\alpha - \sin \alpha}{2 \pi} \right)^{2/3}. \quad (4.2)
\]

Here \( z_{\text{vir}} \) is the collapse epoch of the “top-hat” density perturbation, at which the density diverges to infinity. In reality, the density never diverges, of course. Instead, the dark matter halo undergoes the violent relaxation phase, and forms virialized halo. On the other hand, the “average” density of the virialized halo is

\[
\rho_{\text{vir}} \equiv 1.9 \times 10^{-29} \cdot 18 \pi^2 \Omega h^2 (1 + z_{\text{vir}})^2 \text{g} \cdot \text{cm}^{-3}. \quad (4.3)
\]

Thus, when the density in Eq. (4.1) exceeds \( \rho_{\text{vir}} \) in Eq. (4.3), the evolution of the density is switched to

\[
\rho = \rho_{\text{vir}}. \quad (4.4)
\]

The model of the dynamics of collapsing clouds is defined with the above expressions.
On the other hand, the thermal energy equation of the clouds is integrated with the non-equilibrium chemical reaction equations. The energy equation is given as

$$\frac{d}{dt} \left( \frac{3kT}{2\mu m_p} \right) = \frac{p}{\rho^2} \frac{d\rho}{dt} - \Lambda, \quad (4.5)$$

and when the system comes to the collapse epoch $z = z_{\text{vir}}$, $T$ is assumed to jump to $T_{\text{vir}}$. Here $\mu$ denotes the mean molecular weight. The reaction rate equations are

$$\frac{dy_i}{dt} = \sum_j k_{ij} y_j + n_H \sum_{k,l} k_{kl} y_k y_l + n_H^2 \sum_{m,n,s} k_{mns} y_m y_n y_s, \quad (4.6)$$

where $y_i$ is the fraction of $i$th species, $y_i \equiv n_i / n_H$, with $n_H$ being the number density of hydrogen nuclei, and the $k$ are the coefficients of reaction rates (see Table V). In the present calculations, we take into account six species $e$, $H$, $H^+$, $H^-$, $H_2$ and $H_2^+$.

Figure 7 represents the $T$ and $y_{H_2}$ evolutions of two typical cases ($M = 5 \times 10^6 M_\odot$, $z_{\text{vir}} = 20$ and $M = 10^5 M_\odot$, $z_{\text{vir}} = 10$). For the high mass case, the final fraction of hydrogen molecules is as large as several times $10^{-4}$ and the temperature

![Figure 7](https://academic.oup.com/jpqs/article-abstract/doi/10.1143/PTPS.147.11/1854826/10.1143/PTPS.147.11/1854826)

Fig. 7. Evolution of the temperature and $H_2$ fraction is shown for $M = 5 \times 10^6 M_\odot$, $z_{\text{vir}} = 20$ and $M = 10^5 M_\odot$, $z_{\text{vir}} = 10$. These parameters are shown as labels in the figure. The vertical axis shows the temperature and fraction of hydrogen molecules, and the horizontal axis denotes the redshift. The solid curves are for temperature plot, and the long dashed lines denote the evolution of $H_2$ fraction.
Fig. 8. Shaded region denotes the allowed region to form cooled compact clouds for $\Omega = 0.3, \Omega_A = 0.7, h = 0.7$ universe. The vertical axis denotes the halo mass of the collapsing object and the horizontal axis shows the epoch of virialization. Dashed lines denote the expected collapse epoch of CDM $1\sigma, 2\sigma, 3\sigma$ perturbations.

is much below the virial temperature. On the other hand, for low mass clouds, $H_2$ fraction is smaller than the previous case, and the temperature is almost same as the virial temperature. Figure 8 represents the summary of runs, and it is shown that there are critical mass scales with a given collapse epoch ($z_{\text{vir}}$), above which the formation of cooled clouds is allowed. If we think of the $3\sigma$ perturbations in CDM universe, the epoch of formation of first cooled clouds is around $z_{\text{vir}} \sim 30 - 50$ and the mass of the halo is $M \sim 10^5 - 10^6 M_\odot$. It is also worth noting that the dependence of the minimum mass on the virialization redshift is $M_{\text{min}} \propto (1+z)^{-3/2}$. This dependence is derived from the constant virialization temperature. Thus, it means that the systems have to be heated enough to form a significant amount of hydrogen molecules.

