and (A16):

$$J_1\big|_0^{\pi} = \pi \sin^{-1}\bar{\epsilon} \tag{A20}$$

$$J_2\big|_0^{\pi} = \pi[(1-\bar{\epsilon}^2)^{1/2} - 1] \tag{A21}$$

$$J_3\big|_0^{\pi} = \frac{\pi}{2}[\sin^{-1}\bar{\epsilon} - \bar{\epsilon}(1-\bar{\epsilon}^2)^{1/2}] \tag{A22}$$

References

1 Banerjee, B. N., and Burton, R. A., "An Instability for Parallel Sliding of Solid Surfaces Separated by a Viscous Liquid Film," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Jan. 1976, pp. 157–166.

2 Hughes, W. F., Winowich, N. S., Birchak, M. J. and Kennedy, W. C., "Phase Change in Liquid Face Seals," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 100, Jan. 1979, pp. 74–80.

3 Etsion, I., "Nonaxisymmetric Incompressible Hydrostatic Pressure Effects in Radial Face Seals," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 100, July 1978, pp. 379–385.

4 Etsion, I., "Hydrodynamic Effects in a Misaligned Radial Face Seal," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 101, July 1979, pp. 283–292.

5 Ludwig, L. P., and Gordon, P. A., "Face Seal Lubrication. II. Theory of Response to Angular Misalignment," NASA TN D-8102, Mar. 1976.

6 Haardt, R., and Godet, M., "Axial Vibration of a Misaligned Radial Face Seal under a Constant Closure Force," *ASLE Trans.*, Vol. 18, No. 1, Jan. 1975, pp. 55–61.

7 Griskin, E. N., "The Effect of Dynamics on Fluid Flow in a Face Seal," *Proc. 7th International Conference on Fluid Sealing*, Sept. 1975, Paper B2.

8 Denny, D. F., "Some Measurements of Fluid Pressure Between Plane Parallel Thrust Surfaces with Special Reference to Radial Face Seals," *Wear*, Vol. 4, 1961, pp. 64–83.

9 Etsion, I., "Radial Forces in a Misaligned Radial Face Seal," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 101, Jan. 1979, pp. 81–85.

10 Etsion, I., "The Accuracy of the Narrow Seal Approximation in Analyzing Radial Face Seals," *ASLE Trans.*, Vol. 23, No. 2, Apr. 1980, pp. 208–216.

11 Haardt, R., "Flow Consideration Around the Cavitation Area in Radial Face Seals," *Proc. 1st Leeds-Lyon Symposium on Tribology*, Sept. 1974, Paper VIII(i).

12 Booker, J. F., "A Table of the Journal-Bearing Integral," ASME Journal of Basic Engineering, Vol. 87, June 1965, pp. 533–535.

13 Gradshteyn, I. S., and Ryzhik, I. M., *Tables of Integrals Series and Products*, Academic Press, 1965.

DISCUSSION

R. Metcalfe[1]

It is timely that Dr. Etsion has extended his previous work to include "squeeze effects" between seal faces. As he concludes, these are most important in determining the dynamic motion of an angularly misaligned seal.

From a practical point of view, a clearer physical interpretation of the meaning of the results would be helpful. What is the link between "squeeze" and "hydrodynamic" effects that is alluded to in the discussion section of the paper, and should they really be treated as distinct and independent influences on seal motion?

In measuring and analyzing the dynamic motions of fully lubricated seal rings that are steadily tracking some angular misalignment of the opposing face, i.e. rings that are wobbling with constant amplitude, frequency and seal face separation, this discussor has not found it useful to treat squeeze and hydrodynamic effects separately. Rather, their combined effect is seen to be zero for a half speed wobble of any amplitude and separation; otherwise, their combined effect becomes proportional to the deviation from half speed. There is close similarity to the whirling motion of an unloaded journal. Could Dr. Etsion interpret his findings in these terms?

