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PUMPING FROM LEAKY ARTESIAN AQUIFERS

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When a test pumping from an artesian aquifer has been performed, the result will often exibit a more or less confusing variety of features. The reason for this may, of course, be that the geological conditions of the test field are complicated and inhomogeneous, but another possibilty is that the whole flow process consists of a series of time periods in which quite different physical parameters are of dominating influence.

In the first period after the pumping has started the flow of ground water into the well is small, so that pumping causes a rapid drawdown in the well. The decrease of pore water pressure in the surrounding soil introduces a second period, in which the water stored in the aquifer by compression of the overburden begins to flow into the well, hence counteracting the drawdown of the free water surface. This process is well described by the theory of Theis (1938) and Jacob (1940) For symmetrical conditions the governing differential equation is of the parabolic type

$$c\left(\frac{\delta^2 h}{\delta r^2} + \frac{1}{r}\ \frac{\delta h}{\delta r}\right) = \frac{\delta h}{\delta t} \tag{1}$$

where h is the piezometric head (pressure head), t is time, r distance from well axis, and c a coefficient. This is often written as

$$c = T/S, \tag{2}$$

in which T is the transmissibility and S is the coefficient of storage.

The solution of eq. (1) describes h as a function of r and t, corresponding to a surface of revolution – the so-called depression cone – that is gradually widening with time corresponding to the increasing domain in which the pore water pressure is decreased.

The rapid drawdown of the water surface in the well during the first period induces very large gradients of h near the well surface and consequently a rapidly increasing inflow into the well. Occasionally this inflow may exceed the rate of pumping, so that reverse fluctuation of the water surface in the bore hole may occur at the transition between the first and the second period.

Under certain hydrogeological conditions a third period may be dominated by inflow into the aquifer from the adjacent layers. The decrease in pore water pressure in the aquifer starts a process of consolidation in the neighbouring strata, and this may be a controlling factor of considerable importance. If the pumping does not exceed the infiltration from the adjacent layers, an ultimate steady state flow is a possibility, at least theoretically. If not, the development of the depression cone may continue, for instance until it reaches a reservoir, an infiltration area, or, eventually, until the well goes dry.

One way of facilitating the interpretation of test data is to develop relatively simple mathematical models, from which the main features of flow development may be estimated. The purpose of the present paper is to develop such models for the above-mentioned third period, where the inflow into the aquifer from the adjacent layers is important. The theory will probably make it easier to estimate whether or not this infiltration is important in actual cases.

ONE-DIMENSIONAL CONSOLIDATION

Before we can handle the problem sketched above, it is necessary to carry out the simpler analysis of one-dimensional flow from a uniform aquifer. The problem may be stated as follows, see Fig. 1

As the flow is one-dimensional the well is replaced by a ditch. Until the time $t \equiv 0$, the head is equal to H as in the aquifer. For larger times the free water surface is lowered continuously, and we assume that $s = s(t)$ is an ever-increasing function of time. The problem is to give an approximate description of the variation of the head $h \equiv h(x, t)$ in the aquifer.

When the function $s(t)$ is specified, the Laplace transform technique is an obvious tool. Here we proceed to apply an approximate method, suggested by Terzaghi (1935). Instead of looking for an exact solution of the differential equation

$$c \frac{\delta^2 h}{\delta x^2} = \frac{\delta h}{\delta t}$$

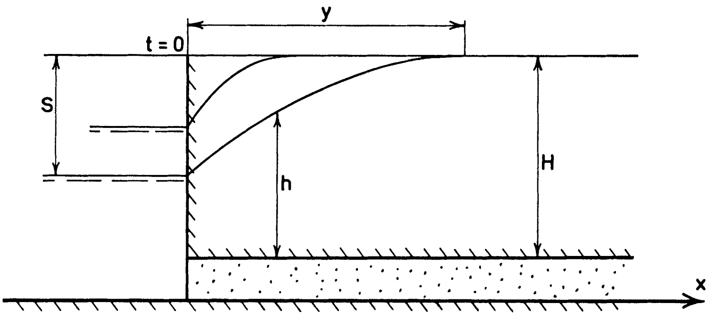

Fig. 1.

we consider the integrated form

$$c\left(\frac{\delta h}{\delta x}\right)_{x=0} = \int_0^y \frac{\delta h}{\delta t} dx \tag{3}$$

in which y is the extension of the depression curve. Then an approximate solution is obtained by substitution of a reasonable estimated expression for h, containing the time-dependent parameter y.

