Air demand in gated tunnels – a Bayesian approach to merge various predictions

Mohammad Reza Najafi, Zahra Kavianpour, Banafsheh Najafi, Mohammad Reza Kavianpour and Hamid Moradkhani

ABSTRACT

High flowrate through gated tunnels may cause critical flow conditions, especially downstream of the regulating gates. Aeration is found to be the most effective and efficient way to prevent cavitation attack. Several experimental equations are presented to predict air demand in gated tunnels; however, they are restricted to particular model geometries and flow conditions and often provide differing results. In this study the current relationships are first evaluated, and then other approaches for air discharge estimation are investigated. Three machine learning techniques are compared based on the flow measurements of eight physical models, with scales ranging from 1:12–1:20, including the fuzzy inference system (FIS), the genetic fuzzy system (GFS), and the adaptive network-based fuzzy inference system (ANFIS). The Bayesian Model Average (BMA) method is then proposed as a tool to merge the simulations from all models. The BMA provides the weighted average of the predictions, by assigning weights to each model in a probabilistic approach. The application of the BMA is found to be useful for improving the design of hydraulic structures by combining different models and experimental equations.

Key words | adaptive network-based fuzzy inference system, aeration, Bayesian model average, fuzzy logic, gated tunnel, genetic fuzzy system

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ABBREVIATIONS AND NOTATION

a	aeration index
ANFIS	adaptive network-based fuzzy inference system
A_g	cross sectional area at gate chamber
BMA	Bayesian model average
F_r	Froude number at *vena contracta*
FCMC	fuzzy C-means clustering
FIS	fuzzy inference system
G	relative gate opening
GA	genetic algorithm
GFS	genetic fuzzy system
GP	grid partitioning
H	upstream reservoir head
MF	membership function
ML	machine learning
Q_a	air discharge
Q_w	water flow discharge
Q^m	measured air discharge
Q^s	simulated air discharge
SC	subtractive clustering
WM	Wang–Mendel
β	aeration ratio
$\mu_A(x)$	membership function

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INTRODUCTION

Gated tunnels are used for the emergency drawdown of reservoirs, regulating the reservoir water level and sediment flushing (Vischer *et al.* 1998). In high-head gated tunnels, flow separation from the gate lip, due to the increased velocity, forms a recirculation zone downstream of the gate.

As a result, significant pressure variations occur, which amplify the risk of cavitation attack.

Air entrainment is found to be an efficient way of eliminating cavitation (Peterka 1953; Russell & Sheehan 1974; Speerli & Volkart 1997; Speerli & Hager 2000). Peterka (1953) showed that an 8% air concentration near the tunnel wall would completely eliminate cavitation damage. The existence of air in the flow field controls and decreases bubble explosions and the resulting pressures (Russell & Sheehan 1974). Flow aeration would also decrease the mean flow density and increase the flow compressibility, which significantly reduces the pressure intensity caused by possible cavitation attack.

In order to supply enough air to the flow, an air vent is commonly placed downstream of the regulating gates. High speed flow issuing from the gate drags and entrains a lot of air. A significant portion of the air flows above the discharged water flow while the rest transfers as an air–water mixture to the downstream pond.

Determination of the air discharge from the air vent is an important factor in designing gated tunnels. Several experiments have been conducted to find relationships between water flow characteristics and tunnel geometry, with the air flow rate. The results are represented by the aeration ratio, $\beta = Q_a/Q_w$, where Q_a is the air discharge through the air vent and Q_w is the water discharge. In most studies the Froude number at the *vena contracta* has shown to be the best parameter to predict the aeration ratio. Kalinske & Robertson (1943) reported their results of air demand when a hydraulic jump forms inside the pipe. They expressed the aeration ratio as a function of the Froude number (F_r) in *vena contracta*, in the form of:

$$\beta = 0.0066(F_r - 1)^{1.4} \quad (1)$$

In another study, Campbell & Guyton (1953) showed:

$$\beta = 0.04(F_r - 1)^{0.85} \quad (2)$$

For a free surface flow, in a partially full conduit with no hydraulic jump, USACE (1964) presented the following equation:

$$\beta = 0.03(F_r - 1)^{1.06} \quad (3)$$

Wisner (1965) also suggested the following relationship:

$$\beta = 0.024(F_r - 1)^{1.4} \quad (4)$$

Sharma (1976) classified the water flow in conduits. For free flow conditions he recommended:

$$\beta = 0.09F_r \quad (5)$$

Haindl & Sotornik (1957) confirmed the expression of Kalinske & Robertson by measurements of air entrainment to the hydraulic jump using gamma radiation, Lysne & Guttormsen (1971) developed an equation using data from several gate installations, Harshbarger *et al.* (1977) compared a prototype and measurements from a scaled model and Baylar *et al.* (2009) suggested an expression relating aeration efficiency and Froude number.

