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An Improved Rational Method for Urban Runoff Application

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In the initial stages of drainage systems planning and for independent tests of advanced runoff model performance there is a need for a simple point flow model. In these cases a method is proposed which is an improved version of the traditional Rational Method. The method, usually regarded as empirical, has a certain relationship with the kinematic wave theory. It is then discussed from both a theoretical and practical point of view based on comparisons with the performance of an advanced continuous model.

Introduction

The analysis and design of urban drainage systems was traditionally, and still often is, executed using the Rational Method. That method has for a long time been regarded as too approximate for many applications. With the introduction of computers several advanced runoff models have been developed and brought into common use. In the initial stages of drainage systems planning and for independent tests of advanced runoff model performance there is a need for a simple point flow model. For these applications the Rational Method, properly used, is a very useful tool despite the extended use of advanced runoff models. In this paper the basic concept of the Rational Method is discussed and compared with the kinematic wave theory. An improved method is proposed and verified in three urban areas.

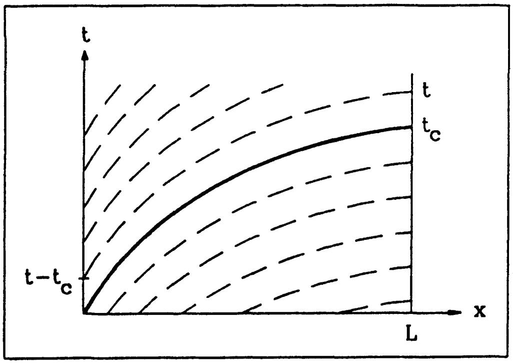

Fig. 1.
A system of kinematic characteristics in the case of lateral inflow only (Lyngfelt 1985).

Basic Deterministic Relationship

The Rational Method is usually referred to as an empirical method. However, the basic concept has a certain relationship with the kinematic wave theory (Newton and Painter 1974). This theory has been proved valid in numerous studies of urban runoff (provided there is no significan backwater). The kinematic wave equations are given by

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} \equiv q \tag{1a}$$

$$Q \equiv a(A)^{b} \tag{1b}$$

where t and x are time and space variables respectively, Q is discharge, A is cross-sectional area, q is lateral inflow and a and b are constants.

Applied to surface flow (length L, width B) with the boundary condition $A(0,t) = 0$, according to Fig. 1, integration of Q along a characteristic line gives Eq. (2) (Lyngfelt 1985)

$$Q(L,t) = a\,B\,\left(\int_{t-t_c}^{t} q(\sigma)\,d\sigma\right)^{b} \tag{2}$$

where the time of concentration t_c is the time for a wave to travel from upstream end at t-t_c to downstream end ($x = L$) at t. In the linear case, $b = 1$, the constant a appears to be the wave velocity defined as $dQ/dA = L/t_c$, giving

$$Q(t) = L\,B\,\frac{1}{t_c}\int_{t-t_c}^{t} i(\sigma)\,d\sigma \tag{3}$$

where i is the rainfall intensity.

Eq. (3) corresponds to an averaging of the rainfall intensities over the time t_c. For each storm event a maximum value of the average intensity can be found

$$i_{\max} \equiv \left[\frac{1}{t_c}\int_{t-t_c}^{t} i(\sigma)\,d\sigma\right]_{\max} \tag{4}$$

The maximum flow is obtained as

$$Q_{\max} = L\,B\,i_{\max} \tag{5}$$

The model (Eq. (5)) expresses the deterministic relation underlaying the Rational Method. Evidently the relation despite its simplicity accounts for not only a transient wave velocity but also detention storage within the flow section. In the case of surface flow (and $b = 1$) the model provides the analytical kinematic wave solution. The relevance of maximum flow values obtained by this model mainly depends on the accuracy of the estimated time t_c and the error introduced by assuming a linear friction relation. In consequence Hager (1985) has shown by numerical experiments that the variation of rain intensities in an interval equal to the time of concentration has little influence on the peak flow value for a rectangular surface (effects of delay were not significant).

The conclusions from this section cannot automatically be generalized to an arbitrary urban drainage system. However, such a system in many cases is only somewhat more nonlinear than a rectangular surface and in addition the inflow pattern to the system may generally be regarded as lateral. Consequently, from a theoretical point of view there is a great potential in the basic model (Eq. (5)).

