

## Cost and reliability comparison between branched and looped water supply networks

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### ABSTRACT

A new design methodology recently introduced for looped networks is now applied to branched networks. The methodology is based on cost analysis minimisation, network mathematical simulation under normal and failure states and reliability estimates. It is capable of obtaining not only an adequate value for the global reliability of a network but also the appropriate total design demand. This paper then presents a deeper insight into the comparison between looped and branched networks by applying the methodology to both types of networks and comparing the results in terms of cost and reliability. Considerations regarding available reliability definitions are also included which seem to favor volume-type definitions in practice. Results provide a rational foundation to reveal the superiority of looped networks over comparable branched networks by applying quantitative cost evaluations.

**Key words** | branched networks, economic analysis, network design, network reliability, optimization, water supply

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### NOTATION

The following symbols are used in this paper:

$C$	constant in Equations (7)	$T$	tolerance index of network
$C_{n1}, C_{n2}$	number of pipes connected to nodes $n_1$ and $n_2$ of broken pipe	$V_f$	volume per day that must be supplied to affected consumers.
$C_v$	coefficient of variation of actual demand	$a$	coefficient of pipe failure frequency formula
$D_{act}$	actual demand as random variable	$c_1$	annualizing factor
$D_{mean}$	mean actual demand	$c_a$	average cost of supplying water to affected consumers in dollars per unit volume
$DD$	design demand	$c_f$	average cost of repair in dollars per day
$D_{ns}$	demand of step ns	$d_k$	diameter of pipe $k$
$K_D$	dimensionless demand in probability curve	$e^{x_s}$	excess pressure in node $s$
$L_k$	length of pipe $k$	$hf$	friction loss
$NN$	total number of nodes	$i$	pipe initial node
$NP$	number of pipes	$j$	pipe final node
$NS$	number of source nodes	$k$	counter
$Q_k$	flow discharge in pipe $k$	$m$	exponent of pipe cost formula
$Q_{break}$	flow to be supplied to affected consumers	$n$	exponent of flow in friction formula
$Q_{n1}, Q_{n2}$	demand flow in nodes $n_1$ and $n_2$ of broken pipe	$ns$	step counter in discretization of probability curve
$R$	volume reliability of network	$pf_0$	probability that network works without failure
		$pf_k$	probability that pipe $k$ will be out of service

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$pm$	required minimum pressure
$p_{\text{ocur}}$	probability of occurrence
$q_i$	exterior flow at node $i$
$q_s$	incoming flow at source $s$
$q^{\text{nf}}$	actual total supply by network without failure
$q^{\text{req}}$	required total supply by network
$q^k$	actual total supply by network with pipe $k$ failed
$r$	exponent of diameter in friction formula
$r_0, r_k$	parameters defined by Equation (3)
$s$	counter for source nodes
$t_f$	average number of days for repair of pipe failure.
$u$	exponent of pipe failure frequency formula
$z$	ground elevation
$\psi$	objective function
$\eta$	coefficient of pipe cost formula
$\lambda$	constant (see Equation (10))
$\mu$	parameter defined by Equation (12)
$\omega$	coefficient associated with each pipe (see Equation (13))
$\zeta$	set of pipes connected to given node

## INTRODUCTION

It is well known that the application of an optimization algorithm to looped networks leads to the opening of the loops into a branched or pseudo-looped network. This has been well recognized in the literature (Watanatada 1973; Alperovits & Shamir 1977; Quindry *et al.* 1981; Rowell & Barnes 1982; Templeman 1982; Bhawe 1985; Lansey *et al.* 1989; Goulter & Bouchart 1990; Awumah *et al.* 1991; Bouchart & Goulter 1991; Loganathan *et al.* 1995; Ostfeld & Shamir 1996; Khomsi *et al.* 1996; Martínez 2001, 2007).

Mathematics so far has supported the notion that a branched network is cheaper; nevertheless, many water supply networks are systematically built with loops. This is because the advantages of loops are recognized even when these advantages have not been quantified explicitly nor expressed in mathematical language.

In a paper by Martínez (2007) the economic advantages of a looped network were explicitly revealed by formulating a new cost objective function (OBF). An example there

showed how a looped network could be less costly than a branched one. By combining this new OBF with a network simulator, a methodology was introduced in that paper to obtain not only an adequate value for the reliability of a looped network but also the appropriate design demand as well, based on cost analysis. The above-mentioned comparison between looped and branched networks was accomplished only with the new objective function. The application of the methodology to this comparison is the main subject of the present paper.

