Proposal for Experiments for Determination of Beta-Decay Interaction and Theory of Triple Cascade Transition

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Four new kinds of experiments on angular correlation in $\beta$-ray and $\gamma$-ray angular correlations, $\alpha$, $\beta$, $\gamma$ rays and also of the other kind of particles.

§ 1. Introduction

The type of the interaction for $\beta$-decay seems to have been determined by the experiments on the $e^-\nu$ angular correlations of He$^6$ and Ne$^{19}$. It is shown to be a linear combination of Scalar and Tensor interactions, (abbreviated as ST hereafter). To ascertain this conclusion by another method, we propose four new kinds of experiments. These experiments are the angular correlations, I, II, III and IV, of a triple cascade $\beta-\gamma_1-\gamma_2$ transition in decay scheme $3_-, 5_-(\beta 1st)$ or $6_+(\beta 2nd)$ or $4_+(\gamma_1 2)$ or $2_+(\gamma_2 2) 0_+$, as explained in § 2. If, besides these experiments, the measurements of the usual $\beta-\gamma$ angular correlation and of the $\beta$-spectrum are performed, the linear combination of $\beta$-decay interactions will be determined definitely. Some other decay schemes are also useful for this purpose.

The rapid development of instruments and methods in the field of $\beta$- and $\gamma$-ray spectroscopy of late years has facilitated to perform the measurements of $\gamma-\gamma$ angular correlation in many nuclei, and anisotropic $\beta-\gamma$ directional correlations have also been found in the elements C$^{14}$, K$^{40}$, As$^{76}$, Rb$^{86}$, Sb$^{124}$, Sn$^{122}$, Tm$^{170}$, Re$^{188}$. In all these cases the differential correlations have been measured. The measurement of the angular correlation in $\beta-\gamma_1-\gamma_2$ cascade transition is, however, much more difficult than that in the above double cascade transitions, for the counting rate is strongly diminished in $\beta-\gamma_1-\gamma_2$ coincidence. In order to remedy this defect as much as possible, T. Hayashi$^{13}$ in-

*) Recently Konopinski$^1$ has reviewed compactly the studies on the type of $\beta$-decay interactions.

***) In the present paper the angular correlation means only the directional correlation.

**** We denote the decay scheme as $j_0$ (forbiddenness of $\beta$-decay) $j_1$ (rank of $\gamma_1$-interaction) $j_2$ (rank of $\gamma_2$-interaction) $j_3$, where $j_0$, $j_1$, $j_2$ and $j_3$ are the spins of the initial, the second, the third and the final nuclear states in a triple cascade transition, respectively. The sign (+ or −) of each spin indicates the parity of the relevant nuclear states, (see Figs. 1 and 3).
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tended to measure the integrated $\beta-\gamma_1-\gamma_2$ angular correlation of Cs$^{134}$ using the $\beta$-counter with an extraordinary wide window. This window subtended a solid angle of about $4\pi/3$ steradian with respect to the centre of the radiation source. Moreover, this $\beta$-counter did not discriminate the energy of $\beta$-particle. The arrangement of three counters was the same one as III in § 2. He obtained the angular correlation of $\beta-\gamma_1-\gamma_2$, which was different from his experimental angular correlation of $\gamma_1-\gamma_2$ without observing $\beta$-ray. The counting rate of $\beta-\gamma_1-\gamma_2$ coincidences was about 10$^5$/minute. The number of coincidences at each angle, $\theta$ equal to 90°, 135°, 157.5° or 180°, was of the order of 1 $\times$ 10^4. Unfortunately, the data of his experiment are not sufficient* for our investigation of $\beta$-decay. However, it seems to us that such an experiment shows the possibility to perform our proposed experiments.

Among the four experiments, I, II, III and IV, the last one may be performed more easily than the other three, because in IV it is unnecessary to observe the intermediate $\gamma_1$-ray. This helps to increase the coincidence counting rate by about factor 10$^2$ in comparison with that of I, II, III, if we use usual $\beta$- and $\gamma$-counters.

Similar experiments have been done in the cases of triple cascade gamma transitions on Pb$^{204}$ by V. E. Krohn and S. Raboy$^{15}$ and on Fe$^{58}$ and Cd$^{110}$ by M. Sakai$^{16}$.

In § 2 we discuss a possibility of unambiguous determination of ST (or VT) as the $\beta$-decay interaction. In order to find the angular correlation functions in the new experiments, we extend the theory of the triple cascade gamma transition, which was developed by Biedenharn, Arfken and Rose$^{17}$, to the case of $\beta-\gamma_1-\gamma_2$ in § 3, and the results are given. The angular correlation functions are useful not only for the above purpose, but also for providing us some information concerning the model of nuclei, if the parameters $x$, $y$, $z$ in these functions can be obtained by the analysis of experimental data. Further we calculate various angular correlation functions for successive triple cascade transitions of $\alpha$, $\beta$, $\gamma$ rays and the other kind of particles in § 4. Suitable nuclei for our experiments, the corrections for parameters $b_{ST}^{ijl}$ of $\beta$-ray, etc., are given in § 5.

§ 2. A possibility of unambiguous determination of ST (or VT) as the $\beta$-decay interaction

A principal difficulty we encounter in the studies of $\beta$-decay is the appearance of several nuclear matrix elements in the transition probability. From the lack of our knowledge of nuclear wave functions, we cannot estimate the exact values of the nuclear matrix elements contained in the probability of the forbidden transitions when we take an arbitrary linear combination of five relativistic invariants as the $\beta$-decay interaction. In actual calculations, therefore, we are forced to treat the ratios among nuclear matrix elements more or less as phenomenological parameters. For ST, there are three such parameters $x$, $y$ and $z$ in the case of the first forbidden $\beta$-decay with $|\Delta J| = 1$, (see reference 18).

*) Hayashi's attempt was to distinguish the excited state (or states), which is (or are) formed as the result of $\beta$-decay of 655 kev (and 683 kev) of Cs$^{134}$, with spin 4 (and/or 3) in Ba$^{131}$.$^{14}$
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\[ G_s \mathcal{M}(\beta r)/G_s \mathcal{M}(\beta \sigma \times r) = -ix, \quad \mathcal{M}(\beta \alpha)/\mathcal{M}(\beta \sigma \times r) = (\alpha Z/2\mu) \gamma, \]
\[ \mathcal{M}(B_{i2})/\mathcal{M}(\beta \sigma \times r) = iz, \]

where \( x, \gamma \) and \( z \) are all real. The notation is as follows: \( G_s \) and \( G_T \) are the coupling constants of Scalar and Tensor interactions, respectively. \( \mathcal{M}(\beta r), \mathcal{M}(\beta \alpha), \mathcal{M}(\beta \sigma \times r) \) and \( \mathcal{M}(B_{i2}) \) are the first forbidden reduced nuclear matrix elements. \( (\alpha Z/\mu) \) is the Coulomb potential of the daughter nucleus. As the number of independent experimental data is less than that of the parameters, the theoretical analysis does not lead to a definite result without further assumptions. This situation prevails in the cases of \( \beta \)-decays for which the final states are the ground or the first excited states of the daughter nuclei.

