The Computer Journal

The usual form of Newton's method for finding the correction to be made at any stage is

\[ x_{i+1} = \frac{1}{2} \left( x_i + \frac{a}{x_i} \right) \]

which can be rewritten as

\[ x_{i+1} = x_i + \frac{a - x_i^2}{2x_i} \]

to show the correction. Mr. Honey's (or Mr. Grabau's) technique is therefore seen to be equivalent to one more step of the Newton process after the single-length result has been obtained.

However, it is necessary to take care when this method is being used in fixed-point arithmetic, as overflow could result if the 'wrong' single-length square root is taken. It is not enough to take the unrounded (rounded down) value, because this leads to a value \( x_i \) satisfying

\[ 0 < a - x_i^2 < 2x_i \]

and this can obviously give overflow. No such difficulty can arise if we take the rounded value, because this satisfies

\[ -x < a - x_i^2 < x \]

with a correction of at most \( \frac{1}{2} \) unit, although it may be of either sign. In Mr. Honey's example, therefore, he should have used 14 as his initial guess at \( V_{192} \), which would have led to 13-86 as the better approximation, instead of 13-88. Since \( (13-86)^2 = 192-0996 \) this gives an error which is about \( \frac{1}{2} \) of that quoted. Indeed, it is not difficult to show that the maximum relative error using the rounded single-length approximation will be about \( \frac{1}{2} \) of the error that could arise from using the unrounded version. Choosing the rounded approximation is thus noticeably more accurate for the same amount of work.

Yours faithfully,

P. A. SAMET

Computer Centre
University College London
19 Gordon Street
London WC1
14 December 1971

Mr. Honey replies:

I am obliged to Professor P. A. Samet for his letter commenting on 'Grabau's Method' for obtaining a double precision result using single precision techniques. I think that I may have misled the readers by the lack of emphasis on the single precision. Professor Samet is quite correct in his observation that Newton's method is involved, although I had not appreciated this fact at the time. My main concern was that single precision techniques are used throughout the process and is something often overlooked by software designers with non-mathematical background.

I am also obliged for Professor Samet's further comment re 'overflow' (again sometimes overlooked), and his development of my worked example in decimal in which a rounded value is taken in preference to an unrounded value—a technique I shall remember in future.

I am sure that several readers will have gained benefit from our minor correspondence—which is its basic purpose. Thank you, Professor Samet, once again.