## DESIGN DEMAND AND REDUNDANCY

Traditionally, many water supply networks (WSN) have been designed with a capacity to fulfill the maximum hourly demand at the end of the design period. For simplicity, no fire flow demand is considered in this paper. Before the computer era, rules of thumb and personal experience were probably the only available tools to engineers in practice. Hand calculations for network analysis represented a tedious and boring task. Demand estimations were less than accurate. Nowadays, a lot of new concepts, databases and computer tools are available, thus allowing the engineer's expertise to be devoted to more relevant undertakings. Different demand projections and network design alternatives can be tested in very little time. In this context, the idea of reliability was probably introduced as one of the new concepts to give a measure of performance and trustworthiness.

Although a definition for reliability can be borrowed from other engineering areas, care must be exercised when applying it to WSN (Khomsi *et al.* 1996). Reliability assessment in WSN is complex because it is a multi-component system, there is no on-off situation and there are a lot of intermediate states between full service and the absence of service. Besides, WSN performance is directly influenced by several factors such as network layout, demand variability, prospective demand growth, pipe ageing and pipe failures.

An important factor in design is network layout and the comparison between branched and looped networks. The concept of redundancy appears in this comparison and has been closely related to reliability (Goulter 1992;

Park & Liebman 1993; Khomsi *et al.* 1996). Redundancy in WSN may be realized in two ways: network connectivity and pipe capacity. To a certain extent—although not well established yet and only for looped networks—these two ways appear to be interchangeable (Duan *et al.* 1990).

A branched network whose pipe design diameters have been decided to cope with the average present-day demand estimation can be regarded as a prototype of a non-redundant network (NRN). It is only logical to think that, if one starts adding new pipes to form loops (ensuring at least two paths from the source to each node) in this NRN, *connectivity redundancy* is introduced and smaller diameters can be expected to be necessary in the former branches because less flow is conveyed through them. Let this looped network be designed again to meet the same demands, new diameters chosen and let it be referred to as a purely connectivity-redundant network (CRN).

Design criteria, largely supported by experience, suggests the convenience of introducing safety factors in most engineering works. In WSN this means, as far as demand is concerned, that design is not based on current average demands but on certain maximum expected values of this demand that will occur at some time in the future. As mentioned before, a common design demand criterion is the maximum hourly demand at the end of the design period.

If now the two above-mentioned networks are re-designed to meet these maximum demands, then *capacity redundancy* is introduced in both of them. The result is that the first network is a connectivity non-redundant and capacity-redundant network and the second is redundant in both senses. With present technology in mind some questions may arise: (1) how can this redundancy be measured?, (2) which is the desirable degree of redundancy? and (3) how large should the safety factor be considered for design demand?

Initial answers to these questions can be found in Martínez (2007), where a cost comparison was made between comparable looped and branched networks based on a new objective function. The results favored the looped one. Besides, assuming that reliability could measure general redundancy, the methodology suggested in Martínez (2007) was able to answer the last two questions for a looped network: from a sequence of *optimal* design alternatives with increasing both design demand and

reliability, the minimum cost alternative indicates affordable levels of design demand and reliability. Better answers to those questions are explored in this paper.

## RELIABILITY

Maybe the first safety factor introduced as common practice in WSN layout is loop formation, even when investment costs are increased. This normally is intended to improve WSN performance during failure time (pipe breakage). Were pipe breakage a never-occurring event or duration of pipe repair would happen to be extremely short, there would be no need for loops. Therefore, loops against failures are needed, which means adding connectivity redundancy. But if loop formation is achieved by closing large-sized branches with small-diameter pipes, then WSN will not behave well in failure time, which in turn points to the need of adding capacity redundancy as well. The conclusion then is that failure time in a WSN can only be properly faced with both connectivity and capacity redundancy.

Another important conclusion is that, if both connectivity and capacity redundancy are needed to handle WSN performance during failure time, then a branched network will hardly ever be able to perform satisfactorily during failure time. Trivial as this conclusion may appear at first, it means that if some reliability parameter gives the same value for comparable branched and looped networks then the parameter is at least doubtful. As far as these conclusions closely relate to the reliability definition, they seem to suggest the convenience of devoting some reliability parameter to measure WSN performance *only during failure time*.

