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# Response Time of Forested Mountainous Watersheds in Humid Regions

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This study proposes an analytical method for the estimation of time lag for forested mountainous watersheds. The water flow in a watershed is separated and analyzed in two phases, the land or hillslope phase and the stream channel phase. In many areas around the globe the flow in a forested high gradient watershed is generated through subsurface pathways as several field experiments have shown. The kinematic wave theory is used to describe the generation of flow from steep forested hillslopes. This hillslope runoff is, then, used as input to the stream channels. The equations were developed by assuming kinematic conditions in the stream channel and that the stream slope and the roughness coefficient i) vary according to a second order polynomial with the distance from the mouth of the watershed, ii) vary linearly with the distance from the outlet of the watershed, and iii) are constant throughout the watershed. Comparison of the results of the proposed equation with data from two experimental watersheds in Coastal British Columbia indicates that the three expressions of the proposed equation, even the simplest one assuming constant stream slope and roughness coefficient, are reliable and give good approximation of the observed time lag.

## Introduction

The estimation of flood flows for a given return period is necessary for the appropriate design of drainage and hydrotechnical structures. Use of the rational method, unit hydrographs and at least some of the commonly used rainfall-runoff models require estimates of the time of concentration or basin time lag. Various definitions of basin time lag, $t_l$, and time of concentration, $t_c$, have been proposed. In this paper, $t_l$, is defined as the time between the centroid of rainfall excess and the hydrograph peak.

Errors in the lag time may cause an error in the design flood flows. Bondelid *et al.* (1982) showed that errors in the time of concentration can cause errors as much as 75% of the total error in an estimate of the peak discharge. Recognizing the importance of the time parameters in hydrologic design, many studies have addressed the problem of estimating the time lag or time of concentration. Most of them have proposed empirical equations which are the result of regression analysis relating $t_l$ to the physiographic parameters of the watershed and some to the rainfall parameters, such as the rainfall intensity or duration (Kiprich 1940; Swenty and Westphall 1989; Sabol 1993; and others). Other studies estimated the response time parameters of watershed by applying simple mathematics based on the kinematic wave approximation of the overland flow (Akan 1986; Aron *et al.* 1991). Evaluation of the above equations for regions different from those used for their development showed that most of them are unreliable (Kibler and Aron 1983; McCuen *et al.* 1984; Goitom 1989). In addition, most of the above methods or equations have been developed mainly for small agricultural and urban watersheds where the overland flow is the dominant runoff generation mechanism. There are few reliable methods for the estimation of time lag or time of concentration of forested mountainous watersheds where the storm flow is, usually, generated through subsurface pathways. Because of a lack of proper methods for the estimation of time lag for forested mountainous watersheds, hydrologists tend to use methods developed for agricultural regions. Application of these methods to mountainous watersheds may result in severe under-estimation of the basin response time and consequently, in the over-estimation of the peak flow. Such an over-estimation of the flood value results in the over-sizing of drainage and other engineering structures and greatly increases the cost of construction.

The present paper is an extension and improvement of previous work on the estimation of time lag of mountainous watersheds (Loukas and Quick 1996). The objectives of this study are to develop a physically-based equation for the estimation of time lag of mountainous forested watersheds by using theoretical formulation and physical observations, and to test the validity of the proposed equation by using data from two experimental watersheds. Such an analysis is aimed at providing the design engineer with a simple reliable method for the estimation of time lag of mountainous watersheds where infiltration losses are much greater than normally encountered.

## Development of the Equations for the Time Lag

The runoff of rainwater in a watershed can be separated into two components; the runoff in the land phase and the flow in the stream system. In this study, separate discussions will be made for the two components of water flow in a watershed.

### Land Phase

To evaluate the hydrologic response of a watershed, certain fundamental knowledge is desirable, for example the knowledge of the pathways that water follows on its way from the hillslopes to the stream channel. Overland flow is unlikely to occur in forested hillsides because of the high infiltration capacity of the soil and the thick organic material. If overland flow does occur, it will probably be concentrated in the riparian areas. Many researchers (Hewlett and Hibbert 1967; Hewlett and Troedle 1975; DeVries and Chow 1978; Mosley 1979; Pearce *et al.* 1986; Sklash *et al.* 1986; Tsukamoto and Ohta 1988; Tanaka *et al.* 1988 and others) have observed the generation of runoff through the soil matrix "translatory flow" and "soil pipes" or "macropores" in mountainous forested hillslopes in humid regions and they confirm that these subsurface runoff mechanisms are capable of producing the observed high response of the mountainous watersheds. More specifically field research on the stormflow generation mechanisms in coastal British Columbia (Cheng *et al.* 1975; 1977) showed that the stormflow in the forested high gradient slopes is generated by the two above mechanisms and especially the "soil pipes" are very important in the stormflow concentration and in the high response of the watersheds in the area.

