Nordic Hydrology, 17, 1986, 151-170

Dispersion in Ice-Covered Lakes

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Dispersion studies in three ice-covered lakes are presented. One of the lakes has a large river through-flow. In the other two lakes, small seiche-induced currents are generated through wind action on the ice cover. Dispersion coefficients are computed from five dye experiments lasting from ½ day to 16 days. These coefficients are related to measured current velocities. The turbulence in the river-flow dominated lake is found to be very low with disperision coefficients less than 1 cm^2/s in directions transverse to the flow. In the lakes where the turbulence is generated by shear from the seiche-induced currents the horizontal dispersion coefficients are two orders of magnitude higher, being almost as high as in ice-free lakes.

Introduction

An environmental engineer, who has to evaluate the effect of an effluent release in ice-covered lakes, has to rely to a large extent on formulas and coefficients derived for ice-free conditions. The ice cover reduces heat losses to the atmosphere and prevents direct generation of wind mixing. Currents and mixing are generated by heat flow from the bottom sediments causing very slow bottom currents, indirectly by the wind causing the ice cover and the water mass below to oscillate with a small amplitude, and by through-flowing rivers. If the through-flow is significant, intuitively the through-flow induced currents should be the dominating ones. In this paper, first a general discussion on circulation in ice-covered lakes is presented, followed by a discussion on dispersion. After that, the problem of dispersion under ice is treated. Experiments from three lakes are reported. The results of the experiments are compared with formulas commonly used in practical applications.

Circulation under Ice

Through-Flow Currents

During the winter, river water has a temperature very close to 0°C. A large river flowing through a rather deep lake gives rise to a pronounced vertical density gradient. The through-flow takes place between the underside of the ice and the developed thermocline. An example of an observed temperature profile in an ice-covered lake with large through-flow relative to the lake water volume is shown in Fig. 1 (p. 156): Råne River with a winter discharge of somewhat less than $10 m^3/s$ is flowing through Lake Prästholm, which has an area of 1 km^2 and a mean depth of about 10 m. When the temperature profile was measured, the through-flow currents were registered to about 1 cm/s in the lake proper, evenly distributed between the ice and the thermocline at 6 m depth, but close to the shores the currents were too low to be measured by the instrument used.

The effect of inflow of river water on the circulation in ice-covered Lake Sperillen, Norway, was discussed by Tesaker (1973) and Stigebrandt (1978). Thinner ice was observed along the right-hand shore relative to other parts of the lake. Tesaker suggested this was due to the upwelling of warm water by secondary currents, which were induced as an effect of the Earth's rotation. Stigebrandt found indications of a concentrated current along the right bank close to the river inlet only. Current measurements taken by an ultrasonic current meter in the lake proper showed a rather uniform flow distribution; the highest velocities, 0.02 m/s, were found in the central part of the lake.

Current measurements with drogues in holes in the ice were carried out in Lake Pyhäjärvi, Finland, by Virtanen, Forsius and Sarkkula (1979). The lake is 9 km long, about 2.5 km wide and the mean depth is 8 m. Due to the relatively large through-flow, 64 m^3/s even during winter months, no stratification was observed. Typical velocities were less than 0.01 m/s and fairly uniform over the cross section. An observed seiche period of 100 min was reported.

Sediment-Heat Generated Currents

Heat is stored in the bottom sediments from summer to winter. During the winter heat is released to the water. In an ice-covered lake the heat loss to the atmosphere is reduced, resulting in increased water temperature throughout the winter. Bottom water which gains heat from the sediments near the shores moves along the bottom towards the deeper parts of the lake. Convective circulation cells develop. Likens and Ragotzkie (1965) have measured vertical currents under ice and found velocities of the order 0.01-0.1 m/d.

Wind-Seiche Induced Curents

When the wind acts on an ice cover, the ice cover as well as the water surface is tilted. When the wind ceases to blow the adjustment of the ice and the water surface takes place as an oscillation, but oscillations are superimposed also on a stationary tilting surface. The period of such oscillations is so short that during a period water particles move only a very short distance back and forth. The net movement at each position is close to nil, but the oscillations may produce a weak circular horizontal circulation pattern. During the back-and-forth movement the water is mixed to a higher degree than what would be expected from the net horizontal currents.

Dispersion Theory

Open Channels

Dispersion in open channels is usually calculated from information about the velocity field using turbulent diffusion coefficients. Often the velocity gradients are excluded from the diffusion equation and instead the advective dispersion is included in the turbulent diffusion terms by using dispersion coefficients. The dispersion coefficients are related to a length and a velocity scale, usually chosen as the hydraulic radius and the friction velocity

$$D \equiv c R U_F \qquad (1)$$

where

D – dispersion coefficient
R – hydraulic radius
U_F – friction velocity
c – a coefficient, which should be 0.07 for isotropic turbulent diffusion.

For horizontal dispersion transverse to the flow in a uniform channel, experiments have given values of the coefficient of 0.1-0.2, cf. Fischer (1973). In the flow direction the coefficient is about 6 for a uniform rectangular channel, but for rivers it may be up to 1,000 depending on the irregularity of the river configuration.