Before we proceed to the next section, it is worth noting that we have to be careful to quote the quantitative results in Tegmark et al.\cite{42} First of all, as mentioned before, they use the cooling function by Hollenbach and Mackee,\cite{20} which is smaller than the latest compilation by Galli and Palla\cite{21} by a factor of $2 \sim 3$. Second, the photodissociation cross section used in their paper significantly underestimated. Thus, the residual fraction of hydrogen molecules is overestimated by two orders of magnitude, since the photodissociation rate by the impacts of CMB photons at $z \gtrsim 100$ is significantly reduced. Thus, the initial fraction of hydrogen molecules is already as large as $10^{-4}$, which means further formation processes of $H_2$ in the
collapsing clouds have less importance than those in our calculations. Actually, the
minimum mass of the cooled clouds obtained by ourselves is ten times smaller than
the original paper, although we use the same method except the new cooling function
and reaction rates,\textsuperscript{21)} and flat-$\Lambda$ cosmology.

In any case, from this very simplified model, we expect that the ‘first lights
of the first stars’ are traveling from ancient objects with $M_{\text{halo}} \gtrsim 10^5 - 10^6 M_\odot$ at
$z_{\text{vir}} \sim 30 - 50$.

4.2. Cooling diagram by time scale arguments

The formation of cold parent clouds that host the first stars, should be basically
understood by the comparison between free-fall time and the cooling time. The
‘cooling diagram’ originally introduced by Rees and Ostriker,\textsuperscript{43)} and Silk\textsuperscript{44)} shows
the region where cooling time is shorter than the free-fall time on $\rho$-$T$ plane, vice versa. Here, we present the cooling diagram on $\rho$-$T$ plane including $\text{H}_2$ cooling based
on the formalism proposed in Nishi and Susa.\textsuperscript{45)} Once we have the cooling diagram,
we can predict whether a cloud virialized at $z = z_{\text{vir}}$ with virial temperature $T_{\text{vir}}$
cools.

In order to estimate the cooling rate at $T < 10^4$ K, we need the fraction of $\text{H}_2$. Number fraction of $\text{H}_2$ (here after denoted as $y_{\text{H}_2}$) is not generally in equilibrium
for $T < 10^4$ K in the epoch of galaxy formation. $y_{\text{H}_2}$ depends not only on $\rho$ and $T$
but also on the initial condition. Consequently, we cannot evaluate the cooling rate
on the $\rho$-$T$ plane without estimating non-equilibrium $y_{\text{H}_2}$. As shown in the previous
section, Tegmark et al.\textsuperscript{42)} calculate $y_{\text{H}_2}$ numerically, however, their primordial $y_{\text{H}_2}$ is about two orders of magnitude over estimated because the destruction rate of $\text{H}_2^+$
by CMB at high-$z$ is under estimated.\textsuperscript{21)} Here we adopt a simplified and generalized
method to estimate the cooling function of $\text{H}_2$, differently from that of Tegmark
et al. We introduce four important time scales, $t_{\text{dis}}$, $t_{\text{form}}$, $t_{\text{cool}}$ and $t_{\text{rec}}$. They represent dissociation and formation time of $\text{H}_2$, cooling time, and recombination
time, respectively. The definitions of these time scales are

\begin{align*}
t_{\text{cool}} & \equiv \frac{1.5kT}{\mu m_p A}, \\
t_{\text{rec}} & \equiv \frac{n_p}{k_{\text{rec}} n_e n_p} = \frac{1}{k_{\text{rec}} n_{\text{H}} y_e}, \\
t_{\text{dis}} & \equiv \frac{n_{\text{H}_2}}{\sum_i k_{\text{dis}}^i n_{\text{H}_2} n_i} = \frac{1}{\sum_i k_{\text{dis}}^i n_{\text{H}_2} y_i}, \\
t_{\text{form}} & \equiv \frac{n_{\text{H}_2}}{\sum_{ij} k_{\text{form}}^i n_i n_j} = \frac{y_{\text{H}_2}}{\sum_{ij} k_{\text{form}}^i n_{\text{H}_2} y_i y_j},
\end{align*}