Statistical Relationship

The design of a network system is basically a statistical problem. In principle, one possible way to balance pipe size against risk is to design the system for each rainfall event in a long series (perhaps 30 years). The return period for the design flows is calculated and a choice between risk levels with corresponding pipe sizes can be made. A more practical but also more approximate approach is based on storms generated by statistical parameters, *i.e.* design storms. By using such a storm a design flow is evaluated which is assumed to have the same return period as the storm.

The traditionally used statistical storm is the Maximum Average Intensity storm (MAI-storm) which is defined by its average intensity $i_{\max}$ corresponding to average time (duration time t_d): Eq. (4) defines $i_{\max}$. Each historical storm can be described by a series of MAI-storms with different durations. From a series of historical storms, frequencies of MAI-storms can be evaluated. For each duration a distribu-

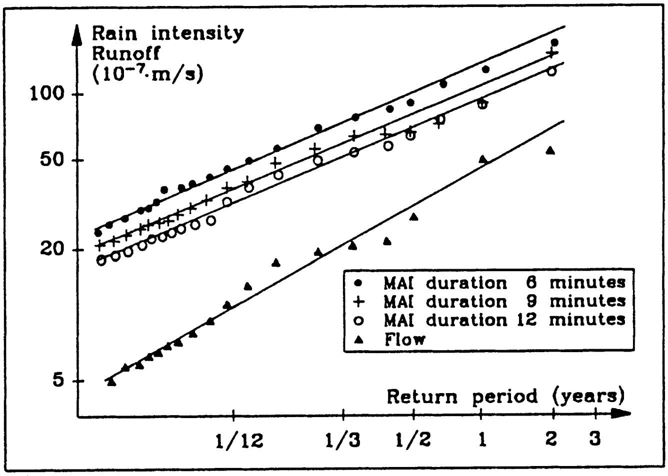

Fig. 2. Distribution functions for MAI-storms and maximum discharge (Arnell and Lyngfelt 1975).

tion function for the intensities can be plotted. Examples of such functions obtained from a two year series are given in Fig. 2 (Arnell and Lyngfelt 1975). The three rain distributions correspond to the durations t_d = 6, 9 and 12 minutes. The frequency is her given as the return period in years (T). The distribution of maximum discharges from a residential area (0.15 km^2) during the same period is also shown in the figure. Assuming parallel intensity and discharge distributions we obtain

$$Q_{max}(T) = c_1\, i\,(T, t_d) \qquad (6)$$

where $Q_{max}(T)$ and $i(T, t_d)$ are flow and MAI-storm distributions respectively, T is the return period, t_d is the duration time and c_1 is a constant. As we can see, all the chosen MAI-storm distributions diverge slightly from this assumption. The storm distributions get closer to the flow distribution with increasing return period. The same tendency can be found in other catchments analyzed in a similar way (Shaake *et al.* 1967).

If the time of concentration is used as the duration of the MAI-storm, the corresponding intensity distribution will have a 'steeper' slope. In a study of five catchments it was found that the distribution $i(T, t_c)$ using constant duration. It was also found that the constant c_1 (Eq. (6)) was close to the estimated contributing area A_c (Lyngfelt 1981). The relation becomes

$$Q_{max}(T) = A_c\, i\,(T, t_c) \qquad (7)$$

where t_c is a function of i.

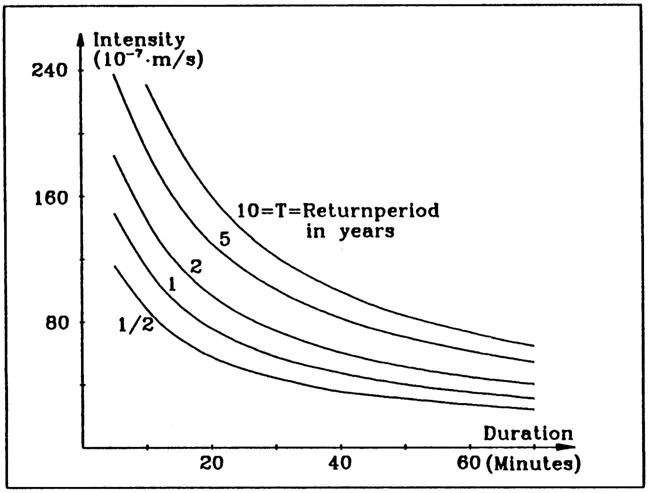

Fig. 3. IDF-diagrams used in Göteborg (VAV 1976).