Different methods have been applied to calculate WSN reliability. Although classifications are always somewhat arbitrary, perhaps the methods can be classified as topological, analytical and simulation-based. Topological reliability emphasizes more on network connectivity than on hydraulic behavior (Goulter & Coals 1986; Su *et al.* 1987; Yang *et al.* 1996). Analytical reliability considers hydraulic behavior with equations relating the various hydraulic parameters (node demand, friction coefficient, nodal pressure, etc) (Xu & Goulter 1997b). Simulation-based reliability is more concerned with estimating demand deficits arising

from the network hydraulic behavior during normal and failure time (Gupta & Bhawe 1994; Xu & Goulter 1997a; Tanyimboh & Templeman 2000).

In many cases probability is used to define reliability. Perhaps probability is not the best way to define reliability, at least not alone. If, for instance, it is said that network reliability is 98%, this is likely to mean that the network will be safe for 98 out of 100 years. But it says nothing about the remaining two years or how unsafe they will be. Volume reliability has also been widely used (Gupta & Bhawe 1994; Tanyimboh & Templeman 2000). It is normally defined as the expected (in a probabilistic sense) average fraction of total demand that will be fulfilled. This definition might seem to give more information about the quality of the unsafe period than probability alone. Nevertheless, this definition of volume reliability averages network behavior during total time, which includes normal (no-failure) and failure time as well. As pipe failure frequency is very low, failure time is very likely to be always a tiny fraction of the total time, say 1% (or even much less), and this 1% of time might be very uncomfortable to users without being reflected in the volume reliability parameter.

This reasoning points to the same conclusion above: some reliability parameter should be defined to measure WSN performance *only during failure time*. Maybe a good definition for reliability in this sense would be the expected average fraction of total demand that will be fulfilled during failure time. This definition has the additional advantage that, if a network behaves well during failure time, it will surely behave even better during normal time while the converse is not necessarily true.

Most reliability definitions and applications found in the literature are not concerned with the network layout. They can be applied to looped networks as well as to branched networks and nothing in their numerical values makes a sharp distinction between these two types of networks. These reliability definitions cannot prevent the opening of the loops resulting from optimization models (Lansley *et al.* 1989; Duan *et al.* 1990; Loganathan *et al.* 1990). While the above suggestion of a reliability definition is not meant to solve the opening of the loops, it will be seen in the example as a noticeable difference when applied to branched and to looped networks.

Yet another argument comes from the first example in Martínez (2007). Almost two identical networks were compared in their total cost. Total cost included expected failure cost (see the section on Model formulation) and this failure cost includes the cost of supplying, by other means, the water deficit arising from pipe failures. The first network was a two-loop network and the second was exactly the same network with the only difference being that one pipe of each loop was disconnected at one of its end nodes so loops were open and this second network was a branched one. Notice that the pipes were not removed and the number of pipes and their diameters in both networks were the same. As a result the branched network was 15% more expensive than the looped one. This difference in cost was mainly due to the difference in water deficit. This example further stresses the importance of the failure time because water deficit occurs at failure time and it made the difference and favored the looped network as the best in this case. The result of this comparison can obviously be generalized for any looped network, provided the branched version is obtained in a similar way.

## TOLERANCE

It seems that Tanyimboh *et al.* (2001) was the first to introduce the notion of *tolerance* which, in essence, is a volume reliability measured only during failure time. The derivation of tolerance follows. Assuming zero probability for two or more simultaneous failures during repair time the probability  $pf_0$  that the network will be working without failures is

$$pf_0 = 1 - \sum_{k=1}^{NP} pf_k \quad (1)$$

where  $k$  is the pipe index,  $NP$  is the total number of pipes and  $pf_k$  is the failure probability of pipe  $k$ .

The value of  $pf_k$  is a function of pipe diameter and length and can be obtained from empirical formulas (Su *et al.* 1987; Bouchart & Goulter 1991; Cullinane *et al.* 1992; Gupta & Bhawe 1994; Khomsi *et al.* 1996) considering an average time for duration of repair.