Although several experimental equations are presented, they are restricted to measurements in models with particular geometries and flow boundary conditions. Using data from eight physical models, the wide range of air discharge estimations is shown in Figure 1. Since the measurements were taken from different models with diverse geometries and flow conditions, discrepancies exist in the estimations of the equations. Therefore one cannot solely rely on them in designing high head gated tunnels. As a result of the

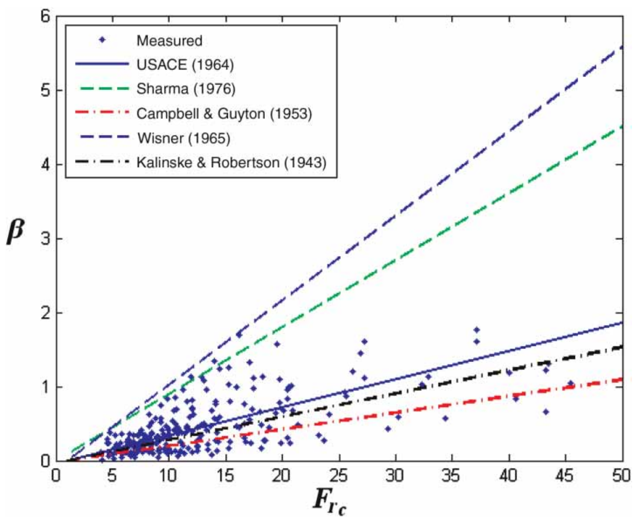

Figure 1 | Aeration ratio versus critical Froude number, comparing experimental equations with measured data.

limitations in the performances of the available air demand expressions – physical scaled models, which are costly and time consuming, are still used to measure the air discharge. Najafi & Zarrati (2010) also showed the application of numerical modelling as an effective tool to estimate air–water flow parameters in gated tunnels.

In the following sections after illustrating the data and predictor variables, three machine learning (ML) techniques are compared: the fuzzy inference system (FIS), which was generated based on the Wang–Mendel technique; the genetic fuzzy system (GFS), based on the Pittsburg algorithm; the adaptive network-based fuzzy inference system (ANFIS), which was initialized with grid partitioning (GP), subtractive clustering (SC) and fuzzy C-means clustering (FCMC). The application of Bayesian model averaging (BMA) is then investigated to merge the predictions of the equations and ML techniques.

DATA

Air demand is affected by several variables. Speerli & Volkart (1997) suggested the upstream reservoir head, gate opening, air vent loss characteristics and downstream tunnel geometry as the main factors. Also the tailrace tunnel length and width affect the air discharge through the air vent. In long tailrace tunnels major air demand is supplied from the air vents (Speerli 1999), while in short tailrace tunnels the air coming through the tunnel exit reduces the air vent flow rate.

In this study, data was obtained from measurements taken from physical scaled models of gated tunnels, ranging from 1:12–1:20, which were constructed at the Water Research Institute of Iran (WRI). The models included the gated tunnels of the Jareh, Alborz, Dasht-e-Abbas, Jegin, Gavoshan, Kosar and Seymareh dams. Measurements take from the Folsom tunnel (California, USA) were also considered. All of the models consisted of rectangular geometries in the gate chamber. A total of 245 data series from eight models were collected. This included the geometry parameters (e.g. the gate height and width, its upstream and downstream tunnel cross-section and the aerator section), air discharge, and the respective hydraulic parameters in the upstream and downstream of the regulating gates.

The following variables were considered as the predictors for air discharge estimation: upstream reservoir head (H), which represents the upstream boundary condition; relative gate opening (G), which controls the output flow condition (spray/free surface flow, or flow with hydraulic jump) as well as the hydraulic parameters; and the cross-sectional area at the gate chamber (A_g), which affects pressure, velocity and flow rate. Another variable was also defined and used as a predictor – the aeration index (a). The aeration index is the ratio of the cross-sectional area just downstream of the regulating gate to one in the upstream. $a < 1$ suggests that aeration is occurring from the top surface of the free jet (Figure 2(a)), while $a > 1$ shows that aeration is also occurring from other surfaces around the jet (Figure 2(b)). The predictor variable ranges are: H (m): [2–12], G (%): [10–100], A_g (m^2): [0.015–0.085] and a: (m^2/m^2) [0.65–5].