Fortunately, the number of independent experiments exceeds that of the above parameters in the cases of \( \beta \)-decays for which the final states are the second excited or more highly excited states of the daughter nuclei. Consider, for example, the successive nuclear transitions, \( 5_-(\beta 1^+) 4_+(\gamma_1 2^+) 2_+(\gamma_2 2^+) 0_+ \) (Fig. 1). In this decay scheme, one can observe the \( \beta \)-ray spectrum, the \( \beta \)-value, the \( \beta - \gamma_1 \) angular correlation as usual, and furthermore the triple angular correlation of the \( \beta \)-ray and the two gamma rays. Theoretically, the last mentioned angular correlation depends on three angular variables (see (4)). In other words, the angular correlation function of two gamma rays changes its value, according to the arrangements of \( \beta \)- and \( \gamma \)-counters. We shall choose the following three cases as typical arrangements (Fig. 2).

I. Three particles are emitted in a plane, \( \beta \) and one of \( \gamma \)'s being emitted in perpendicular directions.

II. Three particles are emitted in a plane, \( \beta \) and one of \( \gamma \)'s being emitted in antiparallel directions.

III. \( \beta \) is emitted perpendicularly to the plane determined by the directions of two \( \gamma \)'s.

In all three cases, it is unnecessary to distinguish the names of two \( \gamma \)'s in experiments. We give the angular correlation functions, \( W(\theta) \), which have different forms for respective cases as shown in § 3. Together with these experiments, one can observe \( \beta - \gamma_2 \) angular correlation without observing \( \gamma_1 \).

IV. \( \beta - \gamma_2 \) angular correlation without observing \( \gamma_1 \)
If these new experiments, I, II, III, IV, and the measurements of a usual $\beta-\gamma_1$ angular correlation and of a $\beta$-spectrum are performed, we can obtain the values of the parameters $x, y, z$ using some of the data and check these values by other data. If a certain set of $x, y, z$ can fit all the data consistently, we have a definite proof to distinguish between ST and VT as the interaction for $\beta$-decay.

For our purpose the following decay schemes are also useful: $3_-, 4_-(\beta 1st) \text{ or } 6_+ (\beta 2nd) 4_+ (\gamma_1 \ 2) 2_+ (\gamma_2 \ 2) 0_+,$ and $2_+, 3_+, 4_+(\beta 1st) \text{ or } 5_-(\beta 2nd) 3_-(\gamma_1 \ 2) 1_-(\gamma_2 \ 1) 0_+.$

§ 3. Theory of triple cascade transition

In order to deduce the various angular correlation functions of a $\beta-\gamma_1-\gamma_2$ transition (Fig. 3), we extend the theory of a triple cascade gamma transition developed by Biedenharn, Arfken and Rose (referred to as BAR, hereafter). This theory is restricted to gamma transitions with a pure multipolarity*. However, this restriction can be removed easily. Thus we consider the successive transitions $j_0 \rightarrow j_1 \rightarrow j_2$ emitting three gamma rays of multiplicities $2^{l_0}, 2^{l_1}$ and $2^{l_2}$, respectively (Fig. 4). The relative probability for this cascade transition is given by (3) of BAR.

$$P(k_0, k_1, k_2) = \sum_{m_s m_3} (j_0 L_0 m_0 m_1 m_2) D_{m_0}^{l_0} (a_0 \beta_0 0) \times (j_1 L_1 m_1 m_2 m_3) D_{m_1}^{l_1} (a_1 \beta_1 0) \times (j_2 L_2 m_2 m_3) D_{m_2}^{l_2} (a_2 \beta_2 0)^2. \text{ BAR(3) **}
$$

Notation is: $p_i$ and $k_i$ are the polarization and the direction of the $i$-th gamma ray, $i=0, 1, 2$. $a_i$, $\beta_i$ and $\gamma_i$ are the Euler angles of $k_i$. $W_{(abcd: ef)}$ is the Racah coefficient$^{10}$. $(j_1 j_2 m_1 m_2 | j m)$ is the Clebsch-Gordan coefficient. It is easily justified that the Euler angles $\gamma_i$ may be set equal to zero. As usual, we define the angular distribution function of the 0-th gamma ray with multipolarity $2^{l_0}$ as follows,

$$P_{\gamma_0}^{m_0} (\beta_0) \delta_{m_1 m_2 m_3} = (1/4\pi) \sum_{j_0} D_{m_0}^{l_0*} (a_0 \beta_0 0) D_{m_1}^{l_1} (a_0 \beta_0 0) d\alpha_0.$$

More generally, $P_{j_l l'} (\theta)$ for a certain particle is defined by$^{50}$

*) They applied their theory also to the double cascade gamma emission following the capture of particles with nonzero orbital momentum.

**) Of course, we assume that the nuclei are not disturbed by extranuclear fields.
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$$F^{m}_{LL'}(\theta) = \sum_{T_L(X_i)T_{L'}(X_j)} \delta \left[ a[T(X_i)]a[T(X_j)] \mathcal{M}^*(X_i) \mathcal{M}(X_j) \right]$$
$$\times \mathcal{Y}^*_{LM}(A_i) \mathcal{Y}_{LM}(A_j) + c.c.,$$

with $\epsilon = \begin{cases} 1/2 & \text{for square terms}, \\ 1 & \text{for cross terms}, \end{cases}$