$n_{\text{H}}$, $n_i$, $y_i$ ($\equiv n_i/n_{\text{H}}$), $k_X$, $\rho$, $\mu$ and $m_p$ are the number density of nucleons, the number
density of the $i$th species, the fraction of $i$th species, the chemical reaction rate
coefficient of the “$X$” process, mass density, mean molecular weight and proton mass,
respectively. Comparing these time scales, we assess the non-equilibrium fraction of
$\text{H}_2$ with given virial temperature, and redshift.

In order to evaluate the fraction of $\text{H}_2$, it is necessary to give the electron fraction
of a virialized cloud. It is assumed as \( y_e = \max(y_{\text{res}, e}, y_{\text{eq}, e}) \). Here \( y_{\text{res}, e} \) is the fraction of cosmologically relic electrons calculated in Galli and Palla.\(^{21}\) It equals to \( 3.02 \times 10^{-4} \) for their standard model. \( y_{\text{eq}, e} \) denotes the chemical equilibrium fraction of electrons.

It is also necessary to find the condition that \( \text{H}_2 \) is in chemical equilibrium. The condition should be

\[
    t_{\text{dis}}, t_{\text{form}} < t_{\text{cool}}, t_{\text{rec}},
\]

since the chemical reaction timescales are shorter than other time scales if chemical equilibrium is achieved. In the present case, the condition reduces to more simplified form. In Fig. 9, the fraction of \( \text{H}_2 \) with chemical equilibrium\( (y_{\text{H}_2}^{\text{eq}}) \), and residual value \( (y_{\text{H}_2}^{\text{res}}) \) for \( z_{\text{vir}} < 100 \) are presented. It is clear that the residual fraction is smaller than the equilibrium value. In other words, the fraction of hydrogen molecules is smaller than the equilibrium value before it reaches chemical equilibrium, i.e. \( y_{\text{H}_2} < y_{\text{H}_2}^{\text{eq}} \).

On the other hand, the relation between \( t_{\text{form}} \) and \( t_{\text{dis}} \) is\(^{46}\)

\[
    t_{\text{form}} = \frac{y_{\text{H}_2}}{y_{\text{H}_2}^{\text{eq}}} t_{\text{dis}}.
\]

Thus, the formation time scale \( t_{\text{form}} \) is smaller than the dissociation time scale \( t_{\text{dis}} \) in the course of the evolution of the system. Consequently, the chemical equilibrium condition reduces to

\[
    t_{\text{dis}} < t_{\text{cool}}, t_{\text{rec}}.
\]

Based upon these arguments, our estimation is summarized below (see Fig. 10).
Where are the First Stars Formed?

Fig. 10. The $y_e - T$ plane is divided into three regions. In the region with higher temperature, $t_{\text{dis}}$ is the fastest and $y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}}$. In the region with lower temperature, $t_{\text{rec}}$ is the fastest and $y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}} t_{\text{rec}}$. Between these two regions, where $t_{\text{cool}}$ is the fastest, $y_{\text{H}_2} \approx y_{\text{H}_2}^{\text{eq}} \frac{t_{\text{dis}}}{t_{\text{cool}} t_{\text{form}}}$. (Eq. (4.12))

\begin{equation}
    y_{\text{H}_2}^{\text{eq}} = y_{\text{H}_2} \frac{t_{\text{dis}}}{t_{\text{form}}}. \tag{4.14}
\end{equation}

Note that $y_{\text{H}_2}^{\text{eq}}$ is the function of temperature and density.