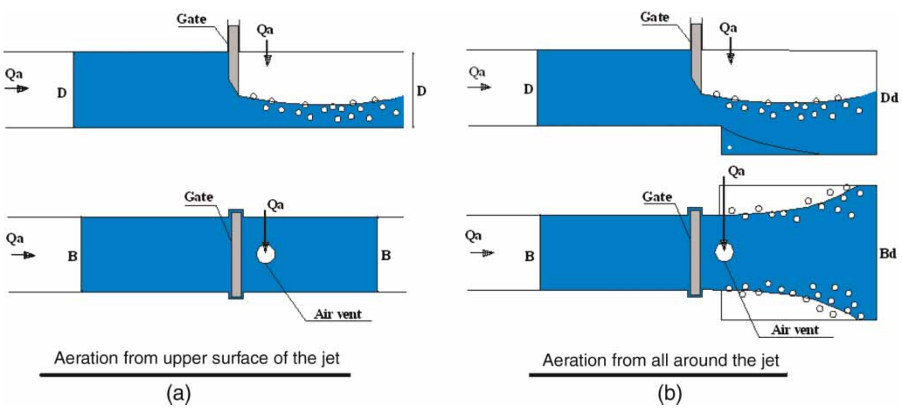

Figure 2 | Schematic view of two mechanisms of flow aeration.

THE MACHINE LEARNING APPROACHES

Three different ML techniques were applied to predict air discharge in gated tunnels. A FIS was first established based on the method proposed by Wang & Mendel (1992). A modified genetic algorithm (GA) was then developed to train the FIS. The combined system was called the genetic fuzzy system (GFS). FIS rules and membership function parameters were optimized by GAs. These two FISs were Mamdani-type fuzzy models. The third method utilized, was the neuro-fuzzy system, also known as the ANFIS, which is a Sugeno-type fuzzy model.

Fuzzy inference system

The introduction of fuzzy sets (Zadeh 1965) brought a powerful tool to deal with variables of an approximate nature. Fuzzy logic has been widely used in several hydraulics/hydrologic applications such as, aeration efficiency analysis, runoff estimation and controlling hydraulic structures like spillway gates, etc. (Kucukali & Cokgor 2007; Barreto-Neto & de Souza Filho 2008; Karaboga et al. 2008; Mujumdar & Ghosh 2008; Gopakumar & Mujumdar 2009; Jimenez et al. 2009).

In a classical or crisp set, the transition for an element between membership and non-membership is abrupt and well-defined. In contrast, a fuzzy set contains elements that have varying degrees of membership (Ross 2004). Considering X as a collection of objects denoted by x, the fuzzy set 'A' in X is defined as a set of ordered pairs: $A = \{(x, \mu_A(x)) | x \in X\}$, where $\mu_A(x)$ is the membership function (MF) of the fuzzy set A. The MF maps each element of X to a membership grade ranging from 0 to 1 (Jang et al. 1997). Gaussian and Bell-shape functions are two commonly used MFs in fuzzy modelling:

$$\text{Gaussian}\,(x;\,c,\sigma) = e^{-\left(\frac{x-c}{\sigma}\right)^2} \tag{6}$$

$$\text{Bell}\,(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \tag{7}$$

where c represents the centre and σ, the width of the Gaussian MF. c and a are the centre and width of the Bell-shape MF, respectively and b represents the slopes at the crossover points.

Figure 3 shows a schematic representation of the 'mamdani' FIS. U and V are the input/output classes. Fuzzification is the process of assigning membership degrees, in fuzzy MFs, to crisp input values. A fuzzy rule base contains a set of if–then clauses which define the system performance. The rule set in a fuzzy system can be generated based on the knowledge and experience of an expert as well as the available data. A fuzzy inference engine contains a set of fuzzy operators (i.e. AND, OR, etc.) which map inputs (i.e. predictors) to the output (i.e. predictand). In contrast to fuzzification, defuzzification is the process of converting a fuzzy quantity to a crisp value. Takagi & Sugeno (1984) proposed another type of FIS in which the output space is represented by several linear equations. In this model there is no need for defuzzification of the outputs.