Time of Concentration

The traditional way of presenting MAI-storm distributions for a series of historical storms is the Intensity Duration Frequency diagram (IDF-diagram). In Sweden IDF-curves have been established at six locations. In Fig. 3 an IDF-diagram from Göteborg is shown.

The curves are characterized by having steep gradients for the durations of most interest in urban drainage design (5-20 minutes). Overestimating the time of concentration by, for example, five minutes may result in an underestimation of the discharge by more than 20 %. The time of concentration is thus a significant parameter and the estimation of the parameter is of great importance in the application of the method.

From the kinematic wave equations it is possible to derive analytical expressions relating the time of concentration to geometric parameters, provided constant rain intensity is assumed. The expressions for surface, gutter and sewer flow may be summarized in Eq. (8) (Lyngfelt 1985)

$$t_c = \frac{K_1}{i^{2/5}} + \frac{K_2}{i^{1/4}} \tag{8}$$

where

$$K_1 \equiv \frac{(n_s L_s)^{3/5}}{S_s^{3/10}} \tag{9}$$

representing the surface flow part (index s) and

$$K_2 = \left[\frac{n_g^2 L_g^2}{S_g}\right]^{3/8} \left[\frac{4(1/z_g+z_g)}{L_s}\right]^{1/4} + \left[\frac{(n_p)^2}{S_p}\right]^{3/8} \left[\frac{4(1/z_p+z_p)}{A_c}\right]^{1/4} L_p \quad (10)$$

representing the gutter flow (index *g*) and pipe flow (index *p*) respectively.

For simplicity, the sewer cross section is assumed here to have a V-shape. *n* is Manning's coefficient of roughness, *L* and *S* are length and slope in flow direction, *z* is the slope factor of side walls. Similar relations also based on the kinematic wave theory have been presented by Singh (1975) and Akan (1984).

Evaluation of the Maximum Flow

The IDF-curves may be expressed by the relation

$$i = \frac{a}{t_d+b} + c \quad (11)$$

where *a, b* and *c* are parameters which vary with location and return period.

Using the Rational Method we are looking for the rain intensity corresponding to the time of concentration estimated by Eq. (8) which is a function of the rain intensity. The intensity is obtained, together with the time of concentration, by solving Eqs. (12a) and (12b)

$$i(T,t_c) = \frac{a}{t_c+b} + c \quad (12a)$$

$$t_c = \frac{K_1}{[i(T,t_c)]^{2/5}} + \frac{K_2}{[i(T,t_c)]^{1/4}} \quad (12b)$$

This may be done by iteration in Eq. (13)

$$(t_c)_{n+1} \equiv \frac{K_1}{[a/((t_c)_n+b)+c]^{2/5}} + \frac{K_2}{[a/((t_c)_n+b)+c]^{1/4}} \quad (13)$$

The intensity $i(T, t_c)$ is obtained from Eq. (12a) for a given t_c and the maximum flow by Eq. (7).

Verification and Application

The improved Rational Method relates the distribution functions of rain intensity and flow in a catchment. It is therefore only possible to investigate the relevance of