The friction velocity can be determined from a friction formula relating friction loss to velocity. Using a friction factor, the friction velocity can be related to stream velocity as

$$U_F \equiv \left(\frac{f}{8}\right)^{\frac{1}{2}} u \qquad (2)$$

where

f – friction factor
u – stream velocity.

Ice-Free Lakes

Also for lakes, dispersion coefficients are estimated from a friction velocity and a length scale. When the dispersion is caused by the wind, the hydraulic radius must be replaced by the depth down to which the wind is acting, which for not very deep lakes is the mean depth of the lake or the depth of the thermocline. The friction velocity is determined from the wind shear on the water surface. Measured horizontal dispersion coefficients in lakes are usually reported to be in the order of 100-1,000 cm^2/s for small lakes, e.g. Ottesen-Hansen (1978), and 1-2 orders of magnitude higher for very large lakes such as the Great Lakes of North America, e.g. Csanady (1975). The vertical coefficients are found to be only 1-10 cm^2/s or less.

Three-dimensional turbulence, including turbulence in lakes, should be characterized by Eq. (1) with a coefficient of about 0.1. From the continuity equation applied to the velocity fluctuations it follows that the turbulent scales for horizontal fluctuations must be of the same order as those for vertical fluctuations. »True« turbulence requires three-dimensional vorticity fluctuations and dissipation, Teenekes and Lumley (1972). The high proportionality coefficients required to fit measured horizontal dispersion coefficients in lakes are probably due to the effect of large horizontal eddies, often called two-dimensional turbulence.

Oscillatory Movements

Oscillatory movements induced by wind setup may be of some importance for the mixing of contaminants. Since oscillatory currents from tidal effects are present in estuaries, it is relevant to consider methods for estimating dispersion coefficients in tidal influenced estuaries.

A one-dimensional dispersion coefficient in the homogeneous tidal zone in an estuary can according to Harleman (1966) be approximated from

$$D = 63\, n\, u\, R^{\frac{5}{6}} \tag{3}$$

where

n – Mannings n.

The equation is based on steady uniform flow involving a logarithmic distribution of flow velocities. It can also be derived by simply combining Eq. (1) and Mannings equation. To derive to a coefficient of 63 in Eq. (3), the coefficient of Eq. (1) must be $c = 20$. This is also the coefficient given in an earlier study by Holley and Harleman (1965).

The dispersion coefficient in the transverse direction to a tidal current is 1-2 orders of magnitude less than in the longitudinal direction. Fischer (1976) has summarized transverse mixing coefficients from some previously published data and has found the coefficient of Eq. (1) to vary only within the range of 0.4-1.7. When Ward (1974) investigated straight and meandering channels, he found that the transverse diffusion coefficient over many tidal cycles could be determined from Eq. (1) with the proportionality factor in the range of 0.21-0.27 and the friction velocity determined from the maximum tidal current. If the mean absolute current velocity is used, the proportionality factor is about 0.4.

Covered Channels

While numerous investigations have been carried out on dispersion in open channels not much attention has been devoted to covered channels. Engman (1977) determined lateral diffusion coefficients in a laboratory flume with simulated ice cover. When accounting for the resistance against the surface cover and the decreased hydraulic radius in the presence of the cover he found the proportionality coefficient of Eq. (1) to range between 0.15 and 0.2.

Ice-Covered Lakes

In an ice-covered lake with large through-flow the turbulent diffusion process should be similar to the one in a covered channel. When the flow is restricted between the ice and a thermocline, the friction velocity must be determined from the shear stress at the underside of the ice and the shear stress at the thermocline. When the turbulence is generated by oscillatory movements, the maximum velocity or the absolute value of the velocity at every moment must be used in estimating the dispersion coefficient but not an average value, which is very close to nil.

The only dispersion study from an ice-covered lake known to the author was conducted in the small Tub Lake, Wisconsin. The lake has a conical shape with a diameter at the surface of less than 100 m and a maximum depth of 8 m. There is no inlet or outlet. Colman and Armstrong (1983) estimated dispersion coefficients using data collected in a radio-isotope tracer experiment. The horizontal dispersion coefficient was found to be 0.47 cm^2/s, which is two orders of magnitude lower than what would have been expected for ice-free conditions.

The Different Dispersion Experiments

Dispersion experiments from three ice-covered lakes are presented. One of the lakes, Lake Prästholm, is dominated by through-flow, one is a deep bay of Lake Mälaren, and the last one is Lake Erken, which during the winter has no inflow. In Lake Erken experiments were conducted in the central part of the lake as well as close to the shores. In all the experiments, the dye Rhodamine B was used as a tracer. The tracer concentration was measured by a Turner flouorometer.

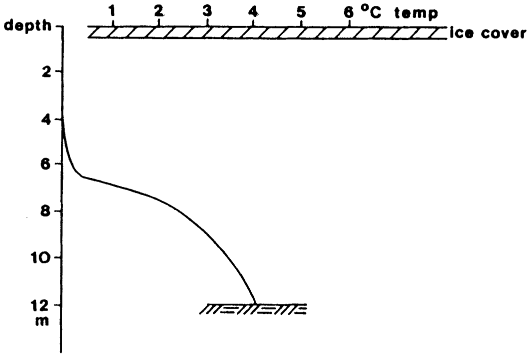

Fig. 1. Temperature profile,
Lake Prästholm,
April 1978.