**a** $t_{\text{dis}} < t_{\text{cool}}, t_{\text{rec}}$ (Region of “$t_{\text{dis}}$ fastest” in Fig. 10):

$\text{H}_2$ is in chemical equilibrium. Therefore $y_{\text{H}_2}$ is determined by the equation, $t_{\text{form}} = t_{\text{dis}}$. In this case, $y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}}$, where $y_{\text{H}_2}^{\text{eq}}$ denotes the fraction of $\text{H}_2$ in chemical equilibrium (solution of $t_{\text{form}} = t_{\text{dis}}$). Since $t_{\text{dis}}$ is independent of $y_{\text{H}_2}$ and $t_{\text{form}}$ is proportional to $y_{\text{H}_2}$, the solution $y_{\text{H}_2}^{\text{eq}}$ is given as (Eq. (4.12))

\begin{equation}
    y_{\text{H}_2}^{\text{eq}} = y_{\text{H}_2} \frac{t_{\text{dis}}}{t_{\text{form}}}. \tag{4.14}
\end{equation}

**b** $t_{\text{rec}} < t_{\text{cool}}, t_{\text{dis}}$ (Region of “$t_{\text{rec}}$ fastest” in Fig. 10):

$\text{H}_2$ is out of chemical equilibrium, and they are formed until the recombination process significantly reduces the electron fraction. Therefore,

\begin{equation}
    y_{\text{H}_2} \sim \int_{t_{\text{rec}}}^{t_{\text{form}}} \frac{dy_{\text{H}_2}}{dt} dt \sim \frac{y_{\text{H}_2}}{t_{\text{form}}} t_{\text{rec}}. \tag{4.15}
\end{equation}

As a result, $y_{\text{H}_2}$ is determined by the equation, $t_{\text{form}} = t_{\text{rec}}$. Combined with the relation $t_{\text{form}} = y_{\text{H}_2}/y_{\text{H}_2}^{\text{eq}} t_{\text{dis}}$ (Eq. (4.12)), $y_{\text{H}_2}$ is obtained as, $y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}} (t_{\text{rec}}/t_{\text{dis}})$. 

c  $t_{\text{cool}} < t_{\text{rec}}, t_{\text{dis}}$ (Region of “$t_{\text{cool}}$ fastest” in Fig. 10):

When the cooling time is the shortest of the three time scales, $y_{\text{H}_2}$ is not in equilibrium, and increases until the system is cooled significantly:

$$y_{\text{H}_2} \sim \int_{t_{\text{cool}}}^{t_{\text{form}}} \frac{dy_{\text{H}_2}}{dt} \sim \frac{y_{\text{H}_2}}{t_{\text{form}}} t_{\text{cool}}.$$  \hspace{1cm} (4.16)

Thus, $y_{\text{H}_2}$ is determined by the equation $t_{\text{form}} = t_{\text{cool}}$. $y_{\text{H}_2}$ is the solution of a quadratic equation:

$$t_{\text{dis}} x = \frac{1.5 n_{\text{H}} k T}{\Lambda_{\text{H}_2}^{\text{eq}} x + \Lambda_{\text{oth}}},$$  \hspace{1cm} (4.17)

where $x = y_{\text{H}_2}/y_{\text{H}_2}^{\text{eq}}$, $\Lambda_{\text{H}_2}^{\text{eq}}$ denote the $\text{H}_2$ cooling rate with $y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}}$, and $\Lambda_{\text{oth}}$ is the cooling rate by other processes (e.g. hydrogen atomic cooling, Compton cooling). After some algebraic calculations,