Volume reliability  $R$  can now be stated as usual (Gupta & Bhawe 1994; Xu & Goulter 1997a; Tanyimboh *et al.* 2001;

Kalungi & Tanyimboh 2003):

$$R = \frac{1}{q^{\text{req}}} \left( q^{\text{nf}} p f_0 + \sum_{k=1}^{NP} q^k p f_k \right) \quad (2)$$

where:

$q^{\text{req}}$  total required demand (summation of all nodal demands)

$q^{\text{nf}}$  total actual supply under no-failure state

$q^k$  total actual supply when pipe  $k$  fails.

Let it be defined:

$$r_0 = q^{\text{nf}}/q^{\text{req}} \quad \text{and} \quad r_k = q^k/q^{\text{req}} \quad (3)$$

and Equation (2) rewritten:

$$R = r_0 p f_0 + \sum_{k=1}^{NP} r_k p f_k. \quad (4)$$

In order to calculate  $q^{\text{nf}}$  and all  $NP$  values of  $q^k$  a head-driven network simulator is needed (see below). Recall in Equation (4) that the first right-hand term represents a part of overall reliability corresponding to normal time (no-failure state) while the second term corresponds to failure time.

The tolerance index as introduced by its authors (Tanyimboh *et al.* 2001) is

$$T = \frac{R - r_0 p f_0}{1 - p f_0}. \quad (5)$$

Considering Equation (4) this can also be written as

$$T = \frac{\sum_{k=1}^{NP} r_k p f_k}{1 - p f_0} \quad (6)$$

where the second right-hand term of Equation (4) has been divided by  $1 - p f_0$  so as to normalize failure time and consider it as the whole 100% of time. This tolerance  $T$  is the expected average fraction of  $q^{\text{req}}$  (total demand) that the network can fulfill during failure time. In other words, it measures how well the networks behaves on average when any one pipe is broken. It is a volume reliability measured only during failure time.

An important conclusion can be drawn from Equation (6): the value of tolerance  $T$  is not influenced by  $r_0$ , which means that  $T$  is independent of how well the network behaves during no-failure time. It can be seen that  $r_0$  is the volume reliability of no-failure time. If a network behaves well during no-failure time then  $r_0 = 1$ . In this case the value of  $T$  alone is a better measure of reliability than  $R$ , otherwise both indices can be used.

Kalungi & Tanyimboh (2003) show interesting applications of this tolerance concept. Although tolerance is not a direct measure of redundancy it seems to reflect very well redundancy *impact*. As stated by its authors, tolerance appears to be a good inverse measure of vulnerability to failure: the more tolerance the less vulnerability.

As mentioned above, if the main reason for looping networks is to handle pipe breakage, then the tolerance index  $T$  seems to be a good one for looped networks. This will be illustrated further in the example. The tolerance concept could be extended to the scheme of three reliability indices suggested by Gupta & Bhawe (1994), which could be re-defined and calculated only within the duration of failure time but, for simplicity, it will not be considered here.

## SIMULATOR

In this paper the same simulator as in Martínez (2007) is applied. The nodal equation for the head-driven simulator is the one used by Xu & Goulter (1997a). The simulator in this paper gives a straightforward (non-iterative) solution such as the classical demand-driven algorithm does.

An important issue to be considered is the concept of segment and valve location as introduced by Walski (1993). In the simulation runs it is assumed that a valve exists at each end of every pipe so a single pipe can be isolated when it fails. In practice this is not true and a criticality analysis should be introduced as in Walski *et al.* (2006). It is hoped that the results of the methodology shown below might be useful in a posterior valve location analysis and decision.

## MODEL FORMULATION

A formulation by Chiong (1985) as cited by Martínez (2007) is first considered. The model formulates network pipe

sizing decisions for a given layout within the planning or early design stages. The issue of gradual construction over the life of the system is usually dealt with after the design has been decided upon.