Field Site – Lake Prästholm

One of the investigated lakes was Lake Prästholm in the Råne River basin about 20 km upstream the Gulf of Bothnia. The lake is 1.3 km long, 0.8 km wide, and has an average depth of about 10 m. The river inlet is somewhat less than 100 m wide and about 1.5 m deep. The outlet was very shallow and narrow, about 0.35 m deep and 20 m wide, at the time the experiment was conducted. The flow through the lake during winter is 6-10 m^3/s. The dye experiment was carried out in late April, 1978. The ice thickness of the lake proper was about 0.6 m, which was also observed at the very edge of the ice-cover at the outlet. There was a sharp thermocline at a depth of about 6 m. The temperature profile from the day of the measurements is shown in Fig. 1. Current measurements were made using gelatine pendulums, cf. Haamer (1974), and an ultrasonic meter. Currents of magnitude less than 0.005 m/s could not be detected. In the central part of the lake, where the dye was released, the current velocity was about 0.01 m/s from the bottom of the ice cover down to the thermocline, but closer to the shores no measurable currents were observed. The magnitude of the current was confirmed by the fact that the travel time of the dye from the central part of the lake to the outlet corresponded to a velocity of 0.01 m/s. The mean velocity over the cross section was determined from the known discharge to 0.002 m/s.

In the central part of the lake, 350 m from the outlet, the dye was continuously released 3 m below the water surface. Downstream the point of release, holes were drilled through the ice with 1 m lateral and 5 m longitudinal spacing to 25 m from the point of release. Some holes were also made upstream the point of release. The concentration of the dye was measured in the holes at depth intervals of 0.5 m. The total number of drilled holes was more than 50, but in most of them no dye was detected.

To maintain a continous release of dye, an insulated tank sheltered from the sun

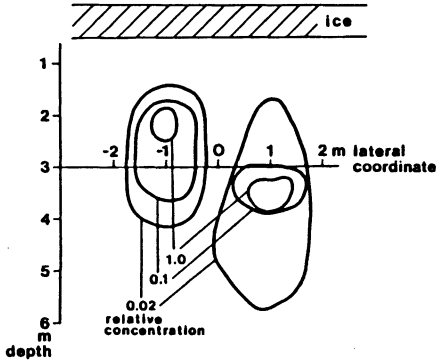

Fig. 2. Concentration distribution of dye 15 m downstream the point of release measured 2 and 2½ hours after the continuous release was started.

and containing a solution of Rhodamine-ethanol and lake water, having a density corresponding to the lake water above the thermocline, was hoisted above the ice. From the tank, the dye was introduced at constant head into the lake water at 3 m depth and directed downstream through a hose with a nozzle at the end. The experiment lasted for about 8 hours. The time interval between concentration measurements in a specific drilled hole was 1-3 hours.

The measured dye concentration in a section transverse to the flow showed high values at one depth in one or two holes and close to background concentration in all the other holes in the cross section. The results were similar for all cross sections regardless of their longitudinal position. An example of measured concentrations is shown in Fig. 2. The dye did not spread much, neither in the lateral nor in the vertical direction, but the position of the point of maximum concentration in a cross section varied during the experiment. The distance over which the point of maximum concentration varied was horizontally about 3 m, and vertically about 1.5 m. These distances are one order of magnitude too large to be explained by back-and-forth movements with a seiche transverse to the lake. The meandering of the centre-line must have been caused by large-scale horizontal eddies.

By relating the concentration at a given time to the point of maximum concentration in a cross section, it is possible to determine the standard deviation of the dye concentration and to estimate dispersion coefficients relative to the centre of the dye plume. The variance of the dye concentration in a direction (z) transverse to the flow was computed as

$$\sigma_z^2 = \frac{\iint c(y,z)\, z^2 dy\, dz}{\iint c(y,z)\, dy\, dz} \qquad (4)$$

where

c – concentration

σ_z – standard deviation of concentration in the indexed direction

z and y being coordinates transverse to the flow.

Down to 25 m, at least, from the point of dye release, the dye in the instantaneous plume was restricted within a width of 2 m and a vertical distance of 4 m. Since the spacing between the grid points was 1 m laterally, it was not possible to calculate a horizontal lateral standard deviation, but only to give a maximum possible value. In the vertical, the spacing between measuring points was 0.5 m, which allowed at least a crude estimate of the standard deviation of the dye concentration. As can be seen directly from Fig. 2, from the fact that it was less than 2 m between points of only background concentration, the lateral standard deviation was less than 1 m. The vertical standard deviation, as estimated from the dye measurements, was found to vary from one occasion to another. The standard deviation increased with the distance downstream the point of release. At 10 m the vertical standard deviation varied between 0.25 and 0.40 m and increased at each downstream measuring point to 0.40-0.60 m at the position 25 m downstream the point of release. The dispersion coefficient can be determined from the standard deviation of the concentration as

$$D = \frac{u}{2} \frac{d\sigma^2}{dx} \tag{5}$$

where

D – dispersion coefficient
u – longitudinal velocity
x – longitudinal coordinate.

At 25 m downstream the point of release the standard deviation of concentration in the lateral direction was less than 1 m. Since the flow velocity was about 0.01 m/s, it is clear that the relative lateral dispersion coefficient was less than 2 cm^2/s.