In the soil matrix "translatory flow" (Hewlett and Hibbert 1967), the infiltrated water in the hillslopes tends to displace most of the water ahead of it, which was stored during previous rainfall events. Except for this mechanism the water in forested hillslopes flows in macropores or "soil pipes" (Jones 1971) which deliver the infiltrated water rapidly to the stream channel. These "soil pipes" are developed by the action of insects, small animals, tree roots, and chemical weathering and are sustained by frequent water passage.

Many studies devoted to the problem of subsurface hillslope soil matrix drainage have made use of Eq.(1) which was first developed by Boussinesq (1877)

$$q = -K\,h\left(\frac{dh}{dx}\cos\theta + \sin\theta\right) \tag{1}$$

where $q$ is the flow rate per unit width of aquifer, $K$ the hydraulic conductivity, $h$ the thickness of the saturated zone measured perpendicular to the impermeable layer, $\theta$ the slope angle of the impermeable layer, and $x$ is the downslope distance. However, in most of these studies (Henderson and Wooding 1964; Beven 1981) the approach was further simplified by the kinematic wave approximation. Also, Sloan and Moore (1984) and Stagnitti *et al.* (1986) used this approximation to describe hillslope drainage as partly saturated flow. Because in the kinematic wave approximation the hydraulic gradient is assumed to be equal to $\sin\theta$ and the $dh/dx$ term in Eq. (1) is neglected, this approximation produces a zero flow for any horizontal aquifer, but this approximation is reasonable for steep mountainous hillslopes where the most important slope is the hill slope.

The dynamics of the "pipe flow" are more complicated than the soil matrix flow but Loukas and Quick (1993) showed that the subsurface pipes can respond, under certain conditions, in a similar way to that of the soil matrix flow. Loukas and Quick (1993) developed an equation for the hillslope outflow from the soil pipes by using kinematic wave dynamics which reassembles the kinematic wave approximation of Eq. (1). Hathorn (1993) suggested that the flow rate per unit width through a soil containing soil pipes can be expressed as

$$q = -K_{av} h \sin\theta \qquad (2)$$

where $K_{av}$ is the average saturated hydraulic conductivity of soil pipes and soil matrix.

According to the previous discussion sinθ can be substituted by the average slope to the hillside $S_H$.

From the kinematic wave theory

$$\frac{dh}{dt} = i_e \qquad (3)$$

where $i_e$ is the effective rainfall intensity and $t$ is the time.

Integrating Eq. (3) and substituting into Eq. (2) gives

$$q = -K_{av} i_e t S_H \qquad (4)$$

Field experiments indicated that the soil matrix hydraulic conductivity always decreases with depth in a given hillslope (DeVries and Chow 1978; Beven 1981). In contrast, the density of soil pipe networks, and thus their hydraulic conductivity, increases with depth (Tsukamoto and Ohta 1988; Uchida *et al.* 1999) and as a result an average value of saturated hydraulic conductivity may be representative of the whole hillslope profile. The value of the average saturated hydraulic conductivity of soil pipes and soil matrix has to be measured in the field. Such a study in a forested watershed in coastal British Columbia (Chamberlin 1972) indicated that the downslope saturated hydraulic conductivity of soil containing soil pipes had a mean value of 350 mm/h which is more than double the 142 mm/h of the saturated hydraulic conductivity of the same soil without soil pipes (O'Loughlin 1972). The values of $K_{av}$ can vary significantly from soil to soil but in this study it was assumed that $K_{av}$ is about 350 mm/h for both study watersheds.

Eq. (4) gives the outflow from an inclined hillslope and is valid when the duration of the storm, $t_d$, is less than the time when steady state conditions have been reached in the hillslope, $t_s$. When a steady state is reached, the flow profile will be given by

$$q_s = i_e x = -K_{av} h_s S_H \qquad (5)$$

where $x$ is the distance from the beginning of the hillslope and $h_s$ is the thickness of the saturated zone at the time when steady state has been reached. At that time all the hillslope contributes to flow and so $x=L_H$, where $L_H$ is the length of the hillslope. Also integrating Eq. (3) results in, $h=i_e t$ and substituting into Eq. (5) and rearranging gives

$$t_s = \frac{L_H}{K_{av} S_H} \tag{6}$$

which is the time at which steady state has been reached in the flow conditions of the hillslope. For any natural and undisturbed hillslope the value of $t_s$ is always larger than the duration of any storm so that Eq. (4) is always valid.

It should be mentioned that overland flow may occur on the hillslope when the saturation level (water table), $h$, reaches the top of the surface of the hillslope. This happens when the soil cannot deliver the infiltrated rainwater downhill to the stream. Field experiments in Jamieson Creek watershed in coastal British Columbia (Loukas 1991) showed that the soil depth at the steep forested hillslopes is about 1.5 m. Furthermore, Intensity-Duration-Frequency analysis in the same watershed and at a large number of rainfall stations in coastal British Columbia (Loukas 1994) showed that even the storms with a return period of 100 years have smaller intensities than the average value of the hydraulic conductivity of the forest floor (200 mm/h) (Plamondon 1972) and that of the soil with and without "soil pipes" (350 mm/h and 120 mm/h, respectively) (O'Loughlin 1972; Chamberlin 1972). Consequently, it is very unlikely that there will be free water on the surface of the hillslope and consequent surface runoff. This is in agreement with experiments in forested hillslopes where no overland flow has been observed (DeVries and Chow 1978).