R. A. Burton[2]

The author is to be congratulated for an excellent paper on the hydrodynamic lubriction of seals. The problem addressed and the related dynamic problem (where seal ring inertia and other restraints are accounted for) have hardly been touched analytically, until now.

By suitable identification of correspondences, the seal dynamics problem is somewhat analogous to the shaft whirl problem, but there are important differences. In the shaft problem one finds a rotating inertial load which varies with eccentricity. In the seal problem, where the stator is self-aligning and the rotor is rigidly fixed to the shaft, the problem is one of a rotating displacement component. It is analogous to a stationary shaft held in a rotating, eccentric bearing. A second difference is that the film thickness is dependent upon speed and operating waviness. We shall eagerly await further developments in the understanding of such phenomena, which may involve whirl-like behavior.

In our observations of simulated seals (B.N. Banerjee, IIT Kanpur, started this work and recently Prof. A. Kistler has carried out some confirmatory studies) we find a broad range of operation that is synchronous. That is the stator motion is at the same frequency as the rotor motion, although there is a phase shift between these.

The onset of thermoelastic instability has been observed in these experiments. Although it is not fully explained, see reference [A-1], I want it to be clear that the phenomenon is not accompanied by asynchoronous behavior. Considerable operation after the transition has been carried out to test for this effect. I mention this because I find some confusion between the phenomena observed and some whirl effects in discussions with persons working in the field.

This does not mean that the thermoelastic instability is not coupled with a synchrous dynamic effect. Indeed Y.T. Wu and I have completed a study of these coupled phenomena which we hope to present within the year. It is described in the report: "Thermoelastic and Dynamic Effects in Seals" and can be obtained from the Mechanical/Nuclear Engineering Dept. at Northwestern University, Evanston, Il. or through DDC (AD No. A061754).

A. O. Lebeck[3]

The authors contribution is useful in that it provides expressions for squeeze effects in seals in a relatively simple form. The significance of the squeeze effects can best be evaluated in the context of a complete seal system which must include the eccentricities, masses and stiffness of the various parts of the seal system. The dynamic equilibrium of the system must be determined. Having made such a formulation, then it is possible to determine whether or not squeeze effects contribute to system stability and under what particular conditions instability results. It would be useful to seal designers if the author would provide an example of the application of his results to a dynamic seal system.

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Author's Closure

The author wishes to thank the discussers for their comments and interest in the paper.

The observations of Dr. Metcalfe and Dr. Burton of a flexibly mounted ring synchronously tracking a misaligned rotating seat are typical in dynamically stable operation which is certainly feasible. In such cases the flexibly mounted ring, when observed from the rotating seat, has a constant tilt γ, constant separation C, and is stationary. The resulting motion observed from an inertial reference is a synchronous tracking of the misaligned rotating seat. Under some conditions half frequency wobble is superimposed on the synchronous tracking, but the nutation $\dot{\gamma}$ and axial velocity v are still zero. In these modes of operation there are no "squeeze effects" as pointed out correctly by Dr. Metcalf. The more general mode of motion, however, is one in which both the nutation $\dot{\gamma}$ and axial velocity v can be nonzero, and the seal ring whirl asynchronously, see reference (A-2). The squeeze effects analyzed in this paper correspond to this general mode and, *therefore, can be separated from shaft speed induced hydrodynamic effects.*

A dynamic analysis, where the present results are already used, was completed recently (A-3), and is hoped to be presented this year. The analysis demonstrates the application of the squeeze (in addition to others) effects to a dynamic seal system and presents the conditions for the various modes of operation mentioned above.

Additional References

A-2 Etsion, I., and Burton, R. A., "Observation of Self-Excited Wobble in Face Seals," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 1, No. 4, Oct. 1979, pp. 526–528.

A-3 Etsion, I., and Dan, Y., "An Analysis of Mechanical Face Seal Vibrations," TME Report 359, Technion, Israel Institute of Technology, July 1979.