In the vertical direction the standard deviation between 10 and 25 m from the point of release increased by a value in the range of 0.15 to 0.35 m, with an average increase of about 0.30 m. The relative dispersion coefficient as computed from Eq. (5) varied in the range of 0.05 to 0.25 cm^2/s, with an average value of 0.18 cm^2/s.

The concentration of dye was also measured close to the outlet of the lake, at 350 m downstream the point where the dye was released. At the outlet one could observe dye emerging in streaks under the edge of the ice cover, which gave an indication that the flow was not a fully developed turbulent flow.

At a position 20 m from the edge of the ice cover, the dye concentration was found to be distributed between the ice cover and a depth of 3 m. The vertical as well as the lateral standard deviations of dye concentration in the cross section were estimated to 0.5-1 m. Using Eq. (5) and $u = 0.01$ m/s, the dispersion coefficient can be estimated to less than 0.3 cm^2/s in both directions transverse to the flow.

Theoretically, a turbulent diffusion coefficient can be determined from a length scale and a friction velocity using Eq. (1). Turbulent mixing excluding shear effects should be described with a coefficient of the order 0.1. However, when Eq. (1) is used to characterize advective mixing or large scale horizontal turbulence the coefficient is much higher. In the ice-covered Lake Prästholm, the dominating shear stress is that against the underside of the ice. The friction factor for the underside of a stable ice cover is about 0.01, e.g. Larsen (1973). The hydraulic radius in Lake Prästholm should be between the half and the full distance between the underside of the ice and the thermocline, which was about 5 m, depending on friction conditions at the thermocline. Knowing the friction factor and the stream velocity, 0.01 m/s, the friction velocity is from Eq. (2) computed to 0.4 mm/s. This value of the friction velocity and a hydraulic radius of 2.5 m inserted into Eq. (1) and using $c = 0.07$ give an isotropic diffusion coefficient of 0.7 cm^2/s. The theoretically computed coefficient is of the same order of magnitude as the ones determined from the experiment.

The estimated dispersion coefficients are approximate. The data only allow the statement that the coefficients are of the order of 1 cm^2/s and probably smaller than that. However, the poor resolution of the observations does not compromise the significance of the low value of the diffusion coefficients in both directions transverse to the flow.

Field Site – Lake Mälaren

The dispersion experiment in Lake Mälaren was carried out near the Bay of Kalmar in late February and early March 1984. The Bay of Kalmar is very deep, 30-50 m, and so are the wide entrance to the bay and the area of Lake Mälaren offshore the bay. The lake water was very cold, about 0.5 °C at 15 m depth increasing to 1°C at 35 m depth. The thickness of the ice cover was about 0.3 m. Dye was released continuously during one day at the sewage plant at Bålsta, from which water is released at a depth of 16 m. The dye, considered as instantaneously released, was traced over a period of 16 days. The extension of the dye cloud as estimated from measurements with the fluorometer is shown in Fig. 3. The centre of the cloud hardly moved at all during the observation period. During the first 10 days the dispersion of the dye was not affected by the shores, but during the last days of the observation period, after having reached the bay itself, the cloud became very elongated.

The Bay of Kalmar is 1 km wide and extends over a length of 2.5 km. At the time of the dye experiment, seiche periods of 20 minutes and – with very small amplitude – of 10 minutes were observed at the innermost part of the lake. At the 500 m wide and 40 m deep entrance to the bay, the currents under the ice were measured by drogues in a wind-protected hole in the ice. The drogues positioned at 2 and 4 m depth moved back and forth with a period of 20 minutes. The observed mean current over half a period was 3.6 mm/s, but is dependent on the amplitude

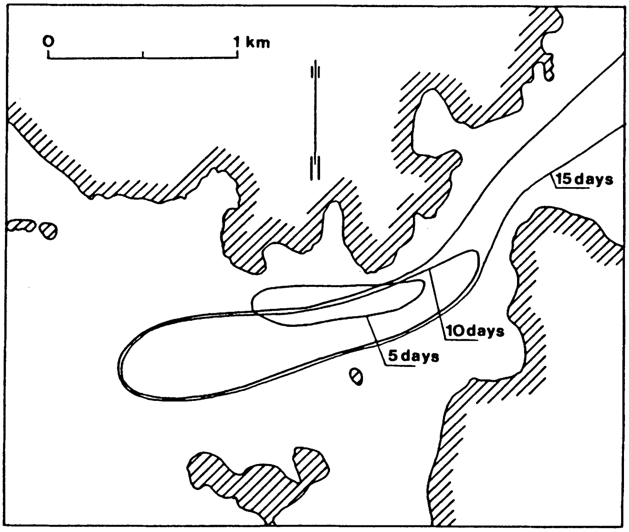

Fig. 3.
Dye cloud extension in Lake Mälaren outside Bålsta, 5, 10, and 15 days after dye injection.

of the seiche.

The vertical dispersion of the dye was found to be small. By comparing the vertical standard deviations computed from Eq. (4) at the cross section of maximum concentration, two and five days after the termination of the dye release, the vertical dispersion coefficient from the equation

$$D \equiv \frac{1}{2}\frac{d\sigma^2}{dt} \tag{6}$$

where t – time coordinate, was found to be about 0.5 cm^2/s. The data were too crude to allow estimates of the dispersion coefficient for the last period of the experiment.