The objective function (OBF) is restricted to account only for costs in the pipe network. The sum  $\psi$ —to be minimized—of annualized capital costs and annual energy costs is then

$$\psi = c_1 \eta \sum_{k=1}^{NP} L_k d_k^m + C \sum_{s=1}^{NS} q_s (pm_s + e^{x_s} + z_s) \quad (7)$$

where  $k, s$ : subscripts for pipes and source nodes;  $L, d$ : pipe length and diameter;  $NP$ : number of pipes;  $NS$ : number of source nodes;  $\eta, m$ : coefficient and exponent of pipe cost formula;  $c_1$ : annualizing factor;  $C$ : constant including annual pumping time, unit price of energy and units conversion;  $q_s$ : inflow (entering) to node  $s$ ;  $pm_s$ : minimum pressure requirement in node  $s$ ;  $z_s$ : ground elevation in node  $s$ ;  $e^{x_s}$ : excess pressure in node  $s$ .

subject to:

$$hf_k = (pm_i - pm_j) + (e^{x_i} - e^{x_j}) + (z_i - z_j) \quad (8)$$

for  $k = 1, \dots, NP$

$$\sum_{k \in \zeta} Q_k + q_i = 0 \quad \text{for } i = 1, \dots, NN - 1 \quad (9)$$

$$hf_k = \lambda_k L_k \frac{Q_k^n}{d_k^r} \quad \text{for } k = 1, \dots, NP \quad (10)$$

where:  $i, j$ : subscripts in Equation (8) for nodes belonging to pipe  $k$ ;  $Q_k$ : flow in pipe  $k$  (positive if leaving the node);  $q_i$ : exterior flow in node  $i$  (outflow is positive);  $\zeta$ : set of pipes  $k$  connected to node  $i$ ;  $NN$ : total number of nodes in network;  $\lambda_k$ : constant including the friction coefficient;  $n, r$ : exponents of the friction formula.

Equation (8) is Bernoulli's law for each pipe, Equation (9) is the node flow continuity and Equation (10) is a generic friction formula. In this model the decision variables (unknowns) are the  $x$  values in nodes (all but one) and one  $Q$  value for each loop. The method of solution is classical differential calculus and the exponential function is used to measure the excess pressures because of the strictly positive character of the function and its derivative, which

considerably enhances convergence of the nonlinear system of equations solved by the Newton–Raphson algorithm.

The existence of an energy cost term allows the hydraulic head in sources to be obtained as a result from the optimization process. Pump efficiency and its operating point are not included here because only network energy consumption is considered. In the case the modeler gives fixed values for hydraulic head in all sources, the energy cost term is constant and it does not influence optimization.

Substitution of Equations (10) into the objective function leads to

$$\psi = c_1 \eta \sum_{k=1}^{NP} \mu_k \frac{Q_k^{m/r}}{hf_k^{m/r}} + C \sum_{s=1}^{NS} q_s (pm_s + e^{x_s} + z_s) \quad (11)$$

where

$$\mu_k = L_k (\lambda_k L_k)^{m/r} \quad (12)$$

In [Chiong \(1985\)](#) a two-step optimization is applied: first, a simple algorithm to calculate pipe flows under maximization of their uniformity and, second, the above formulation with given  $Q_k$  to optimize pipe size for those flows and obtain a global minimum. This means that, for a given set of nodal demands, pipe flows are calculated only once. To maximize flow uniformity the set of pipe flows is treated as a statistical series and flows are calculated to minimize the variance of the series. The final solution gives continuous diameters which must be rounded to available commercial values.

The formulation by [Martínez \(2007\)](#), which considers pipe-failure-associated costs, is applied in this paper. It is assumed that if a pipe goes out of service it can be isolated by closing valves at the extremes and only consumer taps located along the closed pipe are affected. Also, when this happens, affected consumers are supplied by other means. The OBF now is obtained by adding a new term to Equation (7) which accounts for the expected annual cost involved in a pipe breakage. Explicit use is made of an empirical formula to express the frequency of failures ([Su et al. 1987](#); [Bouchart & Goulter 1991](#); [Gupta & Bhawe 1994](#); [Khomsi et al. 1996](#)):

$$\psi = c_1 \eta \sum_{k=1}^{NP} L_k d_k^m + \sum_{k=1}^{NP} \omega_k L_k d_k^{-u} + C \sum_{s=1}^{NS} q_s (pm_s + e^{x_s} + z_s) \quad (13)$$