and $L \leq L'.$

The notation is as follows: $X_i$'s are the vector operators for the nucleus, $A_i$'s are the argument vectors for the emitted particle, $\mathcal{M}(X_i)$'s are the numerical coefficients in the interaction Hamiltonian, $\mathcal{M}^*(X_i)$'s are the reduced nuclear matrix elements and $\mathcal{Y}_{LM}$'s are the polarized solid harmonics. $S$ indicates the sums over the directional arguments (spin etc.) except the colatitude angle $\theta$. $\sum_2$'s denote the sums over the various irreducible tensors retaining the rank $L$ constant. The last summations are unnecessary when we consider only the coexistence of the magnetic (electric) $2^L$ pole and the electric (magnetic) $2^{L'}$ pole radiations. In the case of the forbidden $\beta$-decay, however, these summations become important taking into account the various irreducible tensors with the same rank $L$, e.g., $\mathcal{M}(3r)$, $\mathcal{M}(3a)$ and $\mathcal{M}(3\sigma \times r)$ terms, simultaneously. Then $(j_a L m_a M_{j_a m_a}) (j_b L' m_b M_{j_b m_b}) F^{m}_{LL'}(\theta)$ can be understood as the relative and partial transition probability from $j_a m_a$ to $j_b m_b$ caused by $L$, $M$ and $L'$, $M$ terms in the interaction Hamiltonian. Here we assume an isotropic distribution of magnetic substate $m_a^*$. (M 3) is reduced to

$$F^{m}_{LL'}(\theta) = \sum_n (-)^M (L' L - MM|2n 0) b^{(2n)}_{LL'} P_{2n}(\cos \theta).$$

with $L \leq L'$ and $L' - L \leq 2n \leq L' + L$.

Here $b^{(2n)}$'s are parameter depending upon the properties of the emitted particle. $b^{(2a)}$'s for the $\beta$-ray** and for the $\gamma$-ray up to quadrupole radiations are given in reference 18, and more generally in (34), with $b^{(2a)}$'s for the particles of zero or nonzero spin in (39).

Comparing (1) with $M(3)$ and substituting $M(4)$ in BAR (3), we obtain

$$P(k_0, k_1, k_2) = \sum_{L_0} \sum_{m_0} \sum_{m_1} \sum_{m_2} \sum_{j_0} \sum_{j_1} \sum_{j_2} (j_0 L_0 m_0 m_1 - m_0 |j_1, m_1) (j_0 L_0' m_0' m_1' - m_0 |j_1, m_1')$$
$$\times \left(-\right)^{m_2 - m_0} (L_0' L_0 - m_1 + m_0 m_1 - m_0 |2n 0) b^{(2n)}_{L_0 r} P_{2n}(\cos \beta_0)$$
$$\times \left(\sum_{j_0} (j_1 L_1 m_1 m_2 - m_1 |j_2, m_2) D^{(r)}_{m_2 m_1} (\alpha_1, \beta_1 0) \right)$$
$$\times \left(\sum_{j_2} (j_0 L_2 m_2 m_3 - m_2 |j_3, m_3) D^{(r)}_{m_3 m_2} (\alpha_2, \beta_2 0) \right)^2.$$  \hspace{1cm} (2)

Since $m_0$, $m_0'$, $p_1$, $p_2$, are not measured in experiments, the angular correlation function of the triple cascade transition is obtained by summing over these quantum numbers.

$$W(k_0, k_1, k_2) = \sum_{m_0 m_0'} P(k_0, k_1, k_2).$$  \hspace{1cm} (3)

*) For an anisotropic distribution of magnetic substates, e.g., in an aligned nucleus, the investigation was done by Rose, Steenberg, Tolhoek et al.\textsuperscript{20} and is being carried on by us\textsuperscript{21}.

**) $b^{(2a)}$'s are equal to $b^{(2a)}$'s in reference 18.
After summing over \( m_0 \), we choose \( k_0 \) as \( z \)-axis, i.e., we put \( \beta_0 = 0 \). Then we have

\[
W(k_0, k_1, k_2) = W(\theta_1, \theta_2, \varphi)
\]

\[
= \sum_{L_0, L_1, L_2} \sum_{J_0} \sum_{m_0 m_1 m_2} (-)^{J_0} \sqrt{(2J_0 + 1)(2J + 1)} \times b^{(00)}_{J_0 \Delta}(J_0 2n m_1 0|J_1 m_1) W(J_1 J_2 L_0 L_1^J 2n j_0)
\times \left| \sum_{m_0} (j_1 L_1 m_1 m_2 - m_1 j_2 m_2) D^{(L_1)}_{m_2 \rightarrow -m_2}(0 \theta_1 0) \times (j_2 L_2 m_2 m_2 - m_2 j_3 m_3) D^{(L_3)}_{m_3 \rightarrow -m_3}(\varphi \theta_2 0) \right|^2.
\]

In (4), the 0-th particle is not restricted to the \( \gamma \)-ray. The \( m_2 \) dependent factors

\[
D^{(L_1)}_{m_2 \rightarrow -m_2}(0 \theta_1 0) D^{(L_3)}_{m_3 \rightarrow -m_3}(\varphi \theta_2 0)
\]

make the summing over the magnetic quantum numbers very laborious. However, it is quite easy to perform it in special cases indicated by Fig. 2. In the following calculations we do not specify the parity of each state, because the formulae do not depend on whether gamma rays are electric or magnetic. We specify the emitted particles or the arrangements of the counters in the parenthesis of \( W(\theta) \), as \( W(\theta : \alpha - \gamma_0) \) or \( W(\theta : 1) \).

In the decay scheme, \( j_0(\beta) 4(\gamma_1 2) 2(\gamma_2 2) 0 \), it is unnecessary to distinguish the names of two gamma rays in experiments.

1) \( j_0(\beta \text{ 1st}) 4(\gamma_1 2) 2(\gamma_2 2) 0 \).

\[
W(\theta : \text{I, II, III})
\]

\[
= (-)^{-n-\varphi} \left\{ W(4 \quad 4 \quad 0 \quad 0; 0 j_0) b^{(00)}_{01} + W(4 \quad 4 \quad 1 \quad 1; 0 j_0) b^{(00)}_{11} + W(4 \quad 4 \quad 2 \quad 2; 0 j_0) b^{(00)}_{22} \right\}
\times \left\{ 24 + 3 \cos^2 \theta + \cos^4 \theta \right\}
\]

\[
+ \left\{ W(4 \quad 4 \quad 1 \quad 1; 2 j_0) b^{(00)}_{11} + W(4 \quad 4 \quad 2 \quad 2; 2 j_0) b^{(00)}_{22} \right\} W_2(\theta : \text{I, II, III}).
\]

\[
W_2(\theta : \text{I}) = (5/2 \sqrt{77}) \left\{ -24 + 69 \cos^2 \theta - 13 \cos^4 \theta \right\}.
\]

\[
W_2(\theta : \text{II}) = (5/2 \sqrt{77}) \left\{ -24 - 57 \cos^2 \theta + 17 \cos^4 \theta \right\}.
\]

\[
W_2(\theta : \text{III}) = (10/ \sqrt{77}) \left\{ 12 - 3 \cos^2 \theta - \cos^4 \theta \right\}.
\]

Examples. \( W(\theta : \text{I}) \) for \( 5(\beta \text{ 1st}) 4(\gamma_1 2) 2(\gamma_2 2) 0 \).