In the horizontal direction the standard deviations of the dye concentration distribution were estimated from vertically integrated concentration values within the dye clouds elongated in the direction of the seiche current. From the increase of the estimated standard deviation, the lateral dispersion coefficient was computed from Eq. (6) to be 30-40 cm^2/s over the entire period that measurements were made. This is one order of magnitude higher than what is found from Eq. (1) using the seiche current for determining the friction velocity and taking the hydraulic radius as half the depth. The high value of the dispersion coefficient may be attributed to large-scale horizontal circulation caused by the turn of the seiche current or secondary currents caused as a shore effect or as a consequence of the Earth's rotation.

The coefficient describing the dispersion in the direction of the seiche current – the longitudinal direction – was computed to increase with time; from 900 cm^2/s after 5 days to 7,000 cm^2/s after 13 days. The data fit reasonably well to the 4/3-law of diffusion,

$$D = \alpha \sigma^{\frac{4}{3}} \tag{7}$$

with the constant α being about 0.0024 $\text{cm}^{2/3}/\text{s}$. In the literature, the constant for large ice-free lakes and coastal areas has been given values in the range of 0.002-0.01, e.g. Fischer *et al.* (1979) and the diagrams presented by Okubo (1974).

Field Site – Lake Erken

In Lake Erken, which is situated east of Uppsala very close to the Baltic, two dye experiments were carried out in the central part of the lake and two experiments very close to the shore. Lake Erken is about 9 km long and the width is 2-3 km with a maximum depth of 20 m. The eastern part of the lake, where the experiments of the "central part" were made, is a large open surface 2.5 × 5.5 km with no islands and a depth varying only in the range of 12-15 m. The bottom configuration of the lake is shown in Fig. 4.

In Lake Mälaren it was found that seiche currents were induced below the ice cover and that these currents contributed to the dispersion of dye although there was no net current. To investigate movements under the ice in Lake Erken, currents were measured in the same way as in Lake Mälaren by drogues floating in wind-protected holes in the ice. A measure of the magnitude of the current and of the period of the seiche was obtained. The period was also obtained from observations of the vertical movement of the ice cover.

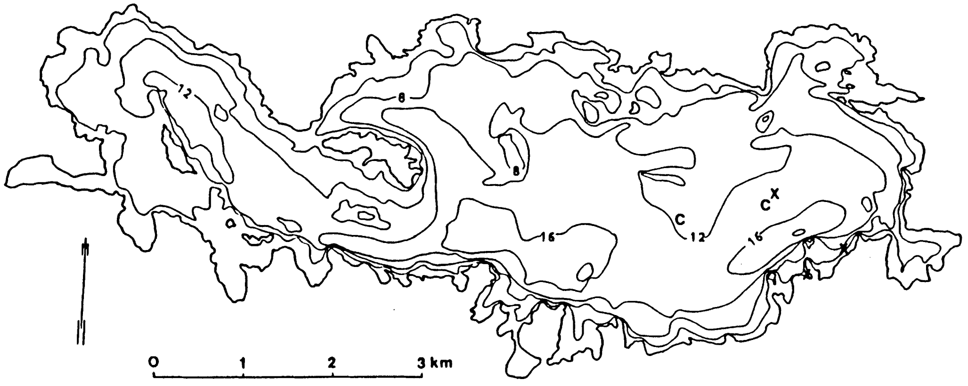

Fig. 4. Bottom configuration of Lake Erken and positions where measurements were made, C for current measurements and X for dye measurements.

Central Part – First Experiment

The first dye experiment in Lake Erken was carried out in April 1984. There was a sharp water temperature gradient just under the 0.3 m thick ice, but from 0.5 m below the underside of the ice down to 7 m the temperature was about 1.6°C. From this level the water temperature increased to 4°C at 15 m.

The observations of the vertical movements of the ice cover revealed a period of 20-22 min. Also the drogues measurements showed a period of about 20 min for the observed seiche current. However, at the positions where drogues measurements were made, cf. Fig. 4, the back-and-forth current was superimposed on a net current. The net current at 1 m depth was 0.5 mm/s, directed eastward, but the mean value of the magnitude of the velocity was 2.4 mm/s. When the drogue measurements were made the wind was blowing from SW at a speed of 8 m/s.

Late in the afternoon, after the drogues measurements had been carried out, dye was instantaneously released in one of the holes in the ice where the drogues measurements were made. The wind had increased and remained SW 8-11 m/s during the dye experiment. The released dye was vigorously mixed by the aid of a pump so that complete mixing was obtained from the ice down to 7 m depth. The dispersion of the dye cloud was followed for 26 hours by measuring dye concentrations in drilled holes in the ice. The dye remained vertically well mixed down to 7 m and did not spread below this level. The dye was found to move north, i.e. transverse the observed seiche current, with a speed of 3 mm/s. The dye cloud had a circular shape. The horizontal concentration distribution 23-26 hours after the dye injection is shown in Fig. 5. The total amount of injected dye was 10 ml Rhodamine B, of which 95% was found above 7 m depth within the marked cloud area in Fig. 5.