where:  $\omega = a t_f (c_f + c_a V_f)$ : coefficient associated with each pipe;  $aLd^{-u}$ : formula giving the expected number of failures per year in terms of pipe diameter and length ( $a$  and  $u$  are known constants);  $t_f$ : average number of days for complete repair of each pipe failure;  $c_f$ : average cost of repair in dollars per day;  $c_a$ : average cost of supplying water to affected consumers in dollars per unit volume;  $V_f = 86,400 Q_{\text{break}}$ : volume per day that must be supplied to affected consumers (86 400 is the number of seconds in one day);  $Q_{\text{break}} = (Q_{n1}/C_{n1} + Q_{n2}/C_{n2})$  for broken pipes in loops;  $Q_{n1}$ ,  $Q_{n2}$ : demand flow as volume per second in nodes  $n_1$  and  $n_2$  of the broken pipe;  $C_{n1}$ ,  $C_{n2}$ : number of pipes respectively connected to nodes  $n_1$  and  $n_2$ ;  $Q_{\text{break}} = Q_k$  for pipes not in loops (the whole flow carried by the pipe).

The value of  $Q_{\text{break}}$  estimates the flow deficit arising when a pipe is broken. Adding a fraction of each one of the nodal demands located at the pipe nodes gives the estimate for loop pipes.

Each fraction is calculated as the inverse of the number of pipes connected to the node. This is a rough approximation of the water deficit to be used in the objective function. In the methodology below more accurate values of water deficit are calculated by simulation. For non-loop pipes the estimate amounts to the whole flow carried by the pipe.

The value of  $c_a$  represents an average cost of supplying water to affected consumers by other means such as the use of potable water trucks. When this is not the current practice it can be the subject of research for its determination as the value of temporal loss of service. Although for simplicity the value of  $c_a$  is held constant it will not be difficult to accommodate any known function of water deficit.

The sequence of solution is similar as before, after substitution of Equations (10) into Equation (13): first, calculate the flows in the pipes by the principle of minimum variance and, second, solve the latter OBF with constraint Equations (8) and (9). Further details can be seen in Martínez (2007).

## METHODOLOGY

The methodology applied in Martínez (2007) is now briefly explained. Actual network demand  $D_{\text{act}}$  is considered

a random variable which follows a given probability distribution with mean  $D_{\text{mean}}$  and coefficient of variation  $C_v$ . The probability distribution is approximated by discretization into a certain number of steps of equal width  $p_{\text{occur}}$  and variable height  $D_{ns}$ . A number of increasing design demand  $DD$  values are to be optimized and compared. The spatial distribution of demands is known and changing  $DD$  simply scales those values up and down.

Methodology steps are:

- Select design demand  $DD$  value in turn.
- Find diameter solution with optimization model explained above.
- Select demand step  $D_{ns}$  to be tested with simulator as demand load.
- Select next network state (no-failure state or one pipe failed).
- Apply simulator to test diameter solution against demand  $D_{ns}$  for given network state.
- Save shortfall data as well as other results from simulator.
- Go to step (d) until all states are done.
- Go to step (c) until all demand steps  $D_{ns}$  are chosen.
- Calculate total costs, reliability and pertinent results for current design demand  $DD$ .
- Go to step (a) until all design demand  $DD$  value are calculated.
- Select minimum cost design demand  $DD$  as the best solution.

In step (i) costs of additional shortfall and energy, arising from simulator results, are averaged and added to objective function costs. Averaging is accomplished over probabilities of network states and demand steps. While objective function costs increase with increasing  $DD$ , costs calculated after simulation are decreasing with increasing

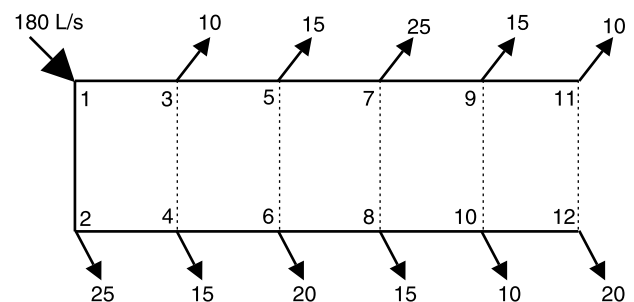


Figure 1 | Example five-loop network.

**Table 1** | General data for example network of (Figure 1)

$Pe^*$ (\$/kW h)	$tp^\dagger$ (h)	$C_1$	$\eta$	$m$	$10^5 a$	$u$	$t_f$ (d)	$c_f$ (\$/d)	$c_a$ (\$/m <sup>3</sup> )
0.05	7000	0.10	160.42	1.5	3.5	1.27	2.0	500	2.00

\*( $pe$  = energy price).

†( $tp$  = pumping time).