To construct the fuzzy model, 80% of the data was randomly selected. The rest of the data was used to evaluate the model performance. The method proposed by Wang & Mendel (1992) was applied to generate fuzzy rules, based on the membership degree of the variable in each data series to a particular membership function. A total of 56 fuzzy rules were generated. The number of input/output MFs was determined by trial and error. The input variables were uniformly partitioned into four fuzzy sets of similar shapes and sizes. The output variable was partitioned into 14 fuzzy sets according to the distribution of the data, that is, the higher frequency of the dataset in a bin, the higher the number of MFs in that bin would be. Gaussian MFs were considered for this FIS.

Regarding the fuzzy operators, 'Product' was chosen for the AND and implication operators, and 'mean average defuzzifier' was selected to defuzzify the output.

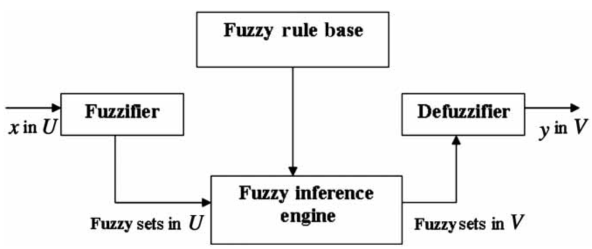

Figure 3 | A Schematic view of the mamdani-type FIS.

Genetic fuzzy system (GFS)

The Wang–Mendel method is an efficient way of generating fuzzy rules. Nevertheless in this approach each data series may define a rule in the fuzzy rule base, therefore it is sensitive to the noisy data. It also does not tune the MF parameters. On the other hand GAs have shown to be a reliable technique to optimize the parameters in complicated search spaces (see e.g. Goldberg 1989; Cho *et al.* 2004; Cordon *et al.* 2004; Fang & Ball 2007). Therefore we applied this algorithm to optimize the MF parameters of the FIS and generate the fuzzy rules.

The GA starts by randomly generating an initial population, which consists of a number of possible solutions called 'chromosomes'. The population evolves during each generation to converge to the optimum parameter set. In each generation some chromosomes merge and produce offspring, which represent new points in the search space. This process is known as crossover. Some chromosomes are then selected, based on their objective function values, and transferred to the next generation. The algorithm tends to converge to a solution (chromosome) which minimizes (or maximizes) the objective function.

Although the algorithm explores the search space from different initial points to find the global optimum, there may be the chance of being trapped in a local optimum. Mutation lets the algorithm explore regions of the search space that were not previously covered, by randomly altering one or more constituents of the chromosomes (also called genes). Crossover and mutation rates determine the number of chromosomes to merge, and the number of genes to alter, respectively.

In order to tune the fuzzy MF parameters, and generate the fuzzy rule base, the Pittsburgh approach (Smith 1980; Cordon *et al.* 2004) was utilized. In this approach the fuzzy rule set is parameterized in one chromosome. Additionally the MF parameters were considered as genes in the chromosomes.

The fuzzy model was initialized by uniformly partitioning the input variables into four fuzzy sets, including the Z shape, S shape and Gaussian MFs. The output variable (air discharge) was uniformly partitioned into nine Gaussian MFs. Considering the number of unknown MF parameters (e.g. two unknown parameters for each Gaussian MF), the number of rules and the degree of trust of each rule, each chromosome contained 170 genes (parameters to optimize). Since the number of unknown parameters was large, the real GA was utilized. The number of fuzzy rules was set to 20.

In order to keep the MFs of each variable in sequence, we substituted the MF mean parameter (c), with a parameter representing the 'distance' between MF means. The range of the parameters representing the MF width was specified so that, in any case, the degree of membership of all the values of the variables in the fuzzy space was more than zero (i.e. each value belongs at least to one MF). Three methods were used to perform the crossover.

