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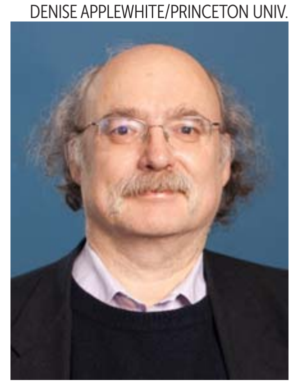

Haldane

As any potter can attest, a ball of clay can be stretched and reshaped into a bowl. But without some cutting, piercing, or attaching different parts together, that ball will never turn into a ring. In the language of topology, which elucidates the properties of an object that remain unchanged even as the object is continuously deformed, the ball and the bowl are topologically equivalent, whereas the ring belongs to a different topological class. As the joke goes, a topologist is someone who can't distinguish between a doughnut and a coffee cup.

In the early 1970s J. Michael Kosterlitz and David Thouless deduced that in two-dimensional systems, topological defects called vortices could drive a phase transition. But the transition was not the ordinary kind accompanied by a change in the system's symmetry. Instead, what changed was the system's topology.[1]

A decade later, F. Duncan Haldane took the ideas of Kosterlitz and Thouless and applied them to 1D spin chains and opened up a new and rich line of research.[2] Around the same time, Thouless once again invoked topology to explain the astonishingly precise quantization of Hall conductivity in 2D electron systems.[3]

The three theorists turned the abstract mathematics of topology into a central tool in the study of low-dimensional systems, and in so doing, they fundamentally altered the landscape of condensed-matter physics. The 2016 Nobel Prize in Physics recognizes their pioneering research. Half of the prize was awarded to Thouless, with the other half split between Haldane and Kosterlitz.

A new kind of transition

Kosterlitz was a particle physicist when he arrived as a postdoc at the University of Birmingham in 1970. After completing several calculations in what might now be called proto-string theory, he found that he had been scooped by competitors and decided a change was in order. By 1971 Kosterlitz was working with Thouless, who was a professor at Birmingham.

During a visit to Bell Labs in 1969, Thouless had learned from Philip Anderson about a puzzle involving phase transitions in certain 1D magnets. Oddly, short-range interactions did not produce a phase transition but long-range ones did, and no one knew what accounted for the difference. Thouless identified the contest between the energy and entropy of magnetic defects as a key difference between the two cases. When Kosterlitz came to him in search of a new research topic, Thouless had been thinking about how similar arguments could be applied to superfluid vortices in liquid helium, particularly in 2D.

In those days, most condensed-matter physicists regarded phase transitions as implying the appearance of long-range order and a change in symmetry. A crystal structure can be rotated only by certain angles and still retain the same appearance, whereas a fluid can be rotated at any angle and look the same. All the spins in an ordered magnet align along specific directions, whereas spins in a disordered magnet point in random directions. A phase transition from a disordered state to an ordered state was presumed to occur through the spontaneous breaking of some symmetry, and the process could be characterized by the evolution of the magnetization or some other "order parameter" as a function of temperature, pressure, or other thermodynamic variable.

In the 1930s, however, Rudolf Peierls argued convincingly that in 2D materials, the thermal motions of atoms would prevent long-range order from being established. N. David Mermin and Herbert Wagner used similar arguments in 1966 to show that an isotropic 2D Heisenberg magnet, in which the magnetic moments can point in any direction, also cannot order. A year later, Franz Wegner showed that the same was true in another 2D magnet, the *xy* model, which constrains the magnetic moments to lie in the 2D plane. Moreover, theoretical work by Pierre Hohenberg in 1967 showed that 2D superfluidity and superconductivity should not exist.[4]

But as the theoretical picture seemed to clarify and converge, puzzling experimental evidence of a superfluid transition in thin liquid He films was beginning to appear. Numerical and other theoretical studies, including Wegner's own, were also finding hints that some sort of transition might occur in 2D atomic or magnetic systems.