From (5) and (6) we obtain \( W(\theta : \text{I}) \).

\[
W(\theta : \text{I}) = \left\{ - (1/ \sqrt{3}) b^{(00)}_{11} + (1/ \sqrt{3}) b^{(00)}_{22} \right\} \left\{ 24 + 3 \cos^2 \theta + \cos^4 \theta \right\}
\]

\[
- \left\{ (1/ 11 \sqrt{6}) b^{(00)}_{11} + (1/ \sqrt{6}) b^{(00)}_{22} \right\} \left\{ -24 + 69 \cos^2 \theta - 13 \cos^4 \theta \right\}.
\]

Substituting the \( b^{(00)}_{ij} \)'s of the first forbidden \( \beta \)-transition into (5'), it becomes in the case of ST as follows:

\[
W(\theta : \text{I})
\]
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\[ \begin{align*}
&= \left( aZ/2p \right)^2 (x-y+1)^2 \\
&\quad + \left( aZ/2p \right) \left\{ -K(x-1) + \left( p/\sqrt{W} \right) (x+1) \right\} (2/3) (x-y+1) \\
&\quad + \{ (1/12) (K^2+p^2) (4x^2-z^2+2) + (2Kp/9W) (x^2) \} \\
&\times \left\{ 24+3 \cos^2 \theta + \cos^4 \theta \right\} \\
&\quad - \left( 1/22 \right) \left\{ (aZ/2p) (p^2/3W) (4x+3 \sqrt{3} x-2) (x-y+1) \\
&\quad + \left\{ - \left( Kp/9W \right) (4x+3 \sqrt{3} x-2) (x-1) \\
&\quad + (p^2/24) (16x^2-5z^2+12 \sqrt{3} x-4) \right\} \right\} \\
&\times \left\{ -24+69 \cos^2 \theta -13 \cos^4 \theta \right\},
\end{align*} \]

(5')

where \( x, y \) and \( z \) are the parameters defined in § 1. \( (aZ/\rho) \) is the Coulomb potential of the daughter nucleus, (unit of \( \rho \) is a Compton wave length of the electron). \( W, p \) and \( K=W_0-W \) are the energy (unit \( mc^2 \)), momentum (unit \( me \)) of the electron and the energy of the neutrino respectively. Suffix zero of \( W \) indicates its maximum value. (5') is the final result to be compared with the experimental data. Similar procedures are applicable to other cases. Here we do not write the explicit dependence on \( x, y, z \) for them to spare the space.

2) \( j_0 (\beta 2nd) 4(\gamma_1 2) 2(\gamma_2 2) 0. \)

\[ W(\theta : I, II, III) \]

\[ \begin{align*}
&= \left( - \right)^n z^n \left\{ W(4 4 2 2 ; 0 j_0) b^{(4)} + W(4 4 3 3 ; 0 j_0) b^{(3)} \right\} \{24+3 \cos^2 \theta + \cos^4 \theta \} \\
&\quad + \{ W(4 4 2 2 ; 2 j_0) b^{(4)} + W(4 4 3 3 ; 2 j_0) b^{(3)} \} \\
&\times W_2(\theta : I, II, III) \\
&\quad + \{ W(4 4 2 2 ; 4 j_0) b^{(4)} + W(4 4 3 3 ; 4 j_0) b^{(3)} \} \\
&\times W_4(\theta : I, II, III) \}. \]

(9)

\[ W_2(\theta : I) = (6), \]

\[ W_2(\theta : II) = (7), \]

\[ W_2(\theta : III) = (8), \]

\[ W_4(\theta : I) = (3/2 \sqrt{2002}) \left\{ -421+1283 \cos^2 \theta -919 \cos^4 \theta \right\}. \]

(10)

\[ W_4(\theta : II) = (3/2 \sqrt{2002}) \left\{ -421+1038 \cos^2 \theta -769 \cos^4 \theta \right\}. \]

(11)

\[ W_4(\theta : III) = (9/2 \sqrt{2002}) \left\{ -67+36 \cos^2 \theta +12 \cos^4 \theta \right\}. \]

(12)

In the decay scheme, \( j_0 (\beta 2nd) 3(\gamma_1 2) 1(\gamma_2 1) 0 \), it is necessary to distinguish the names of two gamma rays in experiments, except in III (Fig. 5).

I. Three particles are emitted in a plane, \( \beta \) and

IA: \( \gamma_1 \) being emitted in perpendicular directions,

IB: \( \gamma_2 \) being emitted in perpendicular directions.
II. Three particles are emitted in a plane, $\beta$ and 
IIA: $\gamma_1$ being emitted in antiparallel directions, 
IIB: $\gamma_2$ being emitted in antiparallel directions. 
III. $\beta$ is emitted perpendicularly to the plane determined by the directions of two $\gamma$'s. 

Fig. 5. Arrangements of $\beta$- and $\gamma$-counters for measurement of $\beta-\gamma_1-\gamma_2$ angular correlation $j_0(\beta)$3($\gamma_1$ 2) 1($\gamma_2$ 1). It is necessary to distinguish the names of two $\gamma$'s in experiments, except in III.

3) $j_0(\beta$ 1st) 3($\gamma_1$ 2) 1($\gamma_2$ 1) 0.

$W(\theta$: I, II, III) 

$$\begin{aligned}
\ &= (-)^{n-\chi} \left[ \{W(3 3 0 0; 0_j) b_{\beta}^{(0)} + W(3 3 1 1; 0_j) b_{\beta}^{(0)} + W(3 3 2 2; 0_j) b_{\beta}^{(0)} \} \times \{29 - 3 \cos^2 \theta \} \\
&\quad + \{W(3 3 1 1; 2_j) b_{\beta}^{(0)} + W(3 3 0 2; 2_j) b_{\beta}^{(0)} + W(3 3 1 2; 2_j) b_{\beta}^{(0)} \\
&\quad + W(3 3 2 2; 2_j) b_{\beta}^{(0)} \} W_2(\theta$: I, II, III) \right].
\end{aligned}$$

(13)

$W_2(\theta$: IA) = $(6 \sqrt{3} / \sqrt{7}) \{5 - 4 \cos^2 \theta \}$. 