### Stream Channel Phase

The watershed can be conceptualized as two planes draining into the main stem of the stream system. In the development of the equation for the time lag several assumptions have been made for the conditions in the stream channel, as follows.

It is assumed that the flow in the stream channel is kinematic which is not an unrealistic assumption for high gradient mountainous streams where there is little storage in the stream and the flow is only translated downstream with minimum attenuation. In addition, the Manning's equation can be assumed to be valid

$$V = \frac{1}{n} R^{2/3} S_e^{1/2} \tag{7}$$

and the stream flow can be written as

$$Q = VA = \frac{R^{2/3} S_e^{1/2}}{n} A \tag{8}$$

where $A$ is the wetted cross sectional area of the stream channel, $R$ is the hydraulic radius, $S_e$ is the friction or energy line slope and $n$ is the Manning's roughness coefficient. The friction slope $S_e$ is usually substituted by the stream slope $S_s$, because the stream slope is easier to measure or estimate. This assumption is weak for short irregular reaches; however, it is acceptable when stream slopes are averaged over longer distances of the order of hundreds of metres (Hughes 1993). Also, this substi-

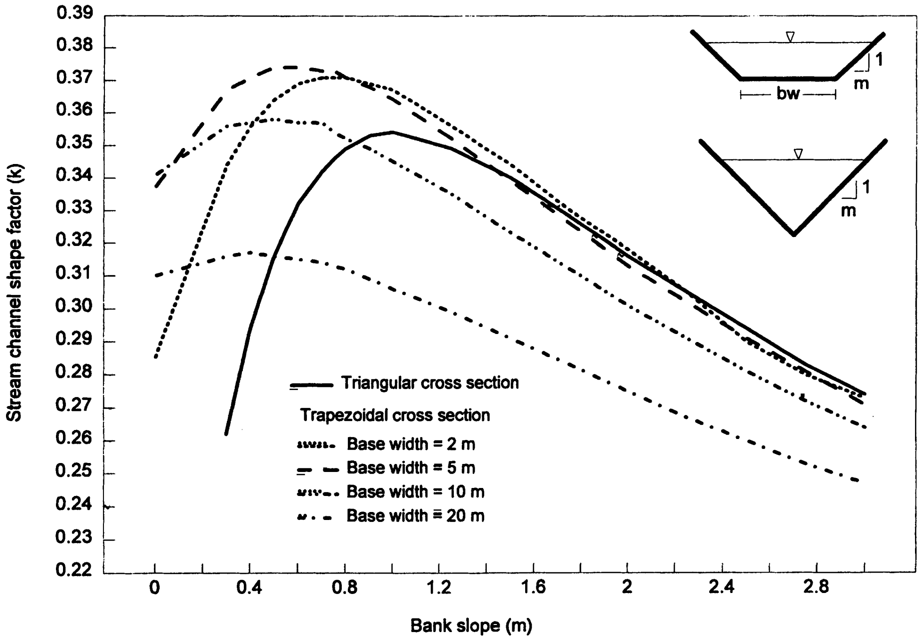


Fig. 1. Variation of stream channel shape factors, $k$, for triangular and trapezoidal cross sections.

tution is in agreement with the kinematic wave approximation assumed in this study for the stream channel flow and the observations in field experiments in mountainous streams (Jarrett 1984).

Also, it is assumed that the hydraulic radius of the stream channel can be expressed as a function of the wetted cross sectional area of the stream channel, $A$

$$R = kA^{1/2} \tag{9}$$

where $k$ is a shape factor.

This assumed relationship was explored further by assuming that the channels may have either a triangular or trapezoidal shape. The hydraulic radii were computed for a range of bank slopes and the shape factors were plotted in Fig. 1. The values of bank slope of the stream channel should be determined by observations in the field for stream segments of similar gradients.

Using continuity for the stream channel flow

$$\frac{\partial Q}{\partial x} = \frac{\partial A}{\partial t} = 2q \tag{10}$$

where $Q$ is the stream flow, $A$ is the wetted area of the channel and $q$ is the lateral hillslope inflow per unit channel length which is defined in Eq. (4).

Application of the method of characteristics and substituting Eqs. (8) and (9) results in the ordinary differentials

$$\frac{dx}{dt} = \frac{dQ}{dA} = \frac{4}{3} k^{2/3} A^{1/3} \frac{S_s^{1/2}}{n} \tag{11}$$

and

$$\frac{dA}{dt} = 2q \tag{12}$$

Substituting Eq. (4) into Eq. (12) and integrating results in

$$A = K_{av} i_e S_H t^2 \tag{13}$$

where $t$ is measured from the beginning of inflow.