The question was, if 2D systems cannot order and no symmetry breaks, how can they undergo phase transitions? "Kosterlitz and Thouless brought the essential solution," comments Wegner. In a radical departure from conventional wisdom, Kosterlitz and Thouless proposed a new type of long-range order, which they referred to as topological. Furthermore, they speculated that the order could exist in 2D solids, neutral superfluids, and the *xy* model.[1]

What Kosterlitz and Thouless showed was that in 2D systems, topological excitations called vortices could exist in addition to more traditional excitations such as phonons in crystals, magnons in *xy* magnets, and surface waves in superfluids. To grasp the topological nature of vortices, first imagine a 2D ferromagnet as an arrangement of arrows all pointing in the same direction. If one traces a counterclockwise circular course around a group of arrows, the arrow directions are trivially the same and the so-called topological charge is 0.

A vortex, illustrated in figure 1a, is a pattern of arrows such that during a full counterclockwise circuit around the vortex core, the arrows successively rotate, yielding a total change of 2π, or one revolution. In that case, the topological charge is 1. An antivortex, shown in figure 1b, also executes a full rotation but through -2π for a topological charge of -1.

If one continuously deforms a vortex—for example, by rotating all the arrows by the same amount—the topological charge remains 1. One could never go from a vortex to an antivortex in that way; a transformation from one to the other must be discontinuous. However, when a vortex and antivortex pair up, their topological charges cancel to zero. Vortex–antivortex pairs can therefore form continuously out of the ordered state, which also has a toplogical charge of zero.

Kosterlitz and Thouless calculated that the energy cost of making a vortex or antivortex and the entropy of the vortex or antivortex both depend logarithmically on the size of the system. The free energy of the system is $F = E - TS$, where E is energy, T is temperature, and S is entropy. At low temperatures, the energy term dominates, and free vortices and antivortices do not exist. However, the energy of a vortex–antivortex pair is a function of the separation between the two, and tightly bound pairs can appear even at low temperature. As the temperature increases, more pairs are excited, and the vortex–antivortex distances grow larger. At some critical temperature, the entropy term overcomes the energy term, the vortices and antivortices become unbound, and free vortices and antivortices roam the system. The system undergoes a topological phase transition since dubbed the Kosterlitz–Thouless (KT) transition. Sometimes the transition is given as BKT, where B is for the late Vadim Berezinskii, who in 1970 published similar ideas in a Russian journal.

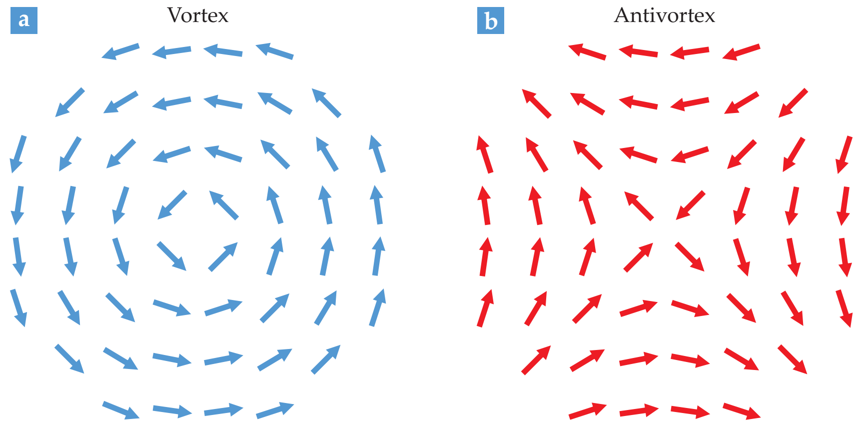

FIGURE 1. VORTEX CONFIGURATIONS. (a) On a counterclockwise circular path around a vortex in a magnet, each successive spin is rotated so that a full trip gives a total rotation of 2π. **(b)** On a similar trip around an antivortex, the total rotation is -2π.

From the beginning, Kosterlitz and Thouless understood that their ideas applied to 2D solids, magnets, and liquid He. However, their original 1972 paper focused on the 2D solid–fluid transition, for which the vortices are point defects called dislocations. Thus the KT transition describes the melting of a 2D crystal. The statistical mechanics of 2D crystals continues to be a lively topic of research, particularly in colloidal systems. William Irvine of the University of Chicago notes that in those systems the KT transition "is the only good theory of melting that we have."