It was found that the dye spread in the same way in all horizontal directions relative to the cloud centre. The standard deviation of dye concentration was found to grow approximately linearly with time, indicating that the horizontal dispersion coefficient was constant. From Eq. (6) a value of 120 cm^2/s was computed. This value is higher but of the same order as the lateral dispersion coefficient found in Lake Mälaren, but very different from the longitudinal values for Lake Mälaren.

The fact that the dispersion coefficient is constant enables the use of an analytical solution to the diffusion equation

$$c = \frac{M/d}{4\pi t D} \exp\left(- \frac{r^2}{4Dt} \right) \qquad (8)$$

where

M – total released mass
d – depth over which the mass is distributed
r – distance from centre of mass.

Best agreement with the analytical Gaussian concentration distribution was found

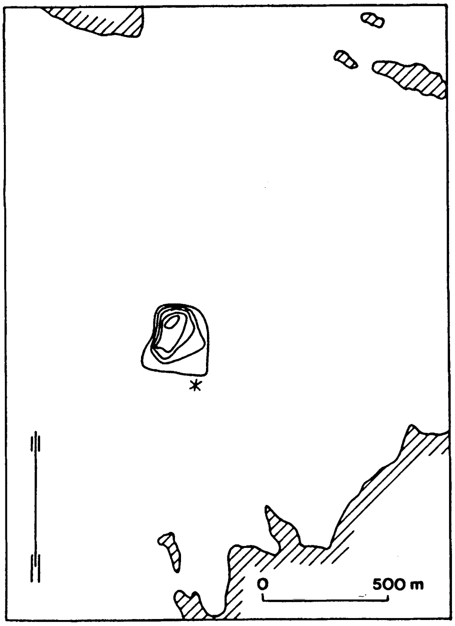

Fig. 5.
Dye concentration distribution in Lake Erken,23-26 hours after the injection at the position marked by a star. Relative concentrations 1.0, 0.5, 0.35, 0.2, and 0.1 are marked.

for $D = 180\ \mathrm{cm^2/s}$. The Gaussian distribution showed too high concentrations close to the centre of the cloud and too low concentrations far from the centre when compared to the observed dye distributions.

From the net current 3 mm/s a friction velocity can be estimated to 0.11 mm/s. Taking the vertical extension of the dye cloud as the hydraulic radius or as a length scale and using a coefficient of 0.15, the dispersion coefficient computed from Eq. (1) is 1 $\mathrm{cm^2/s}$. This is two orders of magnitude less than the horizontal dispersion coefficient estimated from the dye experiment. However, Eq. (3) using $n = 0.025$, cf. Larsen (1973) gives a dispersion coefficient of 240 $\mathrm{cm^2/s}$, which is of the same magnitude as the experimentally determined coefficient. The horizontal dispersion was probably affected by secondary currents or large-scale horizontal gyres. The fact that the depth integrated dye cloud movement was transverse to the observed seiche current at 1 m depth indicates a complex circulation pattern. It should, however, be noted that the drogues measurements were made prior to the dye release.

Central Part – Second Experiment

The dye experiment and the drogues measurements in the central part of Lake Erken were repeated in late February 1985. The water temperature profile under the 0.4 m thick ice cover was as shown in Fig. 6. There was a rather sharp temperature gradient at about 12 m. The temperature increased from 0.9°C at 10 m to 2.0°C

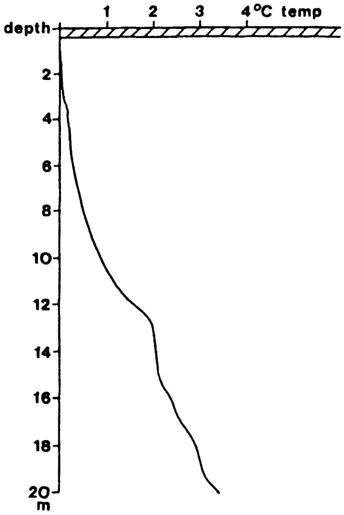

Fig. 6. Temperature profile in Lake Erken, late February 1985.

at 13 m. Drogues measurements were carried out approximately at the same place as where the drogues measurements of the previous year were made and where dye was released. The wind was WSW 2-3 m/s. The measured current velocity fluctuated with a period of about 20 min, but the current was at every moment directed towards SSW. The mean current velocity was 1.6 mm/s. Dye was released instantaneously in the hole where and at the day when the drogues measurements were made. The dye was mixed vertically during the injection. Complete mixing was obtained between 2 and 5 m depth, within which levels the dye remained during the experiment. The dye was traced for two days during which the wind was SSW about 3 m/s. The centre of the dye cloud moved in a SW direction at a fairly steady speed of 1.3 mm/s, which is in agreement with the drogues measurements.

The dye cloud was, as in the previous experiment, found to spread relative to the cloud centre at about the same rate in all horizontal directions. The horizontal distribution of dye concentration one and two days after the injection is shown in Fig. 7. The horizontal dispersion coefficient was computed to be about 15 cm^2/s. This is less than the value of the dispersion coefficient obtained in the first experiment in Lake Erken. The lower value can be explained by the fact that the wind speed was much lower, giving rise to lower seiche currents, and the water more, though still very little, stratified.