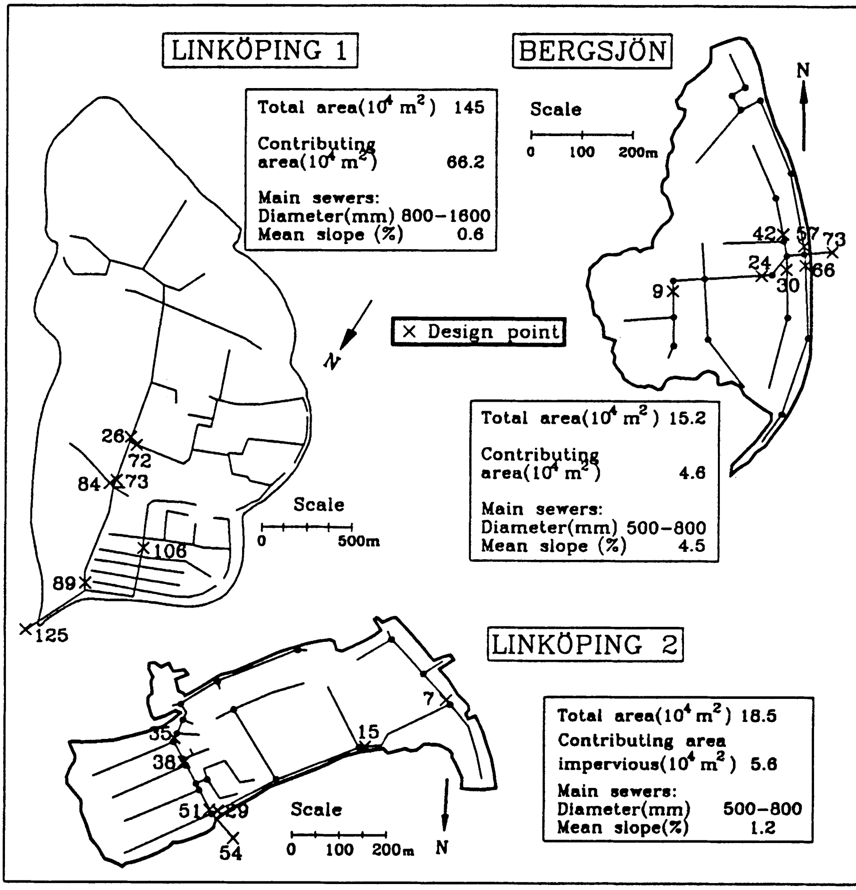

Fig. 4. The three urban areas used in the study with calculation points (sub catchments).

the model in catchments where such functions have been established. Arnell (1982) used an advanced continuous runoff model and a series of historical storms to evaluate distribution functions for discharge and rain intensity in 21 catchments and sub catchments in three different urban areas. The continuous model used was calibrated and verified by measurements of the outflow from the three areas. The study was basically done in order to compare different design storms, used for the design of sewer nets, by detailed runoff models. In Fig. 4 the main sewer network of the three areas Bergsjön, Linköping 1 and Linköping 2 are shown with calculation points.

In this study Arnell's distribution functions were used for comparisons with statistical peak flows evaluated by the Rational Method as described in the last two sections. The intensity-duration-frequency relation (Eq. (11)) used in the Rational Method was evaluated from the above mentioned set of historical storms. In Fig. 5

