Pergamon

Wat. Sci. Tech. Vol. 37, No. 11, pp. 15–19, 1998.
IAWQ

0273–1223/98 $19·00 + 0·00

PII: S0273-1223(98)00311-4

REGIONAL ESTIMATION OF SHORT DURATION RAINFALL EXTREMES

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ABSTRACT

The present study proposes a method for estimating the distribution of short-duration (e.g., 1 hour) extreme rainfalls at sites where data for the time interval of interest do not exist, but rainfall data for longer-duration (e.g., 1 day) are available (partially-gaged sites). The proposed method is based on the recently developed "scale-invariance" (or "scaling") theory. In this study, the scaling concept implies that statistical properties of the extreme rainfall processes for different temporal scales are related to each other by a scale-changing operator involving only the scale ratio. Further, it is assumed that these hydrologic series possess a simple scaling behaviour. The suggested methodology has been applied to extreme rainfall data from a network of 14 recording raingages in Quebec (Canada). The Generalised Extreme Value (GEV) distribution was used to estimate the rainfall quantiles. Results of the numerical application have indicated that for partially-gaged sites the proposed scaling method is able to provide extreme rainfall estimates which are comparable with those based on available at-site rainfall data.

KEYWORDS

Extreme rainfalls; generalized extreme value distribution; missing data; scaling method; statistical modeling; urban hydrology.

INTRODUCTION

For planning and design of various hydraulic structures, extreme rainfall for a given return period is required. In particular, rainfall extremes with high temporal resolution (e.g., one hour or shorter) are necessary for the design of urban drainage systems because urban areas are generally characterised by fast response. However, in most cases, either extreme rainfall records at the study location are unavailable (ungaged site), or the data samples for the time interval of interest at the study site are limited or missing (partially gaged site). On the other hand, daily rainfall data are often widely available. This would suggest that if techniques could be developed to estimate short time interval extreme rainfalls from daily rainfall data, these estimates could be used at locations where only daily rainfall records are available.

Several probability models have been developed to describe the distribution of extreme rainfalls at a single site (see, e.g., Wilks, 1993). Unfortunately, these models are accurate only for the specific time frame associated with the data used; that is, the inference and deduction based on the proposed models have to be restricted to the particular time scale of the rainfall data from which these models were developed. Hence, it

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was argued that the usefulness of a model should lie in its potential ability to adequately describe the rainfall process at other time scales which are not included in the building of its mathematical structure. It has necessitated the need for formulating models whose mathematical structure will follow the main statistical features of the past history through a continuum of levels of aggregation. This implies that the suggested model should statistically and simultaneously match various properties of the physical process at different levels of aggregation; whether or not these properties are included in the model. The most important practical implication of such models is that, from a higher aggregation model we could infer the statistical properties of the process at the finer resolutions that may not have been observed.

Further, for ungaged or partially gaged sites regionalization methods (e.g., Schaefer, 1990) are frequently used to transfer rainfall information from one location to the other where the data are needed but not available, or to improve the accuracy of the estimates where available records are too short. Nevertheless, traditional regionalization techniques are often criticised for the obvious subjectivity, in particular in the definition of hydrologically similar sites or regions, and the lack of theoretical justifications. Further, the accuracy of these techniques are often limited because they cannot take into account some observed "scale-independent" properties of the rainfall series considered.

In view of the above problems, the main objective of the present study is to propose a new method for estimating extreme rainfalls for partially gaged sites using the "scale-invariance" (or "scaling") concept which is currently a popular tool in the modelling and analysis of various geophysical processes (see, e.g., Gupta and Waymire, 1990; Tessier et al., 1996). In this study, the scale invariance implies that statistical properties of extreme rainfall variables for different time scales are related to each other by a scale-changing operator involving only the scale ratio. Extreme rainfall data from a network of 14 raingages in Quebec were used to illustrate the application of the proposed method. The Generalised Extreme-Value (GEV) distribution was used to estimate the extreme rainfall quantiles at different locations. Results of this numerical application have indicated that for sites where data for the time interval of interest are missing the proposed scaling approach can provide extreme rainfall estimates which are comparable with the empirical ones when at-site observed rainfall data are available .