1. A number was randomly selected between one and the length of the chromosomes '170'. The gene of the two parent chromosomes (P_{ma} and P_{da}) corresponding to that number, would then generate a new gene (Haupt & Haupt 2004):

$$\begin{aligned} &\mathrm{Parent}_1 = [\mathrm{P}_{m1}, \mathrm{P}_{m2} \ldots \mathrm{P}_{ma} \ldots \mathrm{P}_{mNvar}];\\ &\mathrm{Parent}_2 = [\mathrm{P}_{d1}, \mathrm{P}_{d2} \ldots \mathrm{P}_{da} \ldots \mathrm{P}_{dNvar}];\\ &\mathrm{P}_{new1} = \mathrm{P}_{ma} - \mathrm{r}[\mathrm{P}_{ma} - \mathrm{P}_{da}];\\ &\mathrm{P}_{new2} = \mathrm{P}_{da} + \mathrm{r}[\mathrm{P}_{ma} - \mathrm{P}_{da}] \end{aligned} \tag{8}$$

   $0 < r < 1$ is a random number. The parent genes on the right of the selected bit were switched afterwards.
2. All of the genes in the 'right' side of the selected bit were merged according to Equation (8).
3. All of the genes in the 'left' side were merged according to Equation (8).

The generated chromosomes were added to the population and the selection was performed from the extended population. The roulette wheel and tournament methods were compared in the selection process. The tournament method was chosen due to its better performance and its flexibility to adjust the selection pressure. The final tournament size was three.

The objective function was defined as the mean squared error:

$$\mathrm{MSE} = \left(\frac{\sum_{i=1}^{N} (Q_i^m - Q_i^s)^2}{N}\right) \tag{9}$$

where N is the number of training data and Q_i^m and Q_i^s represent the measured data and the model simulations respectively. The algorithm was considered converged when the change in the value of the objective function was small in several consecutive generations (Figure 4).

As shown in Figure 4, the algorithm improves considerably in the first 200 generations and shows less progress thereafter. The characteristics of the GA are presented in Table 1.

Adaptive network-based fuzzy inference system (ANFIS)

ANFIS, which was proposed by Jang (1993), is a Sugeno-type fuzzy model, that combines FIS with the training capabilities of neural networks. This method has been applied in various hydraulics and hydrologic problems, for example, characteristics of scour downstream of stilling basins (Farhoudi *et al.* 2010) and around piers (Bateni *et al.* 2007), rainfall-runoff modelling (Gautam & Holz 2001), prediction of flow condition, and estimation of aeration efficiency in stepped spillways (Baylar *et al.* 2007; Hanbay *et al.* 2009).

Figure 5(a) shows a Sugeno-type fuzzy system with two inputs, reservoir height (H) and relative gate opening (G), and one output, air flow rate (Q_a). Two fuzzy if–then rules are defined as:

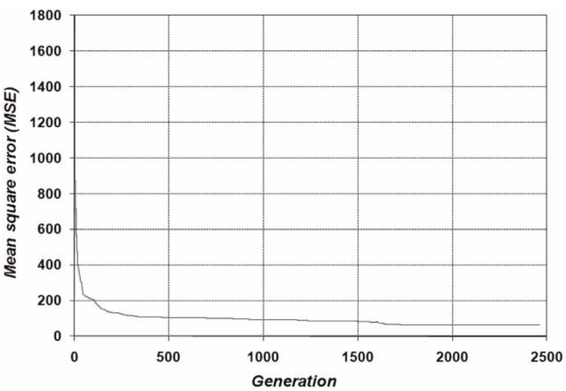

Figure 4 | The change of the objective function in the GFS optimization process.

Table 1 | The proposed GFS

No. of generations	2,462
Initial population size	100
Chromosome length	170
Crossover rate (P_c)	0.9
Mutation rate (P_m)	0.002

Rule$_1$: If H is high, and G is high Then $Q_{a1} = p_1{\cdot}H + q_1{\cdot}G + r_1$
Rule$_2$: If H is low, and G is low Then $Q_{a2} = p_2{\cdot}H + q_2{\cdot}G + r_2$

Parameters p, q and r are optimized during the ANFIS training process. The adaptive network for this system is shown in Figure 5(b). The network consists of nodes and connecting vectors. The training process of ANFIS is illustrated in the Appendix.

Three methods were used to construct the ANFIS model structure (type and number of MFs): Grid partitioning (GP), subtractive clustering (SC) and fuzzy C-means clustering (FCMC). In GP the feature space is divided into equally spaced partitions. The type and number of MFs for each model are determined by trial and error. To optimize the ANFIS structure, the technique presented by Jang (1996) was adopted, which is based on the assumption that the ANFIS model having the lowest residual in the first training epoch, provides the lowest final residual after convergence. Therefore the model, with a particular structure, was trained for one epoch utilizing the least square method. Cross validation was also implemented, in which the model was trained 20 times, each time with a different train and check dataset. The test dataset remained unchanged. The average of the 20 results, for a model with particular type and number of MFs, was taken to evaluate the model's performance. Two Bell-shape MFs for each input variable resulted in the lowest model residual.