Combining Eqs. (11) and (13) results in

$$\frac{dx}{dt} = \frac{4}{3} k^{2/3} (K_{av} i_e S_H)^{1/3} t^{2/3} \frac{S_s^{1/2}}{n} \tag{14}$$

Rearranging Eq. (14) and integrating along the length of the main stem of the stream from 0 to $L$ will require integration of the time from 0 to time lag, $t_l$,

$$\int_0^L \frac{n}{S_s^{1/2}} dx = \frac{4}{3} k^{2/3} (K_{av} i_e S_H)^{1/3} \int_0^{t_1} t^{2/3} dt = \frac{4}{5} k^{2/3} (K_{av} i_e S_H)^{1/3} t_1^{5/3} \tag{15}$$

The left hand side of Eq. (15) can be written as

$$B = \int_0^L \frac{n}{S_s^{1/2}} \tag{16}$$

Substituting back into Eq. (15) and rearranging

$$t_1 = \left( \frac{5B}{4 k^{2/3} (K_w i_e S_H)^{1/3}} \right)^{3/5} \tag{17}$$

Eq. (17) takes the form, when expressing length in metres, rain intensity in millimetres per hour, and time in minutes

$$t_1 = 4.32 \frac{B^{0.6}}{k^{0.4} (K_{av} i_e S_H)^{0.2}} \tag{18}$$

Eq. (18) computes the time lag of watersheds if the topographic characteristics, the effective rainfall intensity, $i_e$, and the average saturated hydraulic conductivity, $K_{av}$, are known, so that it links the physical characteristics of a watershed to its time response through an analytical mathematical procedure. In this study, the effective rainfall intensity is assumed to be equal to the maximum value of the effective rainfall intensity. Also, the average saturated hydraulic conductivity of the soil containing macropores is assumed to be equal to the average value measured in the field (350 mm/h) (O'Loughlin 1972; Chamberlin 1972).

An assumption that is usually made in studies of the time response of watersheds is that both $n$ and $S_s$ are constant over the watershed which is questionable because both $n$ and $S_s$ are likely to increase upstream. In this study $n$ and $S_s$ are kept either constant or varied with the distance from the outlet of the watershed. Three cases were studied: 1) $n$ and $S_s$ are varied according to a second order polynomial with the distance from the outlet 2) $n$ and $S_s$ are varied linearly with the distance from the mouth of the watershed, and 3) $n$ and $S_s$ are assumed to be constant for all the stream length. Also, the channel geometry is assumed constant throughout the stream. Hence, three expressions for time lag have been developed and they will be presented next.

### Non-linear Variation of $n$ and $S_s$

Analysis of the main stream channel slope of 24 mountainous watersheds in coastal British Columbia indicated that the longitudinal slope of the main stem of the stream increases with the distance from the outlet of the watershed and can be described by a second order polynomial

$$S_s = a + b\,x + c\,x^2 \tag{19}$$

in which $a, b, c$, are constants derived by curve fitting and $x$ is the distance from the outlet of the watershed.

In this study, the stream slope of the main stem has been measured from 1:50,000 scale topographical maps but more detailed analysis can be done if field measurements are available and the watershed is accessible.

The Manning's roughness coefficient $n$ has been related to water surface slope $S_w$. Forty-two measurements from mountainous watersheds across U.S.A. were analyzed. The data were taken from Hughes (1993) who compiled data for natural streams published in two U.S. Geological Survey reports (Barnes 1967; Jarrett 1985). Application of linear regression between Manning's n and the water slope, $S_w$, indicated that the $n$ can be written as a function of $S_w$ (Fig. 2)

$$n = 0.0326 + 1.3041 S_w \tag{20}$$

It is assumed that the water slope, $S_w$, can be substituted with the stream slope, $S_s$. This assumption complies to the same restrictions as the assumption of the substitu-

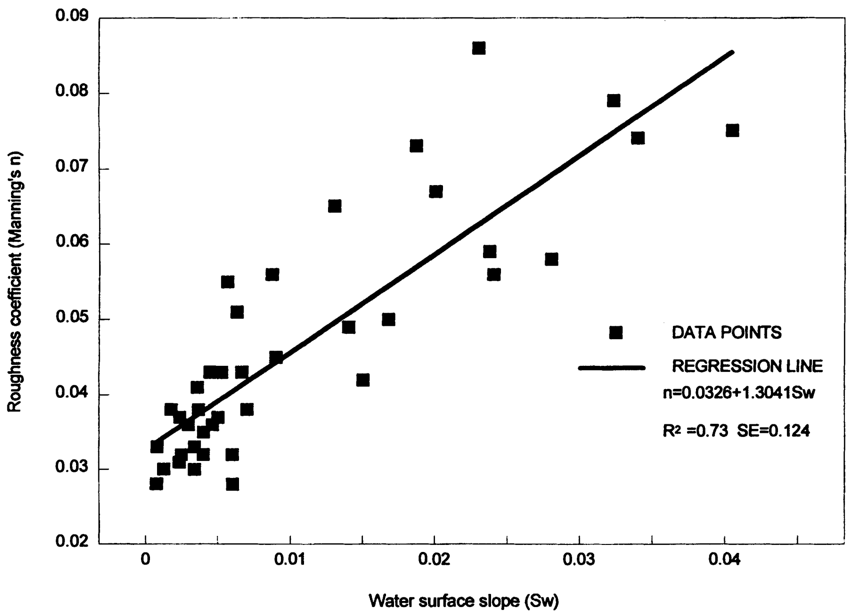


Fig. 2. Roughness coefficient (Manning's $n$) as a function of water surface slope (using data from Hughes, 1993).

tion of the friction slope with the stream slope, discussed previously.