SCALING CONCEPT

By definition (see, e.g., Fedder, 1988), a function $f(x)$ is scaling (or scale-invariant) if $f(x)$ is proportional to the scaled function $f(\lambda x)$ for all positive values of the scale factor λ. That is, if $f(x)$ is scaling then there exists a function $C(\lambda)$ such that:

$$f(x) = C(\lambda) f(\lambda x) \tag{1}$$

It can be readily shown that

$$C(\lambda) = \lambda^{-\beta} \tag{2}$$

in which β is a constant, and that

$$f(x) = x^{\beta} f(1) \tag{3}$$

Hence, the relationship between the non-central moment (NCM) of order k, μ_k, and the variable x can be written in a general form as follows:

$$\mu_k = E\{f^k(x)\} = \alpha(k) x^{\beta(k)} \tag{4}$$

in which $\alpha(k)=E\{f^k(1)\}$ and $\beta(k)=\beta k$. Notice that if the exponent $\beta(k)$ is not a linear function of k, in such cases the process is said to be "multiscaling" (Gupta and Waymire, 1990).

THE GENERALIZED EXTREME VALUE DISTRIBUTION

Application of the Generalised Extreme-Value (GEV) distribution to model the annual series of extreme rainfalls has been advocated by several researchers (see, e.g., Natural Environment Research Council, 1975; Schaefer, 1990). The cumulative distribution function, $F(x)$, for the GEV distribution is given as:

$$F(x) = \exp\left[-\left(1 - \frac{\kappa(x-\xi)}{\alpha}\right)^{1/\kappa}\right] \qquad \kappa \neq 0 \tag{5}$$

where ξ, α and κ are respectively the location, scale and shape parameters. It can be readily shown that the k-th order NCM, μ_k, of the GEV distribution (for $k \neq 0$) can be expressed as (Pandey, 1995)

$$\mu_k = \left(\xi + \frac{\alpha}{\kappa}\right)^k + (-1)^k\left(\frac{\alpha}{\kappa}\right)^k \Gamma(1 + k\kappa) + k \sum_{i=1}^{k-1} (-1)^i \left(\frac{\alpha}{\kappa}\right)^i \left(\xi + \frac{\alpha}{\kappa}\right)^{k-i} \Gamma(1 + i\kappa) \tag{6}$$

where $\Gamma(.)$ is the gamma function. Hence, on the basis of (6), it is possible to estimate the three parameters of the GEV distribution using the first three NCMs. Consequently, the quantiles (X_T) can be computed using the following relation:

$$X_T = \xi + \frac{\alpha}{\kappa}\left\{1 - [-\ln(p)]^{\kappa}\right\} \tag{7}$$

in which p=1/T is the probability of interest.

Further, for simple scaling process, it can be shown that the statistical properties of the GEV distribution for two different time scales t and λt are related as follows:

$$\kappa(\lambda t) = \kappa(t) \tag{8}$$
$$\alpha(\lambda t) = \lambda^{\beta}\alpha(t) \tag{9}$$
$$\xi(\lambda t) = \lambda^{\beta}\xi(t) \tag{10}$$
$$X_T(\lambda t) = \lambda^{\beta}X_T(t) \tag{11}$$

Hence, based on these relationships it is possible to derive the statistical properties of short-duration (e.g., less than 1 day) extreme rainfalls using the properties of daily (t = 1 day) extreme rainfalls. More specifically, if rainfall data for both time scales λt and t are available at the location of interest (i.e., a "gaged" site), the exponent β in (9)-(11) can be readily estimated using the ratio between the means of extreme rainfalls for time intervals λt and t as given in the following expression:

$$\lambda^{\beta} = \frac{\mu_1(\lambda t)}{\mu_1(t)} \tag{12}$$

However, for "partially-gaged" sites, where rainfall data for the short time interval of interest λt are not available, the at-site mean $\mu_1(\lambda t)$ in (12) cannot be estimated. The exponent β is thus computed using a regional ratio which is interpolated from the ratios available at nearby sites. For example, regional rainfall frequency maps have been developed for various regions in Canada (see, e.g., Gray, 1973; National Research Council of Canada, 1989). These maps provide the means of precipitation extremes for different durations (from 5 minutes to 24 hours), and the ratios of short-duration (e.g., 1 hour) rainfall extremes to daily rainfall extremes for different locations in Canada.

NUMERICAL APPLICATION

In the following, to illustrate the application of the proposed approach, a case study was carried out. The proposed scaling method will be used to estimate rainfall extremes for short time intervals (e.g., 1 hour) at locations where rainfall data for longer time scales (e.g., 1 day) are available (partially gaged sites). In this illustrative example, annual maximum rainfall (AM) series for durations ranging from 5 minutes to 4 days from a network of 14 raingages in Quebec (Canada) are considered. The rainfall record lengths vary from 15 to 48 years.