Less than a year later, Kosterlitz and Thouless followed up with a longer paper that detailed how their ideas worked in the *xy* model and superfluid He. In 1977 Kosterlitz and David Nelson (the two met while Kosterlitz was a postdoc at Cornell University) predicted that the KT transition in superfluid He would manifest as a jump in superfluid density. The prediction was quickly confirmed by experiment. By the end of the decade, the KT transition was found to apply to superconducting thin films as well.

From classical 2D to quantum 1D

In 1931 Hans Bethe wrote down an exact solution for the 1D spin-½ chain. (See the article by Murray Batchelor, PHYSICS TODAY, January 2007, page 36.) Known as the Bethe ansatz, the solution gave a gapless spin excitation spectrum: The excitation energies continuously go to zero at low wavenumbers. Because the ansatz led to an answer that superficially resembled that from semiclassical spin-wave theory, physicists tended to skip the difficult mathematics and go straight to the solution. And although there was no proof, the conventional wisdom in the early 1980s was that the situation should not be different for spins greater than ½.

In 1981 Duncan Haldane was working at the Institut Laue–Langevin in Grenoble, France, on the Luttinger liquid model, a perturbation treatment of 1D electron systems. He realized that he could apply the classical statistical-mechanics ideas of Kosterlitz and Thouless to the quantum mechanical 1D spin chain if he turned one of the spatial dimensions into time. Then, the vortices of Kosterlitz and Thouless would become tunneling events between different topological states.

Haldane found that from the topological point of view, the tunneling events wound the spin field around the chain axis by $\pm 2\pi$ in spacetime, much as the vortex does in 2D space. In the path-integral formulation of quantum mechanics, the positive and negative winding exactly cancel for spin-½; that, Haldane showed, leads to gapless excitations.

Once he had worked out the spin-½ case, Haldane considered a spin-1 chain, and he discovered that the cancellation did not occur. "It became immediately obvious that there was a gap," explains Haldane. But, he says, "I presumably didn't

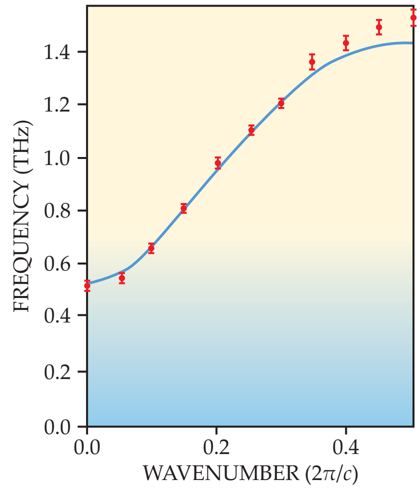

FIGURE 2. THE HALDANE GAP. Duncan Haldane conjectured that in a one-dimensional spin-1 chain, the excitation spectrum would have an energy gap. Three years later neutron scattering results for the quasi-1D compound $CsNiCl_3$, which has magnetic chains with lattice spacing *c*, confirmed the prediction. Shown here is a plot of the excitation spectrum for the chain. The energy gap is evidenced by the fact that the excitation frequency never dips to zero. The blue line is a fit to spin-wave theory. (Adapted from ref. 5.)

explain that well enough in my papers." His original 1981 manuscript was rejected.

By the time he came up with more convincing arguments to support his conjecture, Haldane says, much of the Kosterlitz–Thouless origins of his thinking had disappeared. When his two seminal papers came out in 1983,[2] it didn't take long for experimentalists to take up a search for the Haldane gap. In 1986 William Buyers and his colleagues at Chalk River Laboratories measured[5] a clear gap, shown in figure 2, in the spin excitation spectrum of the quasi-1D system $CsNiCl_3$. "There's nothing like experimental confirmation to quiet the critics," says Haldane.

From real space to momentum space

When a magnetic field is applied perpendicular to a current *I* in a metal plate, the Lorentz force deflects the charges perpendicular to both the field and the current. The accumulation of deflected charges produces a potential difference *V*. The Hall effect is named for Edwin Hall, who discovered it in 1879, and the ratio I/V is called the Hall conductance.