$$y_{\text{H}_2} = y_{\text{H}_2}^{\text{eq}} \cdot \frac{1}{2} \left( -\frac{t_{\text{cool,H}_2}^{\text{eq}}}{t_{\text{cool,oth}}} + \sqrt{\left(\frac{t_{\text{cool,H}_2}^{\text{eq}}}{t_{\text{cool,oth}}}\right)^2 + 4 \frac{t_{\text{cool,H}_2}^{\text{eq}}}{t_{\text{dis}}}} \right),$$

$$\approx y_{\text{H}_2}^{\text{eq}} \frac{t_{\text{cool,H}_2}^{\text{eq}}}{t_{\text{dis}}} \text{ for } t_{\text{cool,oth}} \gg t_{\text{cool,H}_2}^{\text{eq}}.$$  \hspace{1cm} (4.18)

Here, $t_{\text{cool,H}_2}^{\text{eq}}$ represents cooling time scale by $\text{H}_2$ ro-vibrational transitions with $y_{\text{H}_2}^{\text{eq}}$ and $t_{\text{cool,oth}}$ is the time scale by the other cooling processes.

![Fig. 11. Virial temperature vs H$_2$ fraction is plotted for given epoch of collapse (z).](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTPS.147.11/1854826)
In Fig. 9, \( y_{H_2} \) is plotted (solid curve) for \( z_{\text{vir}} \lesssim 100 \). It is clear that the relation \( y_{H_2}^{\text{res}} \leq y_{H_2} \leq y_{H_2}^{\text{eq}} \) is satisfied. In Fig. 11, \( y_{H_2} \) is plotted with given virial temperatures for four different redshifts. For low redshift (\( z \lesssim 100 \)) and high temperature (\( T \simeq 10^4 \text{K} \)), \( H_2 \) is in chemical equilibrium with given ionization degree. As the temperature drops, \( H_2 \) gets out of equilibrium because the cooling time becomes shorter than the other time scales. Below 2000 K, recombination time scale is the shortest, and \( y_{H_2} \) becomes relic value.

For high redshift (\( z \gtrsim 100 \)), the destruction of \( H^- \) and \( H_2^+ \) by CMB reduces \( y_{H_2} \) significantly.

We are able to assess the cooling rate with the given \( H_2 \) fraction evaluated in the previous subsection. We compare the time scale of collapse (\( t_{\text{ff}} \)) with the cooling time (\( t_{\text{cool}} \)) which include the contribution from the \( H_2 \) cooling. These time scales are expressed as

\[
    t_{\text{ff}} = \left( \frac{3\pi}{32G\rho_{\text{vir}}} \right)^{1/2}, \quad t_{\text{cool}} = \frac{1.5\mu^{-1}kT_{\text{vir}}}{n_{\text{vir}}\Lambda (y_{H_2}, T_{\text{vir}}, n_{\text{vir}})}.
\]

(4.19)

Fig. 12. Cooling diagram is plotted on the \((1 + z_{\text{vir}}) - T_{\text{vir}}\) plane for \( \Omega = 0.3, \Omega_{\Lambda} = 0.7, h = 0.7 \) universe. Two thick solid lines divide the plane into three regions. In the region denoted as \( t_{\text{ff}} > t_{\text{cool}} \), cooling proceeds faster than the collapse. The narrow region denoted as \( t_{\text{ff}} < t_{\text{cool}} < H^{-1} \), the cloud cools faster than the Hubble expansion, but cannot catch up with the free-fall. The clouds virialized into the lower region (\( H^{-1} < t_{\text{cool}} \)), cannot cool and will not be the luminous objects. The lower right region (\( T_{\text{CMB}} > T_{\text{vir}} \)) is the forbidden region of cooling, due to the Compton heating. Dashed lines denote the loci of constant mass \( 10^7 M_\odot \) and \( 10^5 M_\odot \). Three lines labeled as 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) represent the location of the virialized temperature and redshift of the given magnitude of density perturbations in CDM cosmology.
Here, $\rho_{\text{vir}} \equiv 1.9 \times 10^{-29} \cdot 18\pi^2\Omega h^2 (1 + z_{\text{vir}})^3$ and $n_{\text{vir}} \equiv \Omega_b\rho_{\text{vir}}/m_p$.