(14)

$W_2(\theta$: IB) = $(6 \sqrt{3} / \sqrt{7}) \{-5 + 5 \cos^2 \theta + \cos^2 \theta \}$. 

(15)

$W_2(\theta$: IIA) = $(6 \sqrt{3} / \sqrt{7}) \{-5 + 3 \cos^2 \theta \}$. 

(16)

$W_2(\theta$: IIB) = $(6 \sqrt{3} / \sqrt{7}) \{5 - 6 \cos^2 \theta - \cos^4 \theta \}$. 

(17)

$W_2(\theta$: III) = $(6 \sqrt{3} / \sqrt{7}) \cos^4 \theta$. 

(18)

4) $j_0(\beta$ 2nd) 3($\gamma_1$ 2) 1($\gamma_2$ 1) 0.

$W(\theta$: I, II, III) 

$$\begin{aligned}
\ &= (-)^{n-\chi} \left[ \{W(3 3 2 2; 0_j) b_{\beta}^{(0)} + W(3 3 3 3; 0_j) b_{\beta}^{(0)} \} \{29 - 3 \cos^2 \theta \} \\
&\quad + \{W(3 3 2 2; 2_j) b_{\beta}^{(0)} + W(3 3 2 3; 2_j) b_{\beta}^{(0)} + W(3 3 3 3; 2_j) b_{\beta}^{(0)} \} W_4(\theta$: I, II, III) \\
&\quad + \{W(3 3 2 2; 4_j) b_{\beta}^{(0)} + W(3 3 2 3; 4_j) b_{\beta}^{(0)} + W(3 3 3 3; 4_j) b_{\beta}^{(0)} \} W_4(\theta$: I, II, III) \right].
\end{aligned}$$

(19)

$W_2(\theta$: IA) = (14), 

$W_4(\theta$: IA) = $(3 / \sqrt{22}) \{-2 - 11 \cos^2 \theta \}$. 

(20)

$W_3(\theta$: IB) = (15), 

$W_4(\theta$: IB) = $(3 / \sqrt{22}) \{-12 + 89 \cos^2 \theta - 90 \cos^4 \theta \}$. 

(21)

$W_3(\theta$: IIA) = (16), 

$W_4(\theta$: IIA) = $(2 \sqrt{2} / \sqrt{11}) \{-9 - 17 \cos^2 \theta \}$. 

(22)
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\[ W_2(\theta : \text{IIB}) = (17), \]
\[ W_4(\theta : \text{IIB}) = (\sqrt{2}/\sqrt{11}) \{ -3 + 96 \cos^2 \theta - 145 \cos^4 \theta \}. \quad (23) \]
\[ W_2(\theta : \text{III}) = (18), \]
\[ W_4(\theta : \text{III}) = (3/\sqrt{22}) \{ -7 - 6 \cos^2 \theta \}. \quad (24) \]

To find \( W(\theta) \) for the experiment IV, we generalize (4) further to the case of gamma rays with mixed multipolarities \( 2^{\ell_1}, 2^{\ell_2}, \cdots \) \((i=1, 2)\).

\[ W(\theta, \theta_2, \varphi) = \sum_{L_0=L_0'} \sum_{L_1=L_1'} \sum_{L_2=L_2'} \sum_{M_0=M_0'} \sum_{M_1=M_1'} \sum_{M_2=M_2'} b_{I_0,i_0}^{(n)} \alpha_1 \alpha_2^* \cdot i^{L_0-L_1-L_2+6a-6d} \]
\[ \times P_2^{L_2+6d} \sqrt{(2L_2+1)(2L_2'+1)(2L_1+1)(2L_1'+1)} \]
\[ \times (-)^M (L_0', L_0-MM|2n0) (j_0 L_0 m_1-MM|j_1, m_1) (j_0 L_0' m_1-M M|j_1, m_1) \]
\[ \times (j_1 L_1 m_1 m_2-m_1 m_2\cdot j_2 m_2) (j_1 L_1' m_1 m_2'-m_1 m_2'\cdot j_2 m_2') \]
\[ \times (j_2 L_2 m_2-m_2\cdot j_3 m_3) (j_2 L_2' m_2'-m_2'\cdot j_3 m_3) \]
\[ \times D_{m_1-m_3}(0 \theta_1 0) D_{m_2-m_4}(0 \theta_1 0) D_{m_4-m_2}(\varphi \theta_2 0) D_{m_3-m_1}(\varphi \theta_2 0), \quad (25) \]

with \( \delta_e = 0 \) for magnetic radiation,
\( 1 \) for electric radiation.

\( \alpha_i \) is the reduced matrix element of \( \gamma \)-ray transition*.

5) \( \beta - \gamma_2 \) angular correlation, \( \gamma_1 \) unobserved (Experiments IV, Fig. 3).

We average (25) over the polarization and the angle variables of the first gamma ray and put \( \varphi=0, \theta_2=\theta \). Then, (25) becomes:

\[ W(\theta : \beta - \gamma_2) = \sum_{L_0=L_0'} \sum_{L_1=L_1'} \sum_{L_2=L_2'} \sum_{M_0=M_0'} \sum_{M_1=M_1'} \sum_{M_2=M_2'} b_{I_0,i_0}^{(n)} |\alpha_1|^2 \alpha_2^* \cdot i^{L_0-L_1-L_2+6a-6d} P_2^{L_2+6d} \sqrt{(2L_2+1)(2L_2'+1)} \]
\[ \times (-)^M (L_0', L_0-MM|2n0) (j_0 L_0 m_1-MM|j_1, m_1) (j_0 L_0' m_1-M M|j_1, m_1) \]
\[ \times (j_1 L_1 m_1 m_2-m_1 m_2\cdot j_2 m_2) (j_1 L_1' m_1 m_2'-m_1 m_2'\cdot j_2 m_2') \]
\[ \times (j_2 L_2 m_2-m_2\cdot j_3 m_3) (j_2 L_2' m_2'-m_2'\cdot j_3 m_3) \]
\[ \times D_{m_1-m_3}(0 0) D_{m_2-m_4}(0 0), \quad (26) \]