$DD$ ; thus an overall minimum cost  $DD$  is found. This best solution gives optimal values for design demand and reliability.

## EXAMPLE

A five-looped network is depicted in Figure 1. This is the same example as in Martínez (2007). General example data are in Table 1 and nodal topography appears in Table 2. Pressure requirement is 20 m for all nodes, length of all pipes is 400 m, the Hazen–Williams coefficient is 100 for all pipes and commercial diameters are available from 100 mm on, in increments of 50 mm. Actual demand follows the Pearson type III distribution ( $Cs = 2Cv$ ) with  $D_{mean} = 90l/s$  and  $Cv = 1/3$ . Discretization of the probability curve involved 10 steps which means  $p_{ocur} = 0.10$  and  $D_{ns} = K_D D_{mean}$  with  $K_D$  values shown in Table 3. Demand values shown in Figure 1 are for  $DD = 180l/s$ . Nodal demands for other values keep the same proportion with total demand.

Optimizations were made for  $DD$  values ranging from 110l/s to 160l/s with an interval of 10l/s. Table 4 has a

**Table 2** | Nodal topography

Node number	Elevation (m)
1	40
2	40
3	42
4	42
5	44
6	44
7	46
8	46
9	48
10	48
11	50
12	50

summary of cost results and Table 6 shows volume reliability  $R$  and tolerance  $T$  results. From Table 4 it can be seen that costs from the OBF are always increasing and the opposite occurs for the additional costs. The grand total cost in the last column of Table 4 reaches a minimum and then increases again. The minimum corresponds to  $DD = 150l/s$ . Reliabilities in Table 6 are all increasing with  $DD$ . The optimal values are, of course, those belonging to the minimum.

From Figure 1 a branched network is formed by removing pipes represented by dotted lines. This network is subjected to the same process described in the methodology. Recall that the branched network is also optimized with the same model. The cost results are given in Table 5 and reliabilities in Table 6.

Comparison of capital costs for looped (Table 4) and branched (Table 5) networks for each design demand show that they are higher for the branched network even when it has five pipes less. This means it has larger diameters with respect to the looped one. This is the effect of optimization trying to increase its capacity redundancy.

Similar comparisons of total cost from objective function (fourth column in Table 4 and Table 5) show that in all cases the looped network is less expensive. The same occurs

**Table 3** | Demand steps

Step	$K_D$
1	0.4998
2	0.6598
3	0.7571
4	0.8409
5	0.9211
6	1.0032
7	1.0931
8	1.1989
9	1.3516
10	1.6744



**Table 4** | Costs results for example looped network

Demand (l/s)	Costs from objective function (\$/yr)				Costs after simulation (\$/yr)			Grand total
	Capital	Energy	Failure	Total	Additional shortfall	Additional energy	Total additional	
110	12,108	12,819	5,289	30,216	18,455	6.05	18,461	48,677
120	13,062	13,866	5,237	32,165	9,384	2.25	9,386	41,551
130	13,290	15,171	5,438	33,899	6,100	1.09	6,101	40,000
140	13,582	16,459	5,683	35,724	3,048	0.34	3,048	38,772
150	14,606	17,441	5,716	37,763	240	0.06	240	38,003
160	15,096	18,620	5,797	39,513	99	0.04	99	39,612

**Table 5** | Cost results for example branched network

Demand (l/s)	Costs from objective function (\$/yr)				Costs after simulation (\$/yr)			Grand total
	Capital	Energy	Failure	Total	Additional shortfall	Additional energy	Total additional	
110	12,008	12,223	7,899	32,130	15,143	16.5	15,160	47,290
120	13,308	13,320	7,966	34,594	9,874	14.4	9,888	44,482
130	13,584	14,444	8,400	36,428	8,043	13.7	8,057	44,485
140	14,188	15,503	8,661	38,352	6,116	13.0	6,129	44,481
150	14,756	16,543	8,892	40,191	5,001	12.5	5,014	45,205
160	15,032	17,758	9,243	42,033	4,864	12.2	4,876	46,909

for the last column of the grand total in all cases except when demand is 110 l/s, in which the branched is slightly better by a margin of 3%. Observe that, in this particular case, the cost of additional shortfall is the one making the difference in favor of the branched network. By looking deeper into the results it was found that the major part of the shortfalls in this case were due to random demand exceeding capacity while minor shortfalls were due to pipe failure. The branched network, with larger diameters, behaved better under the higher  $D_{ns}$  demand steps.