To assess the scaling behaviour of these AM series, the log-log plots of rainfall NCMs against duration were prepared for all 14 stations. For purpose of illustration, Figure 1 shows the plot for Dorval station. The linearity exhibited in the plot indicates that the rainfall statistical moments follow two different scaling regimes. The first one ranges from 5 minutes to 1 hour, and the second from 1 hour to 4 days. Notice that the slope of the straight line segment (starting from t = 5 minutes to t = 1 hour, or from t = 1 hour to t = 4 days) for the NCM of order $k = 1$ as shown in Figure 1 represents an estimate of the scaling exponent β defined in (12). Hence, for a given location, if β is known it is possible to determine the NCMs (or the distribution) of rainfall extremes for short durations (e.g., 1 hour) using available rainfall data for longer time scales within the same scaling regime (e.g., 1 day).

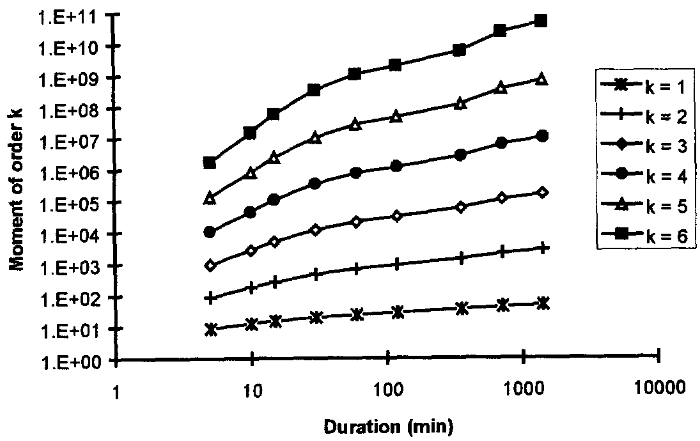

Figure 1. The log-log plot of maximum rainfall non-central moments versus rainfall duration for Dorval station.

As mentioned above, given the wide availability of rainfall data at the daily scale, the present study is concerned with the estimation of the distribution of 1-hour maximum rainfalls for partially-gaged sites (where hourly rainfall data are not available) using the fitted GEV distribution for 1-day maximum rainfalls. However, for comparison purposes, the estimation of hourly maximum rainfall distribution for gaged sites is presented as well. More specifically, two scenarios are considered: (i) for a gaged site, where data of rainfall extremes for both 1-hour and 1-day scales are available, the scaling exponent β is computed based on the ratio of the means of the observed hourly and daily rainfall extremes (see (12)); and (ii) for a partially gaged site, where only daily rainfall data are available, the exponent β is calculated using the regional value of β which is interpolated from values obtained at nearby sites. For illustrative purpose, Figure 2 shows the comparison between empirical (observed) and estimated distributions of 1-hour rainfall extremes at Dorval station for the case where hourly rainfall data are available (at-site curve), and for the case where hourly rainfall data are missing (regional curve). It can be seen that the regional estimate of hourly rainfall extreme distribution for partially-gaged sites are comparable with the at-site estimate. Further, the good agreement between the estimated (both at-site and regional) and empirical distributions for 1-hour rainfall extremes as shown in this case study has indicated the feasibility of the proposed scaling method.

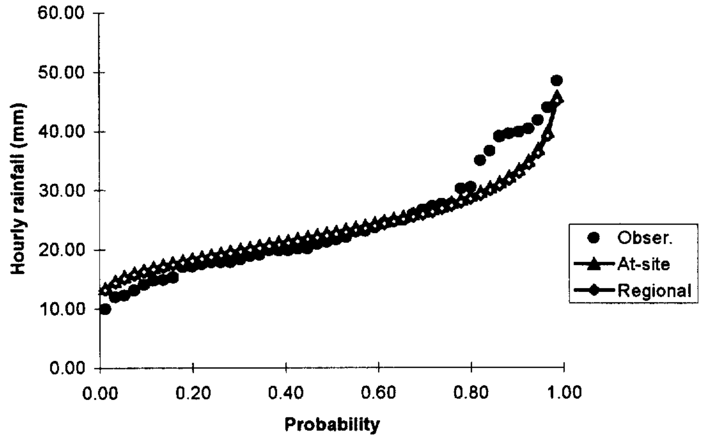

Figure 2. Empirical (observed) and estimated distributions of maximum hourly rainfalls using at-site and regional data.

CONCLUSIONS

The major findings of the present study can be summarised as follows:

By considering the scaling of statistical properties of extreme rainfall processes, a new method has been proposed for the estimation of short-duration extreme rainfall distribution for partially-gaged sites where data for longer time scales are available. Results of an illustrative application have indicated that the proposed scaling approach could provide extreme rainfall estimates which are comparable with the empirical ones.

The annual maximum rainfall series for 14 raingages in Quebec were found to exhibit two different scaling regimes for durations ranging from 5 minutes to 1 hour, and for durations from 1 hour to 4 days.

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