Substituting Eq. (19) into Eq. (20) results in

$$n = p + q\,x + m\,x^2 \tag{21}$$

where $p$=0.0326+1.3041$a$, $q$=1.3041$b$ and $m$=1.3041$c$.

Substituting Eqs. (19) and (21) into Eq. (16) results in

$$B = \int_0^L \frac{p+q\,x+m\,x^2}{(a+b\,x+c\,x^2)^{1/2}}\,dx = \int_0^L \frac{p\,dx}{(a+b\,x+c\,x^2)^{1/2}} + \int_0^L \frac{q\,x\,dx}{(a+b\,x+c\,x^2)^{1/2}} + \int_0^L \frac{m\,x^2\,dx}{(a+b\,x+c\,x^2)^{1/2}} \Rightarrow B = B_1 + B_2 + B_3 \tag{22}$$

Integration of Eq. (22) results in

$$B_1 = p\,B_k \tag{23}$$

where

$$B_k = \frac{1}{c^{1/2}} \left(\ln\{2c^{1/2}(a+bL+cL^2) + 2cL + b\} - \ln(2a^{1/2}c^{1/2}+b)\right) \quad \text{and}$$

$$B_2 = q\left(\frac{(a+bL+cL^2)^{1/2}}{c} - \frac{b}{2c}B_k - \frac{a^{1/2}}{c}\right) \tag{24}$$

$$B_3 = \left(m\left\{\frac{2cL-3b}{4c^2}(a+bL+cL^2)^{1/2} + \frac{3ba^{1/2}}{4c^2} + \frac{3b^2-4ac}{8c^2}B_k\right\}\right) \tag{25}$$

$B_1$, $B_2$, $B_3$ can be found from the known parameters of a given watershed.

### Linearly Varied $n$ and $S_s$

In many small high gradient mountainous watersheds the stream slope may vary linearly with the distance from the mouth of the watershed. In this case the stream slope can be approximated by

$$S_s = a+b\,x \tag{26}$$

in which $a$, $b$ are constants derived by curve fitting and $x$ is the distance from the outlet of the watershed.

Substituting Eq. (26) into Eq. (27) results in

$$n = p+q\,x \tag{27}$$

where $p$=0.0326+1.3041$a$, $q$=1.3041$b$.

Substituting Eqs. (26) and (27) into Eq. (16) and rearranging results in

$$B = \int_0^L \frac{p}{(a+b\,x)^{1/2}}\,dx + \int_0^L \frac{q\,x}{(a+b\,x)^{1/2}}\,dx \Rightarrow$$

$$B = B_1 + B_2 \tag{28}$$

Integrating Eq. (28) results in Eq. (29) where $B_1$ and $B_2$ are

$$B_1 = p\left(\frac{2(a+bL)^{1/2} - 2a^{1/2}}{b}\right)$$

$$B_2 = q\left(\frac{2(bL-2a)}{3b^2}(a+bL)^{1/2} + \frac{4a^{3/2}}{3b^2}\right) \tag{29}$$

The value of $B=B_1+B_2$ can be substituted in Eq. (18) to compute the time lag of a watershed.

### Constant $n$ and $S_s$

The assumptions that the stream slope and the Manning's coefficient are constant in a watershed are easier to apply although hardly apply to mountainous watersheds. Combining Eqs. (11) and (13) and integrating, along the length of the main stem of the stream from 0 to $L$ will require integration of the time from 0 to lag time, $t_l$, leads to Eqs. (17) and (18), where

$$B = \frac{n}{\sqrt{S_s}} L$$

The three expressions that have been developed will be tested against measured data from two experimental watersheds located in coastal British Columbia, in the next section.

## Comparison of the Derived Equations of Time Lag with Measurements

Eq. (18), in its general form derived for the estimation of the time lag was applied to two experimental watersheds to test its validity and the results were compared with the observed values of time lag.

### Study Watersheds

The two experimental watersheds are Carnation Creek and Jamieson Creek. Both watersheds are located on coastal British Columbia. The Carnation Creek watershed is located on the west coast of Vancouver Island, British Columbia, Canada. The basin area of 9.5 km$^2$ features rugged terrain from sea level to 880 m elevation with hill slopes of 40% to over 80%, and a relatively wide valley bottom. Slope soils are coarse colluvial materials of gravelly loam to loamy sand texture with a moderately thick organic layer and are underlain by bedrock of volcanic origin (Hetherington 1982). The average saturated hydraulic conductivity is estimated to be equal to 350 mm/h.