SC: Given a collection of n data points $\{x \ldots x_n\}$ in a p-dimensional space, the density measure at a data point X_i is defined as:

$$D_i = \sum_{j=1}^{n} \exp\left(-\frac{\|X_i - X_j\|}{(r_a/2)^2}\right) \tag{10}$$

r_a is a positive constant. A data point with a largest number of neighbouring points has a high density value. The data point with the highest density is then selected as the first cluster centre. Considering X_{c1} as the point selected with the density measure D_{c1}, the density measure for each data point X_i is recalculated:

$$D_i' = D_i - D_{c_1} \exp\left(-\frac{\|X_i - X_{c_1}\|^2}{(r_b/2)^2}\right) \tag{11}$$

r_b is a positive constant, which is commonly chosen as $r_b = 1.5r_a$ to prevent closely spaced cluster centres. This

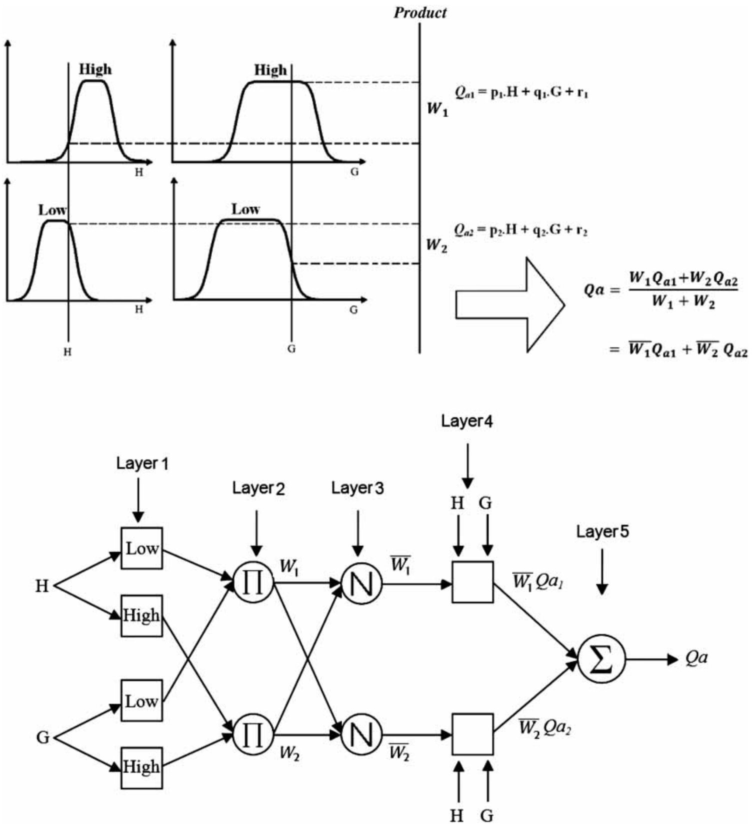

Figure 5 | (a) A typical Sugeno-type fuzzy system with two inputs: reservoir height (*H*) and relative gate opening (*G*) and one output: air flow rate. (b) The corresponding ANFIS structure.

constant defines a neighbourhood to be reduced in the density measure. The points near the first cluster centre will have significantly reduced density measures, making them unlikely to be selected as the next cluster centre. The data point with the highest remaining density measure is selected as the second cluster centre. This process is repeated until the stopping criterion is met (Chiu 1994).

In this study the SC was examined with several radii assigned to the data dimensions, with values between 0 and 1. In order to optimize the radius values for each model, the same procedure performed in grid partitioning was implemented. The Gaussian MF and radii of [0.6, 0.7, 0.7, 0.7, 0.7] corresponding to *G*, *H*, *Ag*, *a* and *Qa*, were finally selected.

FCMC: This clustering algorithm (Bezdek & Ehrlich 1984) is a multivariate data analysis technique that partitions a dataset into overlapping clusters. The cluster centre identifies each cluster. The number of clusters is specified manually.