If, for instance, we assume the relation

$$H - h = s\left(1 - \frac{x}{y}\right)^2 \text{for } x \leqq y, \tag{4}$$

we find from eq. (3) the following ordinary differential equation

$$\frac{6cs}{y} = \frac{d}{dt}(sy)$$

or

$$\frac{dy^2}{dt} + \frac{2}{s}\,\frac{ds}{dt}y^2 = 12c$$

The solution of this equation in terms of s is

$$y^2 = \frac{12c}{s^2}\int_0^t s^2\,dt \tag{5}$$

if $y \equiv 0$ and $s \equiv 0$ for $t \equiv 0$.

As a simple example of the application of eq. (5) let us consider an instantaneous drawdown to $s = s_0$. Then we get

$$y = \sqrt{12\,ct}$$

and the discharge q from the aquifer

$$q \equiv -kd\left(\frac{\delta h}{\delta x}\right)_{x \equiv 0} \equiv \frac{2\,kd\,s_0}{y} \equiv \frac{2\,kd\,s_0}{\sqrt{12\,ct}},$$

an expression that differs only 2 per cent in magnitude from the exact solution.

PUMPING FROM A WELL

Next we will develop a simple approximate theory for the pumping from an artesian layer in which the storage is negligible, so that the supply of water originates from the adjacent strata while the aquifer itself merely acts as a conduit. The situation is illustrated in Fig. 2.

For the sake of simplicity we may assume the underlying layer to be impervious, so that the infiltrations come from the upper layer only. If, for instance, the underlying layer is equal to the upper, we let d denote half of the thickness.

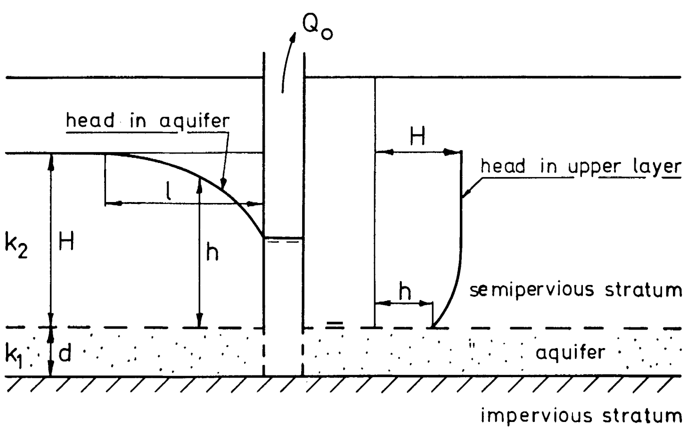

Fig. 2.

When the flow is supposed to be symmetrical, the discharge Q in the aquifer is determined by the equation of continuity

$$\frac{\delta Q}{\delta r} = -2\,\pi r k_2 \left(\frac{\delta h}{\delta x}\right)_{x\,=\,0} = -2\,\pi r k_2 i \tag{6}$$

in which the left-hand side is the change in aquifer discharge per unit increment of radius and the right-hand side is the inflow due to consolidation of the upper layer; k_2 is the permeability of the upper layer. The rate of pumping from the well is assumed to be constantly Q_0.

We are now faced with the difficulty that the gradient i of the upper layer at the interface is determined by the time-dependent development of the hydraulic head h in the aquifer. This, in turn, depends on the consolidation process in the upper layer. To obtain an approximately correct idea of the flow development, we assume that the head h changes according to the following expression

$$H - h = f\left(\frac{r}{l}\right),$$

in which H is the undisturbed head far from the well, and l is the radius of the depression cone, as indicated in Fig. 2. The function f is unknown, but a simple approximation would be to choose a logarithm for r $\geqq l$:

$$H - h = \frac{Q_0}{2\,\pi k_1 d}\, ln \frac{l}{r} \tag{7}$$

in which k_1 is the permeability of the aquifer, d the thickness, and l is a function of time.

Considering a point in the aquifer, say for $r = r_0$, it may now be argued that the pressure is unchanged until the point is reached by the depression cone, that is until $l = r_0$. Suppose that this happens at the time $t = t_0$, then the consolidation takes place, when the following condition is fulfilled

$$l(t) \geqq r_0,$$

or, what amounts to the same thing, when

$$t \geqq t_0.$$

As an assumption to be verified later, we put

$$l(t) \sim t^n, \tag{8}$$

in which n is a fixed exponent. Then eq. (7) may be written

$$H - h \equiv \frac{Q_0 n}{2\,\pi k_1 d}\, ln \frac{t}{t_0} \tag{9}$$

valid for $t \geqq t_0$.