The second study watershed, the Jamieson Creek, is located about 30 km north of Vancouver. The basin has an area of 3 km$^2$ and its elevation ranges from 305 to 1,310 m above mean sea level. Jamieson Creek is characterized by high gradient slopes with an average hillslope gradient of 48%. The soils of the watershed are shallow and very permeable coarse-textured sand and gravelly, sandy loam underlain by bedrock (Cheng 1975). The average saturated hydraulic conductivity is measured to be equal to 350 mm/h (O'Loughlin 1972; Chamberlin 1972).

The climate of the coastal region of British Columbia is influenced by the adjacent Pacific Ocean and features mild, wet winters, characterized by frequent frontal storms, and mild summers. Annual precipitation is mostly rain and ranges from about 2,100 m to over 4,800 m, over 75% falling in October to March.

The data from Carnation Creek consist of hourly streamflow and hourly precipita-

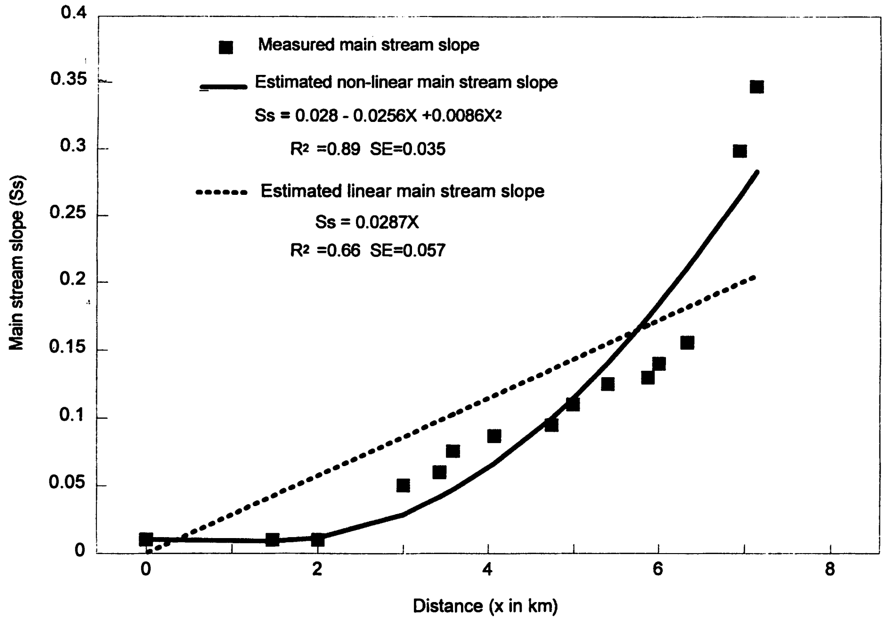


Fig. 3. Variation of the main stream slope with distance from the outlet of the Carnation Creek watershed.

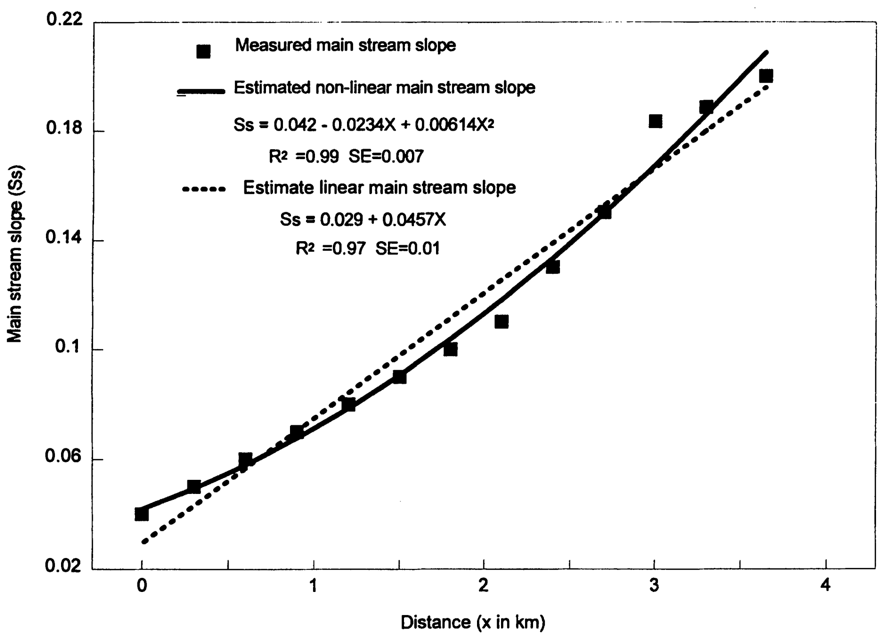


Fig. 4. Variation of the main stream slope with the distance from the outlet of the Jamieson Creek watershed.

tion from one recording station. In addition, there are six high elevation storage gauges that have been used to calculated the ratios of the long term daily precipitation to that of the recording station. In Jamieson Creek there are five recording precipitation stations which measure hourly precipitation and a streamflow gauge at the mouth of the watershed which records hourly flows.