In order to optimize the number of clusters for each model the same procedure performed in GP and a subtractive algorithm was implemented. The number of six fuzzy clusters was finally selected. The Gaussian MF was also used in this algorithm.

After the model structures were identified, in order to train the model, data was divided into train (50%), check (30%) and test datasets (20%). The check data set was utilized to prevent the risk of overtraining. The optimized model parameters were chosen based on the minimum residual of the check dataset (e.g. iteration No. 140 according to Figure 6).

BAYESIAN MODEL AVERAGING (BMA)

The Bayesian model averaging (BMA) method (Hoeting *et al.* 1999) provides a probabilistic tool to merge several simulations. It assigns weights to each model prediction

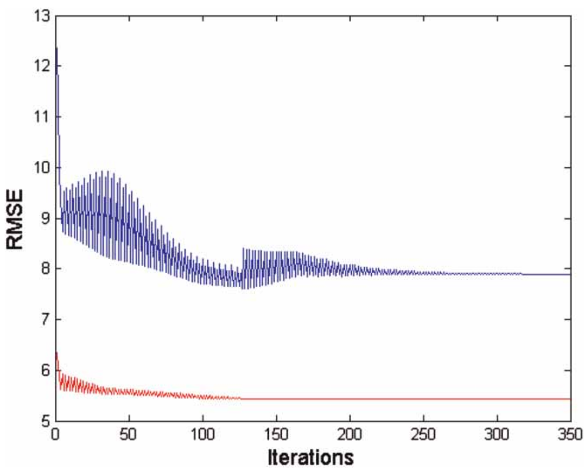

Figure 6 | ANFIS train based on two sets of data; upper: check dataset and lower: train dataset.

and generates a weighted average result, which is more reliable than each single simulation. BMA has been applied recently in some climatologic and hydrologic studies, including the weather forecast using different climate models (Raftery *et al.* 2005), and multi-model streamflow prediction (Duan *et al.* 2007). In this study its application in the design of aerators in gated tunnels, by merging the simulations from several models is analysed. This approach can be extended to combine any other hydraulic structure design method.

One can estimate the prediction variable q_a (e.g. air discharge) by the law of total probability:

$$p(q_a) = \sum_{n=1}^{N} p(q_a|Q^s, Q^m) \cdot p(Q^s|Q^m) \tag{12}$$

where Q^m and Q^s are the measured and simulated air flow respectively. Given the measured data (Q^m), the posterior probabilities of the models, i.e., $p(Q^s|Q^m)$ represent the BMA weights. According to (Raftery *et al.* 2005), $p(q_a|Q^s, Q^m)$ can be simplified by a Gaussian distribution like $g(q_a|Q^s, \sigma_n^2)$. Q^s and σ_n are the mean and standard deviation of the distribution which can be obtained using the expectation maximization (EM) algorithm. For non-Gaussian observations Duan *et al.* (2007) transformed the observed and simulated hydrologic data to a semi-Gaussian distribution, by the use of Box–Cox transformation. In this study, since the distributions of the air flow measurements were skewed, a similar process was performed.

As discussed in previous sections, the experimental equations are limited to physical models with particular characteristics. Therefore the predictions using these equations may be biased. Before merging the results using BMA, all equations and ML simulations were bias corrected using the linear regression between each simulation versus the measurements. As a result Q^s in Equation (12) was converted to $a_n + b_n Q^s$. The intercept (a_n) and slope factor (b_n) of the linear bias corrected simulations are shown in Table 2. The parameters of the regression equations can be used to bias correct further predictions. The results were then transformed to a Gaussian distribution using the Box–Cox equation.

Considering the unknown values of $w_n = p(Q^s|Q^m)$ and σ_n, the log-likelihood function is defined as:

$$l(w_1, \ldots, w_n, \sigma^2) = \sum_{n=1}^{N} w_n \cdot \sum_{t=1}^{T} g(q_{at}^{obs}|Q_t^s, \sigma_n^{Iter}) \tag{13}$$

The unknown values are calculated using the iterative procedure of EM, which swaps between expectation and maximization steps utilizing an underlying variable z_{nst} (Raftery *et al.* 2005). The algorithm is completely illustrated in Raftery *et al.* (2005) and Duan *et al.* (2007).