In 1980 Klaus von Klitzing and his colleagues discovered that the Hall conductance of a 2D electron gas is quantized in integer multiples of e^2/h (e is the electron charge and h is Planck's constant).[6] The quantization of the Hall conductance, for which von Klitzing received the 1985 Nobel Prize in Physics, can be observed to better than 1 ppb, regardless of sample-dependent properties such as the number of defects. (See PHYSICS TODAY, December 1985, page 17.) So exquisitely precise is the measurement that $R_K = h/e^2$, the von Klitzing constant, now defines the SI unit of resistance. Soon it will also be used to define the SI unit of mass. (See the article by David Newell, PHYSICS TODAY, July 2014, page 35.)

In 1982 Thouless, by then at the University of Washington, and three of his postdocs—Mahito Kohmoto, Peter Nightingale, and Marcel den Nijs—explained the quantization of the Hall conductance using topological considerations. (See the article by Joseph Avron, Daniel Osadchy, and Ruedi Seiler, PHYSICS TODAY, August 2003, page 38.)

"Here the topology is in a little more abstract setting because one's not talking about some topological configuration in real space the way the vortex is," the University of Pennsylvania's Charles Kane explains. "The topology in the quantum Hall effect is really a topology in a quantum state."

What Thouless and his colleagues figured out, elaborates Allan MacDonald of the University of Texas at Austin, was that the integer in the QHE "is actually a topological index of the band structure." In essence, the quantized jumps in Hall conductance are transitions between different topological states.

It turns out that the QHE was just the tip of the iceberg in terms of topological effects in momentum space. For instance, topological insulators—materials that are insulating in the bulk but host conducting states at the surface—have emerged as a major topic in condensed-matter physics.[7] (See the article by Xiao-Liang Qi and Shou-Cheng Zhang, PHYSICS TODAY, January 2010, page 33.)

Kane explains the topological surface states this way: The defining characteristic of insulators and conductors is their electronic band structures. Insulators have a gap in their band structure that separates occupied valence states from unoccupied conduction ones. Metals conduct precisely because they don't have such a gap. If a topological insulator is joined to an ordinary insulator, one can try to smoothly interpolate between the band structures of the two at their interface. “But if you did that, somewhere along the way, the [energy] gap has to go to zero, because if it didn’t go to zero, that would mean that the two are topologically the same.” The consequence is that the boundary state is a gapless conducting one. (See the article by Nick Read, PHYSICS TODAY, July 2012, page 38.)

Such states, because they are topological in nature, are robust to perturbations, much as the quantum Hall states are. “You can mess with it, you can tickle it, you can change things,” explains Kane, “but there are certain things that can’t go away.” Thus topological insulators have been suggested as a host for robust quantum computation.

“I think the Nobel Committee wanted to point out that topology will play an increasing role in physics,” comments von Klitzing, “and they wanted to go back to the roots of topological ideas.” Indeed, Wegner and others add that this year’s Nobel Prize will likely not be the last to recognize topological physics.

The laureates

David Thouless was born in 1934 in Bearsden, Scotland. After receiving his undergraduate degree from the University of Cambridge, he earned his PhD from Cornell University in 1958. Following a postdoc at the University of California (UC), Berkeley, he went to the University of Birmingham in 1965. He did a short stint at Yale University before joining the University of Washington in 1980, where he is now an emeritus professor.

Michael Kosterlitz was born in Aberdeen, Scotland, in 1943. He did his bachelor’s and master’s work at the University of Cambridge before moving to the University of Oxford, where he received his DPhil in 1969. He did a one-year postdoc at the Institute of Theoretical Physics in Turin, Italy, followed by another postdoc at the University of Birmingham. After one more postdoc at Cornell University, he returned to the University of Birmingham as a lecturer and later became a reader. In 1982 he moved to Brown University, where he is now the Harrison E. Farnsworth Professor of Physics.

Duncan Haldane was born in 1951 in London. He received his undergraduate education at the University of Cambridge and completed his PhD there in 1978. He worked at the Institut Laue–Langevin in Grenoble, France, before moving to the University of Southern California, Bell Labs, and then UC San Diego. In 1990 he settled at Princeton University, where he is currently the Eugene Higgins Professor of Physics.

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