Watershed modeling of Jamieson Creek runoff response (Loukas and Quick 1993) indicated that, on average, 1.4 mm/h are diverted to the slow or deep groundwater runoff. Also, that study indicated that the Hewlett and Hibbert (1967) hydrograph separation method is valid for that watershed. According to the Hewlett and Hibbert method the hydrograph can be separated into its fast and slow runoff components by drawing a line upward from the point of initial hydrograph rise at a slope of $0.55 \times 10^{-3}$ $m^3 sec^{-1} km^{-2} hr^{-1}$. In this study the rainfall abstractions for the Jamieson Creek watershed were taken as the abstractions computed in the previous rainfall-runoff study (Loukas and Quick 1993) whereas for the Carnation Creek watershed the rainfall abstractions were assumed that is, on average, equal to 1.4 mm/h. The direct runoff hydrograph for both watersheds was calculated by using the Hewlett and Hibbert method. In this study, the direct runoff is the runoff generated through the soil matrix

Table 1 – Characteristic parameters of the two study watersheds

| Parameters | Carnation Creek | Jamieson Creek |
|---|---|---|
| Length of stream, $L$ (m) | 7470 | 3645 |
| Non-linear variation of stream slope, $S_s$ (Eq. 19) | $a = 0.028$<br>$b = -2.56 \times 10^{-5}$<br>$c = 86 \times 10^{-10}$ | $a = 0.042$<br>$b = -2.34 \times 10^{-5}$<br>$c = 61.4 \times 10^{-10}$ |
| Linear variation of stream slope, $S_s$ (Eq. 26) | $a = 0$<br>$b = 2.87 \times 10^{-5}$ | $a = 0.029$<br>$b = 4.57 \times 10^{-5}$ |
| Constant mean stream slope, $S_s$ | 0.04 | 0.16 |
| Mean Manning's $n$ | 0.085 | 0.1 |
| Watershed shape factor, $f$ ($m^{-1/5}$) | 0.49 | 0.47 |
| Channel shape factor, $k$ | 0.35 | 0.35 |
| Average soil hydraulic conductivity, $K_{av}$ (mm/h) | 350 | 350 |
| Average hillside slope, $S_H$ | 0.40 | 0.48 |
| $B$ for non-linear stream slope | 3866 | 1178 |
| $B$ for linear stream slope | 4059 | 1188 |
| $B$ for constant stream slope | 3166 | 911 |

and macropores or "soil pipes" discussed earlier in the paper.

The rainfall-runoff events analyzed were single peak hydrograph events. Sixteen events were analyzed for the Carnation Creek watershed and seven events for the Jamieson Creek watershed. The maximum return period of the events analyzed is estimated to be 5 years.

The slope of the main stream of the two study watersheds was plotted against the distance from the outlet of the watershed and it has been approximated by: i) a second order polynomial, ii) a straight line, and iii) an average slope (Figs. 3 and 4) as discussed earlier. The coefficients of the stream slope estimation (*i.e.* $a$, $b$, and $c$) and the average stream slope are shown in Table 1 along with the values of other measured or estimated parameters for the two watersheds.

## Application of the Proposed Equation

The Eq. (18) was applied to the two study watersheds. Sixteen single peak events were analyzed and used from the Carnation Creek watershed whereas seven events used from the Jamieson Creek database. Various statistical criteria used to measure the bias and the precision of the three approximations of the proposed equation. The statistical criterion for bias was the standardized bias

$$B_s = \frac{1}{m} \sum_{i=1}^{m} \frac{\hat{t}_l - t_l}{t_l} \tag{30}$$

in which $m$ is the number of events, and $\hat{t}_l$ and $t_l$ are the computed and measured values of time lag.

The accuracy of a method is also an important criterion. To assess the accuracy of the proposed equations the mean error and the standard deviation of errors were computed. Also, the standard error was used in the comparison which is defined as

$$SE = \left( \frac{1}{m} \sum_{i=1}^{m} \left( \frac{\hat{t}_l - t_l}{t_l} \right)^2 \right)^{1/2} \tag{31}$$

The values of the time lag computed from the proposed equations are compared with the observed time lag in Carnation Creek watershed (Table 2) and in Jamieson Creek watershed (Table 3). Here, it should be mentioned that in Jamieson Creek watershed, the stream channel has a very high gradient with an average slope equal to 0.16. Eq. (20) cannot be used for the estimation of the Manning's roughness coefficient, $n$, since the stream slope takes values out of the validity range of Eq. (20). The stream channel of Jamieson Creek is characterized by large organic debris and boulders, occasional organic debris jams, and small flow depth. For the application of the proposed equation in Jamieson Creek watershed, the roughness coefficient, $n$, is assumed to be constant equal to 0.1 throughout the watershed. This value is the upper limit for mountain streams with boulders and organic debris (Shen and Julien 1993).