Equating $t_{\text{ff}}$ and $t_{\text{cool}}$ in Eq. (4.19), we obtain the boundary between the cooled region during the collapse and the other, which is drawn on the $(1 + z_{\text{vir}})-T_{\text{vir}}$ plane in Fig. 12. The objects virialized into the region denoted as $t_{\text{ff}} < t_{\text{cool}}$ will be cooled by H$_2$ during the gravitational collapse. In this case, the collapsing cloud will be highly dynamical, because the thermal pressure becomes negligible. We remark that the cooling region expands into $T_{\text{vir}} \lesssim 10^4$ K, differently from the classical cooling diagram such as the one in Rees and Ostriker.

We also compare the cooling time scale with the Hubble expansion time ($H^{-1}(z)$). The line $t_{\text{cool}} = H^{-1}(z)$ is also drawn on Fig. 12. The cooling region during the Hubble expansion time is slightly larger than the previous one, because Hubble expansion time is longer than the free-fall time. In this case, the collapse proceeds in semi-statically. As a result, the central region of the cloud will proceed to the run away collapse phase.

4.3. Results obtained by numerical simulations

Several authors have attacked this problem by high resolution numerical simulations. Abel et al. calculated the formation of first structures in CDM universe by 3D adaptive mesh refinement technique, taking into account the non-equilibrium chemistry of H$_2$, radiative cooling, hydrodynamics, and cosmological initial conditions. They use the cooling function by Lepp and Shull in their numerical simulations. They compared the numerical results with the prediction of simple approach used in Tegmark et al. with the cooling function of Lepp and Shull. As a result, it is found that the numerical results are consistent with that of the semi-analytic (Tegmark’s) approach.

Fuller and Couchman investigated this problem with N-body hydrodynamic code HYDRA, simulating the overdense region in the CDM universe. They also find the results of the 3D numerical simulations agree well with the simple semi-analytic approach, however, the numerically obtained minimum mass is an order of magnitude smaller than the result obtained in Tegmark et al. This inconsistency might be caused by the use of different H$_2$ cooling functions in two papers. Remark that our own results shown in Fig. 8 agree well with the results of Fuller and Couchman.

Yoshida et al. performed an extremely huge SPH cosmological simulations with H$_2$ chemistry, and they found the minimum mass is consistent with that of Fuller and Couchman.

In summary, the numerical simulations tell that the minimum mass of the “parent” clouds is basically agrees with the results of simple semi-analytic approach and the time scale arguments. The minimum halo mass at $z \sim 10$ is $10^6 M_\odot$, and it varies as $M_{\text{min}} \propto (1 + z)^{-3/2}$. This means the minimum mass of halo is determined by the virial (shocked) temperature of the system, above which hydrogen molecules are formed enough.

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* In fact, their results are consistent with the results in the previous two sections.
Where are the First Stars Formed?

§5. Summary

In this chapter, the mass and the formation epoch of the pregalactic clouds that host first stars are investigated. Three methods based upon simplified one-zone model, time scale arguments and numerical simulations are introduced. The results of these methods basically agree with others. According to the results, minimum mass scales of the host clouds are found for given formation epochs. The physical reason of the presence of this minimum mass scale is as follows: H\textsubscript{2} have to be formed for the clouds to cool. Since the formation rate of H\textsubscript{2} is larger for higher temperature, a higher H\textsubscript{2} fraction is expected for larger host clouds (higher virial temperature). Consequently, we have the critical mass scale above which the clouds are cooled by hydrogen molecules. If the 3\(\sigma\) density perturbations of CDM universe are regarded as the seeds of “first collapsed halos”, the mass of the very first parent clouds are \(M \sim 10^5 - 10^6 M_\odot\), and they will be found at \(z \sim 30 - 50\).

References

50) N. Yoshida et al., submitted to Astrophys. J.