After summing over the magnetic quantum numbers and the polarization \( p_2 \), we have

\[ W(\theta : \beta - \gamma_2) = \sum_{L_0=L_0'} \sum_{L_1=L_1'} \sum_{L_2=L_2'} b_{I_0,i_0}^{(n)} |\alpha_1|^2 \alpha_2^* \cdot i^{L_0-L_1-L_2+6a-6d} \]
\[ \times (-)^{L_1+L_2+L_2'} (2L_1+1)(2L_2'+1)(2L_1'+1) \]
\[ \times (L_0', L_0-1-1|2n0) W(j_1, j_1 L_0 L_0' ; 2n j_0) W(j_1, j_2 j_2 ; 2n L_0) \]
\[ \times W(j_2, j_2 L_2 L_2' ; 2n j_3) P_{m_2}(\cos \theta), \quad (27) \]

*) \( \alpha_i \) is equal to, e.g., \( \alpha \) or \( \beta \) of \( M(\gamma) \).
where $\sum_{a}$ is omitted, because the parity condition, $L_x + L'_x + 3 \delta_x + \delta'_x = \text{even}$, is always taken into account. Now we take the reduced matrix element of $BR_{22}^{(3)}$, $(j||L_0||j_{+1})$, which is connected by the relation, $(-)^L_j(j||L_0||j_{+1}) = \alpha_i i^{L_x + \delta_x}$. These $(j||L_0||j_{+1})$'s are chosen as real numbers simultaneously $^{20}$ We also use the $F$ coefficient $^{20}$

$$
F_S(L' L' j_a j_b) = F_S(L' L j_a j_b) = (-)^{j_b - j_a - 1} \sqrt{(2j_b + 1)(2L + 1)(2L'_1 + 1)(L_1' - 1|N_0)W(j_b,j_b; L' L') N_{j_a}}.
$$

(27) becomes,

$$
W(\theta : \beta - \gamma) = \sum_0 \left\{ \sum_{L_0 L_0'} \left( - \right)^{L_0 - 2n \theta} b_{L_0 L_0'}^{(3)} W(j_1, j_1; L_0 L_0'; 2n j_0) \sqrt{2j_0 + 1} \right\}
$$

$$
\times \left\{ \sum_{j} \sum_{L_0 L_0'} \left( - \right)^{L_0 + j} (j_2 || L_0|| j_2) (j_2 || L_0' || j_2) F_{n_0}(L_2 L_2' j_2 j_2) \right\} P_{n_0}(\cos \theta),
$$

where we put the same factors $\sqrt{2j_0 + 1}$ into the first and the second curly brackets.

6) $\beta - \gamma$ angular correlation, $\gamma$ unobserved $^{6,7}$ (Figs. 3).

This angular correlation is the usual double cascade correlation of $\beta$- and $\gamma$-rays. Averaging over the polarization, the angle variables of the second gamma ray and $m_{3\gamma}$ (25) becomes,

$$
W(\theta : \beta - \gamma_1) = \sum_{L_0 L_0'' L_1 L_1'} \sum_{n \delta_1 \delta_1'} \sum_{p_1} \sum_{m_0 M_{10}} \sum_{m_1 M_{20}} b_{L_0 L_0'}^{(3)} \alpha_i \alpha_i' i^{L_1 - L_1'} + 3 \delta_1 - 3 \delta_1' \sqrt{(2L_1 + 1)(2L_1' + 1)}
$$

$$
\times \left\{ \right. (-)^I M (L_0' L_0 - MM) (2n 0) (j_0 L_0 m_0 - MM j_1 m_1) (j_0 L_0' m_1 - MM j_1 m_1)
$$

$$
\times (j_1 L_1 m_1 m_0 - m_1 j_1 m_0) (j_1 L_1' m_1 m_2 - m_1 j_1 m_2)
$$

$$
\times D_{m_0 - m_1}^{(L_0)} (0 \theta 0) D_{m_1 - m_2}^{(L_1)} (0 \theta 0),
$$

(30)

where $\theta = \theta_1$.

After summing over the magnetic quantum numbers and the polarization $p_1$,

$$
W(\theta : \beta - \gamma_1) = \sum_{L_0 L_0'' L_1 L_1'} \sum_{n \delta_1 \delta_1'} b_{L_0 L_0'}^{(3)} \alpha_i \alpha_i' i^{L_1 - L_1'} + 3 \delta_1 - 3 \delta_1' \sqrt{(2L_1 + 1)(2L_1' + 1)}
$$

$$
\times \left\{ \right. (-)^{L_1 - L_1'} (2n 0) W(j_1, j_1; L_0 L_0'; 2n j_0) W(j_1, j_1; L_1 L_1'; 2n j_0) P_{n_0}(\cos \theta)
$$

(31)

This is equal to M(6) or Formula I of M. Moreover (31) is expressed by

$$
W(\theta : \beta - \gamma_1) = \sum_{n \delta_1 \delta_1'} \left\{ \right. \sum_{L_0 L_0'' L_1 L_1'} (-)^{L_1 - L_1'} b_{L_0 L_0'}^{(3)} W(j_1, j_1; L_0 L_0'; 2n j_0) \sqrt{2j_0 + 1}
$$

$$
\times \left\{ \sum_{j, L_0 L_0'} (-)^{L_1 + L_1'} (j_2 || L_0|| j_2) (j_2 || L_0' || j_2) F_{n_0}(L_2 L_2' j_2 j_2) \right\} P_{n_0}(\cos \theta).
$$

(32)
§ 4. Miscellaneous angular correlations

In this section we calculate the angular correlation functions for various decay schemes without observing one of the triple cascade particles.

In order to symmetrize $F_{LL'}^{M}(\theta)$ with respect to $L$ and $L'$, we introduce a new definition:

$$F_{LL'}^{M}(\theta) = \sum_{L} \sum_{L'} \mathcal{A}_{L}(X_{i}) \mathcal{A}_{L'}(X_{j})$$

\[ \times \mathcal{R}_{LM}(A_{L}) \mathcal{R}_{L'M}(A_{L'}) \]  

where the restriction, $L \leq L'$, is removed.

With this definition, $F_{LL'}^{M}(\theta)$ for $\gamma$-ray is

$$F_{LL'}^{M}(\theta) = \sum_{n} \left( j || L || j' \right) \left( j || L' || j' \right) \left( - \right)^{M+1}$$

\[ \times \sqrt{(2L+1)(2L'+1)} \]  

\[ (LL'-MM|2n 0)(LL'-1-1|2n 0)P_{2n}(\cos \theta) \]  

(33)

From M(4) and (33), $b_{\gamma}^{(n)}$ of $\gamma$-ray is

$$b_{\gamma}^{(n)} = \left( j || L || j' \right) \left( j || L' || j' \right) \left( - \right)^{M+1} \sqrt{(2L+1)(2L'+1)} \]  

(34)

1) $\gamma_{0} - \gamma_{2}$ angular correlation, $\gamma_{1}$ unobserved, (Fig. 4).