Table 2 – Goodness-of-fit statistics for the test of time lag formulae – Carnation Creek watershed

| Method | Mean Time Lag (h) | Mean Error (h) | Standard Deviation of Errors (h) | Standardized Bias ($B_S$) | Standard Error |
|---|---|---|---|---|---|
| Observed | 4.64 | - | - | - | - |
| Proposed equation for non-linear stream slope | 4.89 | 0.25 | 1.48 | 0.19 | 0.47 |
| Proposed equation for linear stream slope | 5.03 | 0.40 | 1.48 | 0.22 | 0.50 |
| Proposed equation for constant stream slope | 4.34 | -0.30 | 1.48 | 0.05 | 0.39 |

Table 3 – Goodness-of-fit statistics for the test of lag time formulae – Jamieson Creek watershed

| Method | Mean Time Lag (h) | Mean Error (h) | Standard Deviation of Errors (h) | Standardized Bias ($B_S$) | Standard Error |
|---|---|---|---|---|---|
| Observed | 1.99 | - | - | - | - |
| Proposed equation for non-linear stream slope | 2.46 | 0.47 | 0.32 | 0.27 | 0.32 |
| Proposed equation for linear stream slope | 2.47 | 0.48 | 0.32 | 0.28 | 0.33 |
| Proposed equation for constant stream slope | 2.11 | 0.12 | 0.35 | 0.09 | 0.17 |

The results indicate that the three expressions of the proposed Eq. (18) compute the time lag of the two study watersheds with accuracy and precision. The results in Tables 2 and 3 show that Eq. (18) assuming constant stream slope and Manning's $n$ gives the best results even though it is the simplest one. This approximation has the smallest values of standardized bias and these values of standardized bias are close to zero which indicates that this proposed method does not severely over-estimate or under-estimate the observed time lag (Tables 2 and 3). This equation approximates the mean observed time lag with a mean error of -0.30 hours for the Carnation Creek watershed and 0.12 hours for the Jamieson Creek watershed. Also, the other statistics indicate that the proposed equation assuming constant mean stream slope and Manning's $n$ better estimates the observed time lags of the two watersheds.

From the other two expressions of the proposed method, the one which assumes non-linear stream slope gives the second best results even though it best describes the channel slope variation. The standardized bias is 0.19 for the Carnation Creek watershed (Table 2) and 0.27 for the Jamieson Creek watershed (Table 3). In addition to that, the method approximates the observed mean time lag within 0.25 hours for the Carnation Creek watershed and 0.47 hours for the Jamieson Creek watershed. In general, the statistics support the conclusion that the proposed Eq. (18) approximates well the observed values of time lag.

## Conclusions

An equation has been derived to compute the time lag of forested mountainous watersheds in humid areas. In the development of this equation, the flow in a watershed is separated into two phases, the land phase which represents the generation of flow in the hillslopes and the stream channel phase. In the land phase the runoff is generated through subsurface pathways either soil matrix flow or flow through macropores or soil pipes. Using observations and findings reported in field experimental studies and assuming kinematic conditions an equation for the outflow from the base of a hillslope has been presented. This outflow is used as inflow to the stream. For the solution of the flow equations in the stream channel, kinematic conditions have been assumed.

Three approximations have been proposed and the above statements apply to all three cases. The three approximations of the derived Eq. (18) differ in the assumption for the variation of the main stem stream slope and the Manning's roughness coefficient. In the development of the first approximation, it has been assumed a non-linear variation of the two stream channel parameters with distance from the mouth of the watershed, according to a second order polynomial. In the derivation of the second approximation, it was assumed that the Manning's $n$ and the stream slope linearly vary with distance from the outlet of the watershed. Finally, the third proposed approximation is derived with the assumption of constant stream slope and Manning's $n$ throughout of the watershed. All three approximations for the estimation of time lag have been derived by solving the resulting partial differentials.

Application of the method to two experimental study watersheds in coastal British Columbia indicated that, in general, all three proposed approximations are reliable and approximate well the observed time lag of the watersheds. However, the equation assuming constant values of Manning's $n$ and stream slope throughout the watershed gave the best results even though it is the simplest one. This finding is useful because this equation requires only an estimate of the mean slope of the main stem of the stream instead of a detailed stream slope variation which is required by the other two proposed approximations.

The main advantage of the proposed equation is that it has been derived by using observations from field experiments and by applying simplified mathematics whereas the empirical equations are, usually, the product of regression analysis between the observed time lag and the topographic and rainfall parameters. Application of the proposed method minimizes the errors in time lag computation and so it results in better, more reliable prediction of the design flood. Also, the proposed equation have been derived for forested mountainous watersheds and because its analytical derivation, it may be applicable to broad geographical and climatic regions. However, the validity of the proposed equation should be thoroughly tested with additional data when these become available.

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