Comparing the best cost alternative of each network it can be seen that the looped one is less expensive by some 17%. It is also interesting to note the maximum difference in grand total cost within each alternative, which for the looped network is about 28% while for the branched network it is only 6.3%. This appears to be determined by the drastic reduction in the cost of additional shortfall by the looped network with increasing design demand, in contrast to the modest reduction of this cost by the branched one.

As for reliability and tolerance, the values shown in Table 6 speak for themselves. For all design demands, reliability  $R$  for the looped network surpasses its corresponding one for the branched network. Nevertheless,

it can be seen that a branched network, optimally designed with the same model, can reach high values of reliability due to capacity redundancy. The sharp difference is observed by the tolerance index  $T$ : while the looped network can reach values very near 100%, the branched one always goes below 75%. As a final interesting remark, notice that the branched network—with reliabilities almost as high as the looped network—is particularly as expensive, or more, in its capital cost. For illustration Table 7 shows the diameter solution for both networks and design demands of 140 and 150 l/s; the average diameter comparison in Table 7 explains this last remark.

**Table 6** | Reliability and tolerance results for both networks

Demand (l/s)	Looped network		Branched network	
	$R$ (%)	$T$ (%)	$R$ (%)	$T$ (%)
110	99.7970	98.8280	99.7957	74.1212
120	99.8999	99.3511	99.8569	72.8675
130	99.9353	99.4600	99.8773	73.1015
140	99.9776	99.6305	99.8990	73.3914
150	99.9974	99.7828	99.9112	73.5501
160	99.9989	99.8349	99.9143	73.7636

**Table 7** | Some diameter solutions for both networks

Type		Looped network		Branched network	
Demand (l/s)		140	150	140	150
Pipe nodes					
$n_1$	$n_2$	Diameter (mm)	Diameter (mm)	Diameter (mm)	Diameter (mm)
1	2	350	350	450	450
1	3	350	400	350	350
2	4	300	300	400	400
3	4	200	250	–	–
3	5	300	300	350	350
4	6	300	350	350	400
5	6	200	200	–	–
5	7	300	300	300	350
6	8	300	300	350	350
7	8	200	200	–	–
7	9	250	250	300	300
8	10	250	250	300	300
9	10	150	150	–	–
9	11	200	250	300	300
10	12	250	250	300	300
11	12	200	200	–	–
Average diameter		256	269	341	350

## CONCLUSIONS

The goal of quantifying the economy of comparable branched and looped water supply networks has been addressed. While the economic advantage of a looped network over a comparable branched network by formulating a new objective function was demonstrated by Martínez (2007), the application of his full methodology in this paper confirms, with a detailed comparison, the cost convenience of looped networks over branched networks, even when both network types have been optimally designed with the same model. So far, it seems also that this is the first time in the literature where a branched network is analyzed under failure conditions.

A thorough discussion about reliability seems to point out the convenience of using a volume reliability type index as a preferable definition of reliability rather than other definitions invoking probabilities as the main index.

Nevertheless, the usual definition of the volume reliability index appears to be not fully trustable as was shown in the example, where a branched network can achieve very high values of the index. Some arguments and empirical evidence have been presented to support the use of the tolerance index as an indispensable measure of network performance during failure time and its seemingly irreplaceable role as a measure of performance of looped networks, and for comparing performance between branched and looped networks.

As for the three questions above, let the answers be written in order:

- (1) How can this redundancy be measured? As discussed previously, most reliability definitions found in the literature are not a good measure of redundancy. Following experience by the authors of the tolerance index (Tanyimboh *et al.* 2001; Kalungi & Tanyimboh 2003) and results shown in this paper, it seems that the best measure, existing so far, of the impact of redundancy is tolerance.
- (2) Which is the desirable degree of redundancy? The application of the methodology above leads to a minimum grand total cost design with an associated tolerance index as a fair measure of redundancy impact. This then should be the desirable–affordable–redundancy.
- (3) How large should the safety factor be considered for design demand? The solution just mentioned above corresponds to a given design demand which is again the affordable–optimal–one. If a value of a safety factor is desired, it can be calculated as the ratio between optimal design demand and average demand.

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