Inserting (34) in (29), we obtain

$$W(\theta : \gamma_{0} - \gamma_{2}) = \sum_{n} \left( j || L || j' \right) \left( j || L' || j' \right) \left( - \right)^{M+1} \sqrt{(2L+1)(2L'+1)} \]  

\[ \times \sum_{L} \left( j_{1} || L_{1} || j_{3} \right) \left( j_{1} || L_{1}' || j_{3} \right) F_{2n}(L_{0} L_{1} L_{2} L_{3} j_{1} j_{2}) P_{2n}(\cos \theta) \]  

(35)

which is the generalization of BAR (24).

2) $\gamma_{0} - \gamma_{1}$ angular correlation, $\gamma_{2}$ unobserved, (Fig. 4).

Inserting (34) in (32), we obtain $W(\theta : \gamma_{0} - \gamma_{1})$ which is equal to $W(\theta)$ of the usual double cascade gamma transition, BR (64)\(^{29}\).

$$W(\theta : \gamma_{0} - \gamma_{1}) = \sum_{n} \left( j_{0} || L_{0} || j_{1} \right) \left( j_{0} || L_{0}' || j_{1} \right) F_{2n}(L_{0} L_{0}' j_{0} j_{1})$$

\[ \times \sum_{L} \left( - \right)^{L_{1}+L_{1}'} \left( j_{1} || L_{1} || j_{2} \right) \left( j_{1} || L_{1}' || j_{2} \right) F_{2n}(L_{1} L_{1}' j_{1} j_{2}) \]  

\[ \text{P}_{2n}(\cos \theta) \]  

(36)

3) $\gamma_{1} - \gamma_{2}$ angular correlation, $\gamma_{0}$ unobserved, (Fig. 4).

$$W(\theta : \gamma_{1} - \gamma_{2}) = (36), \text{in which each suffix } i \text{ except } n \text{ is replaced by } i+1. \]  

(37)

In (35), (36) and (37) the sign functions $(-)^{L_{1}+L_{1}'}$ are preferable in analysing the three gamma rays in connection with $\gamma_{0} - \gamma_{1}, \gamma_{1} - \gamma_{2}, \gamma_{0} - \gamma_{0}$.

In the case of the particle with spin zero, e.g., $\alpha$-particle, $\pi$ meson, etc., $F_{LL'}^{M}(\theta)$ is given by
\[
F_{LL'}^M(\theta) = \sum_n (j||L||j') (j||L'||j') (-)^M \times \sqrt{(2L+1)(2L'+1)} \left( (LL-MM|2n 0) (LL' 00|2n 0) P_{2n}(\cos \theta) \right),
\]
(38)
where \(L\) and \(L'\) indicate the \(L\)-th and \(L'\)-th partial waves.

\[
b_{LL'}^{(2n)} = b_{L'L}^{(2n)} = (j||L||j') (j||L'||j') \sqrt{(2L+1)(2L'+1)} (LL' 00|2n 0).
\]
(39)

Comparing (34) with (39), the equations for \(\gamma\)-ray are also useful for the particle with spin zero if we replace \((LL' 1-1|2n 0)\) by \(-(LL' 00|2n 0)\) for relevant \(\gamma\)-ray. When the particle has nonzero spin, we calculate the equations with channel spin formalism. For example, in the capture of the particle with nonzero spin, we should replace \((L_0L_0' 1-1|2n 0)\) by \(-\sum_s |A(s)|^2 (L_0L_0' 00|2n 0) W(j_1, j_0 L_0 L_0'; 2n j_0)\), where \(A(s)\) is the reduced matrix element for the capture process of channel spin \(s\).

4) \(\alpha-\gamma\) angular correlation (Fig. 6).

\[
W(\theta: \alpha-\gamma) = \sum_n \left\{ \left[ \sum_{L_0L_0'} (-)^{L_0-L_0'} (j_0||L_0||j_1) (j_0||L_0'||j_1) \sqrt{(2L_0+1)(2L_0'+1)(2j_1+1)} \times (L_0 L_0' 00|2n 0) W(j_1, j_0 L_0 L_0'; 2n j_0) \right] \times \left\{ \sum_{L_1L_1'} (-)^{L_1-L_1'} (j_1||L_1||j_2) (j_1||L_1'||j_2) F_{2n}(L_1 L_1' j_2 j_2) \right\} \right\} P_{2n}(\cos \theta).
\]
(40)

5) \(\alpha-\gamma_3\) angular correlation, \(\gamma_3\) unobserved (Fig. 7).

\[
W(\theta: \alpha-\gamma_3) = \sum_n \left\{ \left[ \sum_{L_0L_0'} (-)^{L_0-L_0'} (j_0||L_0||j_1) (j_0||L_0'||j_1) \sqrt{(2L_0+1)(2L_0'+1)(2j_1+1)} \times (L_0 L_0' 00|2n 0) W(j_1, j_0 L_0 L_0'; 2n j_0) \right] \times \left\{ \sum_{L_1L_1'} (-)^{L_1-L_1'} (j_1||L_1||j_3) (j_1||L_1'||j_3) F_{2n}(L_1 L_1' j_3 j_3) \right\} \right\} P_{2n}(\cos \theta).
\]
(41)
6) $\beta-\alpha$ angular correlation (Fig. 8).

$$W(\theta : \beta-\alpha)$$

$$= \sum \left\{ \sum_{L_1, L_2} (-)^{j_1-j_0} W(j_1, j_1, L_0, L_0'; 2n_j_0) \sqrt{2j_1+1} \right\}$$

$$\times \left\{ \sum_{L_1, L_2} (-)^{L_1+L_2} W_{L_0, L_0}^{(n)} W(j_1L_1L_0; j_1L_0'j_2) \right\}$$

$$\times \sqrt{(2L_1+1)(2L_2+1)} (2j_1+1)(2j_2+1) W(j_1, j_1, L_0, L_0') \} P_{\nu n}(\cos \theta). \quad (42)$$

7) $\beta-\alpha$ angular correlation, $\gamma$ unobserved (Fig. 9).

$$W(\theta : \beta-\alpha, \gamma \text{ unobserved})$$

$$= \sum \left\{ \sum_{L_1, L_2} (-)^{j_1-j_0} W(j_1, j_1, L_0, L_0'; 2n_j_0) \sqrt{2j_1+1} \right\}$$

$$\times \left\{ \sum_{L_1, L_2} (-)^{L_1+L_2} W(j_1L_1L_0; j_1L_0'j_2) \right\}$$

$$\times \sqrt{(2L_1+1)(2L_2+1)} (2j_1+1)(2j_2+1) W(j_1, j_1, L_0, L_0') \} P_{\nu n}(\cos \theta). \quad (43)$$

Fig. 8. $W(\theta : \beta-\alpha)$. Fig. 9. $W(\theta : \beta-\alpha, \gamma \text{ unobserved})$.