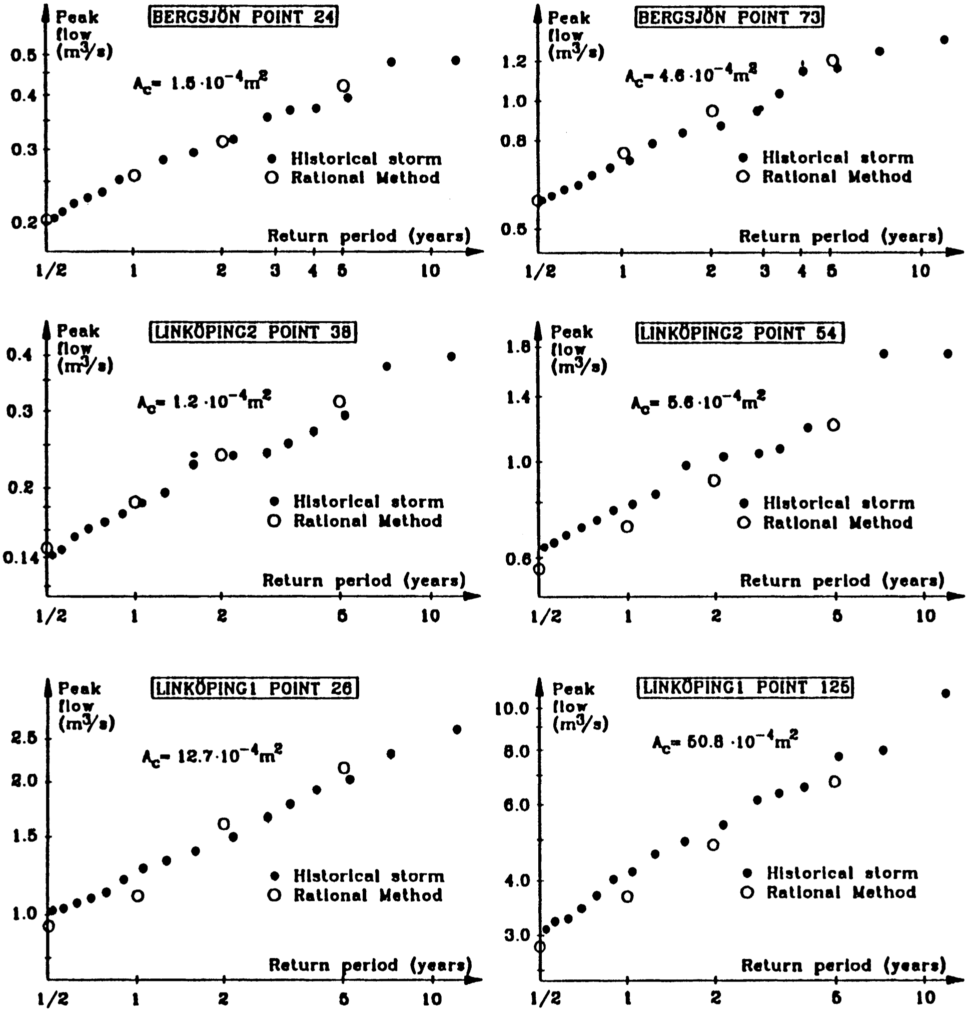

Fig. 5. Distribution functions for peak flows in 6 of the 21 analyzed catchments with corresponding statistical peak flows obtained by the Rational Method.

Table 1 – Mean ratio λ_p, standard deviation σ_p and absolute error ε_p for peak flows evaluated by the detailed model and the Rational Method

Selected catchments	Mean ratio λ_p	Standard dev. σ_p	Abs. error ε_p %	Number of peaks
All	1.05	0.04	5	21
$<1\times10^{-4}m^2$	1.00	0.05	2	9
$1\text{-}5\times10^{-4}m^2$	1.01	0.06	3	7
$12\text{-}50\times10^{-4}m^2$	1.16	0.08	15	5

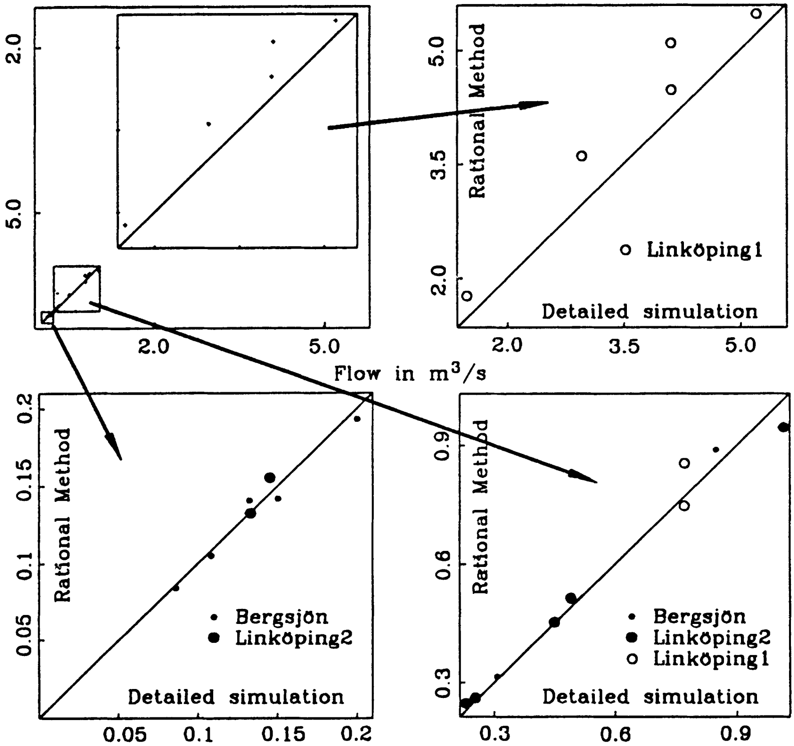

Fig. 6. Statistical flow peaks estimated by the advanced continous model and the improved Rational Method plotted together.

the distribution functions for six of the 21 catchments or calculation points are shown together with corresponding peak flows evaluated by the improved Rational Method. The performance appears by visual inspection of the plots to be satisfactory. This is also evident when commonly used statistical parameters for the peaks, namely the mean ratio λ_p, the mean standard deviation σ_p and the absolute error ε_p, are compared with values which in the literature generally are taken as evidence for a good model performance (Colyer 1977, Price 1980), see Table 1.

In the study a wide range of catchment sizes have been used from the three smallest, $\approx 0.6 \times 10^{+4}$ m^2, to the greatest $50 \times 10^{+4}$ m^2 contributing area. The deviation in flow values between the catchments is shown in Fig. 6 where the peak flows from distributions and the Rational Method are plotted together (return period 2 years). In Table 1 the statistical parameters evaluated for three classes (sizes) of catchments are given. Because of the few values in each class there should not be drawn too many conclusions from the material. It is, however, clearly shown in the table that the deviation increases with increasing catchment area or time of concentration.