Shore Experiments

The two dye experiments close to the shore were performed simultaneously in mid March 1985 at positions marked in Fig. 4. The dye solution was injected close to the bottom at 2.5 m depth at a density exceeding that of the bottom water in order to enable studies of the heat-flow induced bottom currents and of the convective

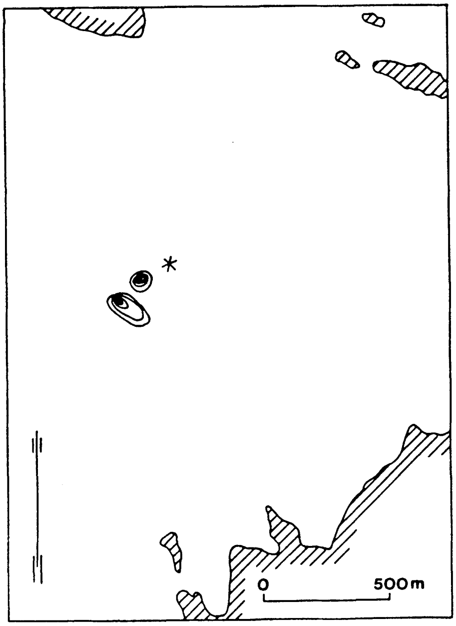

Fig. 7.
Horizontal dye concentration distribution (rel. conc. 2.0, 1.0, 0.5, 0.2, 0.1) approximately 1 and 2 days after injection at the X-marked position, Lake Erken, February 1985.

vertical mixing. However, it was not possible to inhibit rather intense vertical mixing when the dye solution was introduced into the lake water. At both shore positions the injection resulted in complete vertical mixing over the 2.5 m depth. Therefore, only the horizontal dispersion was studied.

The dye at the shore positions was traced for two and three days, respectively. The wind was E about 3 m/s, never exceeding 5 m/s, during these three days. At the western shore position where the dye was enclosed in a bay, the dye spread at a rather uniform rate in all horizontal directions. The dye cloud three days after the dye injection is shown in Fig. 8. Already half a day after the injection the point of maximum concentration had moved 40 m offshore from the point of release, but stayed there until the measurements were terminated. The horizontal dispersion coefficient was 15 cm^2/s, not growing with time.

At the eastern shore injection position the centre of the dye cloud moved very slowly, 0.4 mm/s, along the shore in a westerly direction. The cloud was elongated in the longitudinal direction of the lake as shown in Fig. 9. In the direction lateral to the movement of the cloud centre the dispersion coefficient was computed to be constant 5-10 cm^2/s, but in the longitudinal direction the dispersion coefficient was found to increase with time. The concentration relative to the centre of the cloud decreased faster in the »downstream« direction than in the »upstream« direction. The dispersion of the dye was affected by a seiche transverse to the lake, which was clear from 10 minute period fluctuations of dye concentration.

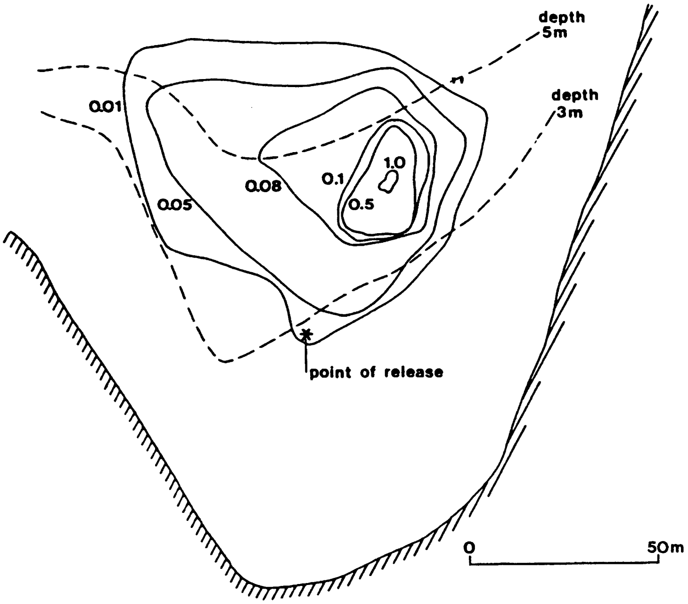

Fig. 8. Concentration distribution at the western shore position in Lake Erken approximately 3 days after injection, March 1985.

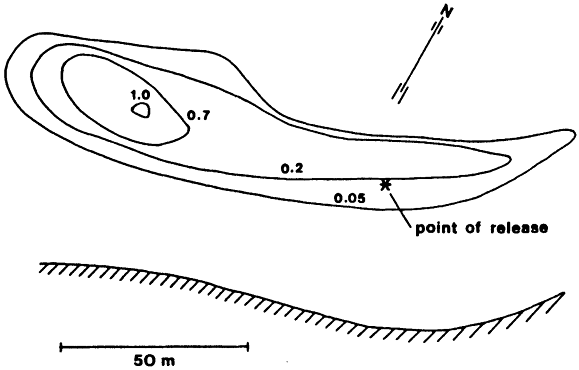

Fig. 9. Concentration distribution at the eastern shore position in Lake Erken approximately 2 days after injection, March 1985.