§ 5. Concluding remarks

A set of the experiments on the angular correlation $\beta-\gamma_1-\gamma_2$, I, II, III, IV, together with the measurements of $\beta-\gamma_1$ angular correlation and of $\beta$-ray spectrum, is expected to determine the linear combination of $\beta$-decay interaction definitely. $W(\theta)^{(n)}$'s given in § 3 and $b_{LL}^{(n)}$'s for $\beta$-ray spectrum may be useful in analysing the experimental data theoretically. It seems that the $\beta$-decays of odd-odd nuclei are suitable for our purpose, especially, 3_ or 5_ ($\beta$ 1st) $4_+ (\gamma_1 2) 2_+ (\gamma_2 2) 0_+$, because the even-even nucleus has a well-known rotational level structure $4_+ - 2_+ - 0_+$. Unfortunately, there has not been tried any experiment on $3_-$ or $5_-(\beta 1st) 4_+ (\gamma_1 2) 2_+ (\gamma_2 2) 0_+$ so far. It is also possible to use the decay schemes, $4_-(\beta 1st)$ or $6_+(\beta 2nd) 4_+ (\gamma_1 2) 0_+; 2_+, 3_+, 4_+(\beta 1st)$ or $5_-(\beta 2nd) 3_+ (\gamma_1 2) 1_-(\gamma_2 1) 0_+$ and any other $\beta-\gamma_1-\gamma_2$ process, for which, however, more laborious calculation is needed. We will pick up suitable nuclei for our purpose from references 23, 26 and 27 and show them in the following.
If it is certain that the second excited state of Mo$^{94}$ is $4/5^+$, $6^+$, and $(\gamma_1, 2) 2_+ (\gamma_2, 2) 0_+$ is available. But, if it is $3_+$ as described in reference 26 and 27, this decay scheme is useless, for $\beta$-decay is caused by $S_{\ell\ell}$ term only.

2) $^{208}Tl^{208} \rightarrow ^{208}Pb^{208}$: log $\rho t = 6.5$, $W_0 = 1.02$ Mev.

$6^- (\beta 1st) 6_+ (\gamma_1, 2) 4_+ (\gamma_2, 2) 2^+$ is available. $6^+$ state is the third excited state of Xe$^{130}$. The ground state of Cs$^{131}$ has spin 4, but its parity is not certain. If its parity is plus, $4_+ (\beta 1st) 3_- (\gamma_1, 2) 1_+ (\gamma_2, 1) 0_+$ is available. On the contrary, if its parity is minus, $4_- (\beta 1st) 4_+ (\gamma_1, 2) 2^+ (\gamma_2, 2) 0_+$ is available.

4) $^{81}Tl^{208} \rightarrow ^{82}Pb^{208}$: log $\rho t = 5.6$, $W_0 = 1.79$ Mev.

$5_+ (\beta 1st) 5_- (\gamma_1, 2) 3_- (\gamma_2, 3) 0_+$ is available.

$W(\theta)$'s for the decay schemes, 2) and 4), are not yet given here. They are deducible by the same procedure described in § 3 and will be calculated if the experiments are possible.

If the window of the $\beta$-counter is circularly symmetric with respect to $\zeta$-axis in colatitude angle $\theta$ to the centre of the radiation source, then $b_{\ell\ell}'s$ for $\beta$-ray in $W(\theta)$'s should be replaced by $b_{\ell\ell}'s$ in $W(\theta)$'s should be replaced by $b_{\ell\ell}'s$ in $F(Z, W)K^2pWdW$. In the energy region, where the Fermi function $F(Z, W)$ times $p/W$ is nearly constant, we can replace this integration by $\int_{W_{min}}^{W_{max}} b_{\ell\ell}'s K^2W^2dW$. This reduces the labour of computation very much. For example, in $Z = 42$,

\[
(p/W) F(Z, W) = \begin{cases} 3.475 & : p = 0.0, \\ 3.449 & : p = 1.8, 
\end{cases}
\]

which are given by Rose$^{20}$. In $0.0 \leq p \leq 1.8$, $(p/W) F(Z, W)$ may be regarded as constant with an error of about 0.2 percent.

When some drastic cancellation occurs in $b_{\ell\ell}'s$, we have to take into account the finite nuclear size correction and the finite de Broglie wave length effect. However, the main terms of these corrections to $b_{\ell\ell}'s$ are expressible as the renormalization of the Coulomb potential $\left(\alpha Z/\rho\right)$, i.e., the effects of the corrections are equivalent to the effective change of $Z$ and/or $\mu$.$^{20}$ This guarantees the use of $b_{\ell\ell}'s$ in the usual approximation with great accuracy.$^{30}$ Unfortunately, this fact is verified only for the first forbidden transition with $\alpha Z \ll 1$. We are now engaged in removing this restriction.

*) This approximation is not so bad even in the case of RaE.
Proposal for Experiments for Determination of Beta-Decay Interaction

It is necessary to consider the correlation of $\beta$-spectra $(n=0)$ in $W (I, II, III)$, when the $\beta$-spectra are measured by $\beta-\gamma_1-\gamma_2$ coincidence. If one wants to exclude this correlation in experiments, the $\beta$-counter has to be rotated over the whole solid angle.

To our regret, we cannot determine the relative sign of coupling constants of Scalar and Tensor interactions from our experiments. However, we may obtain a set of accurate and definite values of $x, y, z$; therefore, this relative sign is deducible from these values with a least assumption on the model of nuclei. Further, these values of $x, y, z$ may give us some information concerning the model of nuclei if they are accumulated in a sufficient number for various nuclei. Recent progress of the experiments may make this possible. The author hopes that the experiments will be performed in this field.

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31) If we want to estimate the two corrections for $b_{\alpha\beta_{\alpha}}$'s explicitly, we should use $F_{LL',M}(\theta)$ in terms of $L_i$ and $L_i$, etc., which were given by us (see, references of M).