Now, the pressure release in the aquifer induces a process of consolidation in the confirming bed, causing an infiltration downwards into the aquifer. This flow may to the first approximation be described by the equations of one-dimensional consolidations developed in the previous section, the quantity *H-h* being identified with the drawdown *s* in eq. (4). By substitution of eq. (9) and integration we obtain

$$y^2 = \frac{12\,ct}{ln^2\left(\frac{t}{t_0}\right)}\left[2\left(1 - \frac{t_0}{t}\right) - 2\,ln\frac{t}{t_0} + ln^2\left(\frac{t}{t_0}\right)\right]$$

The significant inflow corresponds to $t \gg t_0$. It is a consequence of the assumption (4) that $i = 2\,s/y$, and for $t > 3\,t_0$ we find with excellent approximation that

$$i = \frac{2\,s}{y} \simeq \frac{2\,Q_0 n}{2\,\pi k_1 d}\,\frac{1}{\sqrt{12\,ct}}\left[ln\frac{t}{t_0} + 1.05\right] \equiv \frac{1}{\sqrt{3\,ct}}\left[H - h + 1.05\,\frac{Q_0 n}{2\,\pi k_1 d}\right]$$

Next, we return to the equation of continuity (6) and substitute the obtained expression for *i* and the Darcy law

$$Q = 2\,\pi r d k_1\,\frac{\delta h}{\delta r} = -\,2\,\pi r d k_1\,\frac{\delta}{\delta r}(H - h)$$

Then we end up with the differential equation

$$\frac{\delta^2}{\delta r^2}(H - h) + \frac{1}{r}\,\frac{\delta}{\delta r}(H - h) = \frac{k_2}{dk_1\sqrt{3\,ct}}\left(H - h + 1.05\,\frac{nQ_0}{2\,\pi k_1 d}\right) \tag{10}$$

Now we introduce the length scale

$$l = \sqrt{\frac{dk_1}{k_2}\,\sqrt{3\,ct}} \tag{11}$$

that is in agreement with eq. (8), when we put $n = 1/4$. Then we obtain the solution for $r \leqq l$:

$$H - h = \frac{Q_0}{2\,\pi k_1 d}\left[\,K_0\left(\frac{r}{l}\right) - 0.26\,\right] \tag{12}$$

where K_0 denotes the modified Bessel function of the second kind and order zero.

As this function does not differ much from the logarithm, we conclude that the previous calculations are reasonably good, except for values of r near to l. This last restriction, however, makes comparison with most field investigations difficult, because observations are often carried out at borings so far from the well that it corresponds to small values of the drawdown.

Our next task is to evaluate a criterion for the occurrence of a third period in the sense defined above. The value of l in the classical case of storage in the aquifer only is given by

$$l = 2\sqrt{c_1 t} \tag{13}$$

index 1 referring to the aquifer, whereas we let index 2 refer to the upper layer. A time scale τ characteristic for the transition between the second and the third period is then obtained by equating this and the length l given by eq. (11):

$$2\sqrt{c_1\tau} = \sqrt{\frac{k_1}{k_2}\, d\sqrt{3\, c_2\tau}}$$

from which

$$\tau = \frac{3}{16}\left(\frac{k_1\, c_2}{k_2\, c_1}\right)\frac{d^2}{c_2} \tag{14}$$

When the time t is large as compared with τ we may assume the inflow from leakage to be decisive for the flow process.

As an illustration, let us consider a case where the compressibility of the soil in the aquifer is equal to that of the surrounding soil. In that case we have

$k_1\, c_2 = k_2\, c_1$, as in general $c = kE/\gamma$,

when the compressibility of water is neglected. γ is the specific gravity of the water.

Then eq. (14) reduces to

$$\tau = \frac{3}{16}\,\frac{d^2}{c_2}$$

If we assume a modulus of elasticity of

$E_2 = 1000$ bar $= 10^8\ N/\mathrm{m}^2$

and permeability

$k_2 = 10^{-7}$ m/sec

we find

$$c_2 = \frac{10^{-7} \times 10^{8}}{9.81 \times 1000} \simeq 10^{-3} \text{ m}^2/\text{sec}$$

If the effective thickness of the aquifer is 10 m we get

$$\tau = \frac{3}{16} \frac{10^{2}}{10^{-3}} = 18\,800 \text{ sec} \sim 5 \text{ hours.}$$

Hence, after few hours pumping the flow is severely influenced by the infiltration from the upper layer.

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