Table 1 – Dye experiments in Lake Erken 1984 and 1985

	central/84	central/85	shore/west	shore/east
depth (m)	15	15	3	3
temperature gradient (°C/m)	–	0.1	–	–
dye cloud thickness (m)	7	3	3	3
wind direction (from)	SW	WSW	E	E
wind speed (m/s)	7-10	2-3	3	3
current direction (towards)	N	WSW	–	WNW
net current velocity (cm/s)	0.3	0.13	–	0.04
dispersion coeff (cm^2/s)	120	15	15	10*

* transverse to the net current, no constant value was determined in the longitudinal direction

Summarizing the Lake Erken, Experiments

The data from the Lake Erken experiments are summarized in Table 1.

As previously argued, dispersion coefficients of constant value should be related to a length scale and a velocity scale. When the scales are described by the hydraulic radius and the friction velocity, a proportionality coefficient of 20 is suggested by Holley and Harleman (1965) to be valid for tidal flows in estuaries. No theory for seiche-induced mixing seems to be available in the literature. The period of tidal flows exceeds the seiche period in Lake Erken by almost 2 orders of magnitude, but the theory for tidal flows should include the increased mixing at the turn of the flow. Actually a coefficient of 20 should be used for characterizing the mixing in the flow direction. If the approach suggested by Holley and Harleman is used also for characterizing the overall horizontal dispersion in the central part of Lake Erken, choosing the hydraulic radius as the depth over which the dye was distributed, the theoretical dispersion coefficients should be 240 cm^2/s for the situation of 1984 and 30 cm^2/s for the situation of 1985, which although twice as large is in reasonable agreement with the dispersion coefficients 120 and 15 cm^2/s determined from the dye concentration distributions.

Generalization of the Dye Experiments

The turbulence in an ice-covered lake is generated through the friction against the underside of the ice or against the bottom from currents induced by a through-flowing river or by wind-induced seiches. Although not explicitly treated in this paper, also buoyancy-generated turbulence through heat flow from the bottom sediments is believed to be of main importance in most parts of a lake. Even if the seiche-generated mixing is indirectly wind-generated, the turbulence is, just as for the through-flow situation, internally shear-induced. A considerable flow distance

Table 2 – Physiographical and meteorological data from the investigated lakes and results of the field experiments

	Lake Prästholm	Lake Mälaren	Lake Erken-84	Lake Erken-85	Lake Erken/shore
length[km]	1.2	4	5	5	
width [km]	0.8	1	2.5	2.5	
depth [m]	10	40	15	15	3
thermocline depth [m]	6	–	7	12	–
dye cloud thickness [m]	4	10	7	3	3
river flow/cross section [mm/s]	2	–	–	–	–
$\delta T/\delta z$ [°C/m]	0	0.02	0.04	0.1	0.1
wind speed [m/s]	0	–	8	3	3
extension of experiment [days]	1/2	16	1	3	3
net current [mm/s]	10	0	3	1.3	0/0.4
seiche period [min]	–	20	20	20	20/10
seiche current [mm/s]	–	3.6	2.4	–	–
D(vert) [cm^2/s]	0.5	0.5	–	–	–
D(lateral) [cm^2/s]	<1	40	–	–	10
D(hor) [cm^2/s]	–	–	120	15	15
D(long) [cm^2/s]	–	increasing	–	–	increasing
α (4/3-law) [$cm^{2/3}/s$]	–	0.0024	–	–	–

is required before the turbulence is developed over the full water depth or down to the depth of the thermocline. Ottesen-Hansen and Rasmussen (1979) suggest that the displacement thickness of a turbulent boundary layer increases proportionally to the flow distance with a proportionality factor of 1/70. This means that 700 m from a river inlet the turbulent flow should reach a depth of 10 m unless it is restricted by bottom or density conditions. Therefore, the turbulent flow in Lake Prästholm should have been fully developed down to the thermocline. In the other lakes there was no indication of a turbulent boundary layer of limited thickness close to the ice overlying a non-turbulent bottom layer.

Turbulent diffusion and dispersion are for engineering purposes described by dispersion coefficients, which are related to a length scale and a friction velocity. The friction velocity is determined from the shear against the bottom and the ice cover. When the spreading is restricted within limited depths far from the bottom and the ice cover, as for the experiment in Lake Mälaren and one of the experiments in Lake Erken, it can be argued that the dispersion and the related coefficients should be determined from the flow and turbulent conditions at these depths. Since the observed data are crude, it was not felt meaningful to try to perform a detailed analysis of the turbulent conditions.

Physiographical data and results obtained from the experiments in all the lakes are summarized in Table 2. The vertical dispersion is seen to be characterized by a dispersion coefficient less than 1 cm^2/s. The lateral dispersion is very small in the river shear induced flow below the ice cover in Lake Prästholm. The dispersion coefficient is almost two orders of magnitude higher for the seiche-induced flow than for the river-induced flow. This high coefficient is lower but not far from what is expected for small ice free lakes.

Conclusions

The findings from the three investigated lakes can be transferred to other ice-covered lakes only in a qualitative way. When through-flow induced currents are present, the turbulence level is very low. Dispersion coefficients are of the order 1 cm^2/s or less. Pollutants may run through the lake with the river water without much mixing. In lakes having no significant through-flow seiches are generated through wind action on the ice cover. The turbulence and the dispersion generated by the seiche currents are lower but may be comparable in intensity with conditions in ice free lakes.

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Received: 13 March, 1986

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