Conclusions and Recommendations

The Rational Method, usually referred to as an empirical method, has a sound theoretical basis with a close relationship to the kinematic wave theory. It appears to be capable of estimating statistical design flows. The accuracy of estimated peak flows decreases with increasing contributing area but the method may still be used in quite large catchments.

It should be stressed that the method, as presented here, requires much the same amount of input data as a kinematic wave model. In addition it is usually advantageous to have the entire hydrograph and not only the design flow as a basis in the design situation. Runoff systems with retention storages or overflows are examples where routing methods are preferred.

The improved Rational Method is a very suitable method for calculating peak flows in the preliminary design stage of a network system, in small or simple systems and also for checking advanced continous models. Basic in the method is the careful estimation of the time of concentration with relations based on the kinematic wave theory.

From a practical point of view the proposed method with careful estimation of the time of concentration seems rather complicated. However, when the method has been used for a while the user will get a better feeling for variations in time of concentration and flow with slope, shape, area and so on. Because of this, there is an important educational aspect of the method; the practicing engineer will improve his skill to a level which is hardly reached when using the traditional method.

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Notation

A	(m^2)	– cross-section of flow
A_c	(m^2)	– contributing catchment area
a		– parameter defined in the nonlinear friction relation (1b)
		– parameter defined in the IDF-relation (11)
B	(m)	– width of channel
b		– parameter defined in the nonlinear friction relation (1b)

i	(m/s)	– rain intensity
i_{max}	(m/s)	– maximum average rain intensity
L	(m)	– length in flow direction
n	($m^{-1/3}s$)	– Manning's coefficient of roughness
Q	(m^3/s)	– flow rate
Q_{max}	(m^3/s)	– maximum flow rates obtained by the Rational Method
S_b	(-)	– slope in flow direction
T	(years)	– retun period
t	(s)	– time
t_c	(s)	– time of concentration
t_d	(s)	– duration of the rain
x	(m)	– space coordinate
z	(-)	– slope factor af side walls
s, g, p		– index denoting surfaces gutters and pipes respectively
ε_p	(%)	– mean absolute error in compared peak flow values
λ_o	(-)	– mean of the ratio between flow peaks
σ_p	(-)	– standard deviation of the ratio between compared flow peaks

References

Akan, A. O. (1984) A Physics-Based Approach to Determine Inlet Concentration Times, Proc. 3rd International Conference on Urban Storm Drainage, Vol. 1, Göteborg.

Arnell, V., and Lyngfelt, S. (1975) Rainfall-Runoff Measurements in Bergsjön 1973-1974, Chalmers University of Technology, Urban Geohydrological – Research Group, No. 13, Göteborg.

Arnell, V. (1982) Rainfall Data for the Design of Sewer Pipe Systems, Chalmers University of Technology, Department of Hydraulics, Report Series A:8, Göteborg.

Chen, C-N., and Wong, T. (1987) Comparison of Kinematic Wave and Rational Methods for Site Drainage Design, Proc. 4th International Conference on Urban Storm Drainage, Lausanne.

Colyer, P. (1977) Performance of Storm Drainage Simulation Models, Proc. Inst. Civ. Engineers, Part 2, No. 63, pp. 293-309.

Hager, W. (1985) Effects of Excess Rainfall Time Distribution on Catchment Area Hydrograph, *Nordic Hydrology, Vol 16, No. 3,* pp. 157-168.

Lighthill, M. J., and Whitham, G. B. (1955) On kinematic waves: I. Flood movement in long rivers, Proc. Royal Soc. of London, Ser. A, Vol. 229.

Lyngfelt, S. (1981) Design of Storm Sewer Systems, The Rational Method, Chalmers University of Technology, Urban Geohydrological Research Group, No. 56, Göteborg.

Lyngfelt, S. (1985) On Urban Runoff Modelling. The Application of Numerical Models Based on the Kinematic Wave Theory, Chalmers University of Technology, Department of Hydraulics, Report Series A:13, Göteborg.

Newton, S. G., and Painter, R. B. (1974) A Mathematical Examination of Urban Runoff Prediction, Proceedings of the Institution of Civil Engineers, Vol. 57, U.K.

Singh, V. P. (1975) Derivation of Surface Water Lag Time for Converging Overland Flow, Water Resources Bulletin, AWRS, Vol. 11, No. 3.
VAV, Swedish Water and Waste Water Works Association (1976) Manual for Design of Sewer Systems, VAV Publ. P28, Stockholm.

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