Other methods like the Markov Chain Monte Carlo (MCMC), can also be applied to approximate the final posterior distribution and find the weights as discussed in Vrugt *et al.* (2008). The mean of the BMA can then be obtained by:

$$E(q_a|Q^m) = \sum_{n=1}^{N} w_n Q^s \tag{14}$$

$a_n + b_n Q^s$ replaces Q^s since the model was bias corrected. The EM algorithm was a very efficient method, and converged after 12 iterations.

RESULTS

Three performance measure functions were used to assess the experimental equations and ML simulations:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (Q_i^m - Q_i^s)^2}{N}} \tag{15}$$

Table 2 | The intercepts and slopes of the linear regressions, which were applied to bias correct the models

Models	FIS	GFS	GP	SC	FCMC	USACE	Sharma	Campbell & Guyton	Wisner	Kalinske & Robertson
Intercepts	−7.674	−3.258	−0.477	−1.733	−3.612	9.834	8.294	7.615	13.917	13.917
Slopes	1.334	0.97	1.07	1.115	1.197	1.769	0.67	2.447	0.782	2.843

Table 3 | Evaluation of the experimental equations in air demand estimation for the test data set

Equations	RMSE	BIAS (%)	NSE
Sharma	18.092	32.837	0.592
Wisner	18.816	38.559	0.559
USACE	27.579	59.340	0.053
Campbell & Guyton	31.565	67.365	−0.241
Kalinske & Robertson	36.773	78.104	−0.684

$$\mathrm{BIAS} = \frac{\sum_{i=1}^{N} |Q_i^m - Q_i^s|}{\sum_{i=1}^{N} Q_i^m} \times 100 \quad (16)$$

$$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{N} (Q_i^m - Q_i^s)^2}{\sum_{i=1}^{N} (Q_i^m - \overline{Q_i^m})^2} \quad (17)$$

NSE (Nash Sutcliffe Efficiency) values are in the range of +1 to −1. NSE = 1 shows that the residuals are zero, while negative NSE values indicate that the predictand mean value is more skillful than the model used. As mentioned in previous sections, all data were divided into two categories, train and test. The test dataset which included 20% of the whole data (about 50 data series) was used to evaluate the equations, and the ML techniques. It should be noted that the test data were resampled randomly. However if the value of the selected variable was outside its range in the train dataset, it was removed and another data series was randomly chosen. Therefore the train data covered all of the variable ranges. Table 3 shows the performance measure of the experimental equations. Those by Sharma (1976) and Wisner (1965) performed significantly better than the others. The equations presented by Kalinske & Robertson (1943), Campbell & Guyton (1953), and the United States Army Corps of Engineers (USACE) (1964) show the lowest performances respectively. As shown in Figure 7, they all underestimate the air entrainment compared with the measurements. This is in agreement with previous studies (e.g. Speerli & Hager 2000) and the assumptions of the equations: Kalinske & Robertson conducted experiments on the air entrainment measurement in pipes with hydraulic jump. The difference between the circular cross-section of the pipes and the rectangular section of the models used in this study is one of the reasons for this bias. The experiments performed by Campbell & Guyton, and the USACE reflect the flow turbulence and therefore show higher air entrainment estimations, nevertheless they do not consider various flow types. Wisner and Sharma classified the flow types into three categories of spray flow, free surface flow and flow with hydraulic jump, which resulted in more generalized equations. In another study performed by Najafi & Zarrati (2010) over 'Gotvand' large-scale gated tunnel, the Wisner equation also showed a good performance.

The ML techniques applied in this study outperformed all the equations (Table 4). ANFIS produced the best estimation while the fuzzy Wang–Mendel (WM) produced the poorest one, though still better than the equations. This shows that the structure of ML tools has a considerable impact on their performance. There exists a major difference between GFS and fuzzy WM compared with ANFIS. The first two are mamdani-type FISs and the last is a Sugeno FIS. On the other hand, although ANFIS applies similar training techniques for GP, SC and FCMC, they show different performances due to their different structures.

Besides the effectiveness of the models, the efficiency of ANFIS was higher than the other two ML methods. It automatically trained the fuzzy system in a shorter time than GFS. However the rules generated by the fuzzy WM and GFS are more easily interpretable than the ones obtained by ANFIS. They relate the input variables to the predictand in a linguistic way, where the importance of each variable is realized by comparing the rules. On the other hand in the Sugeno-type ANFIS, the results of the rules are linear equations, which do not convey much information. Overall these ML methods can be considered as a black

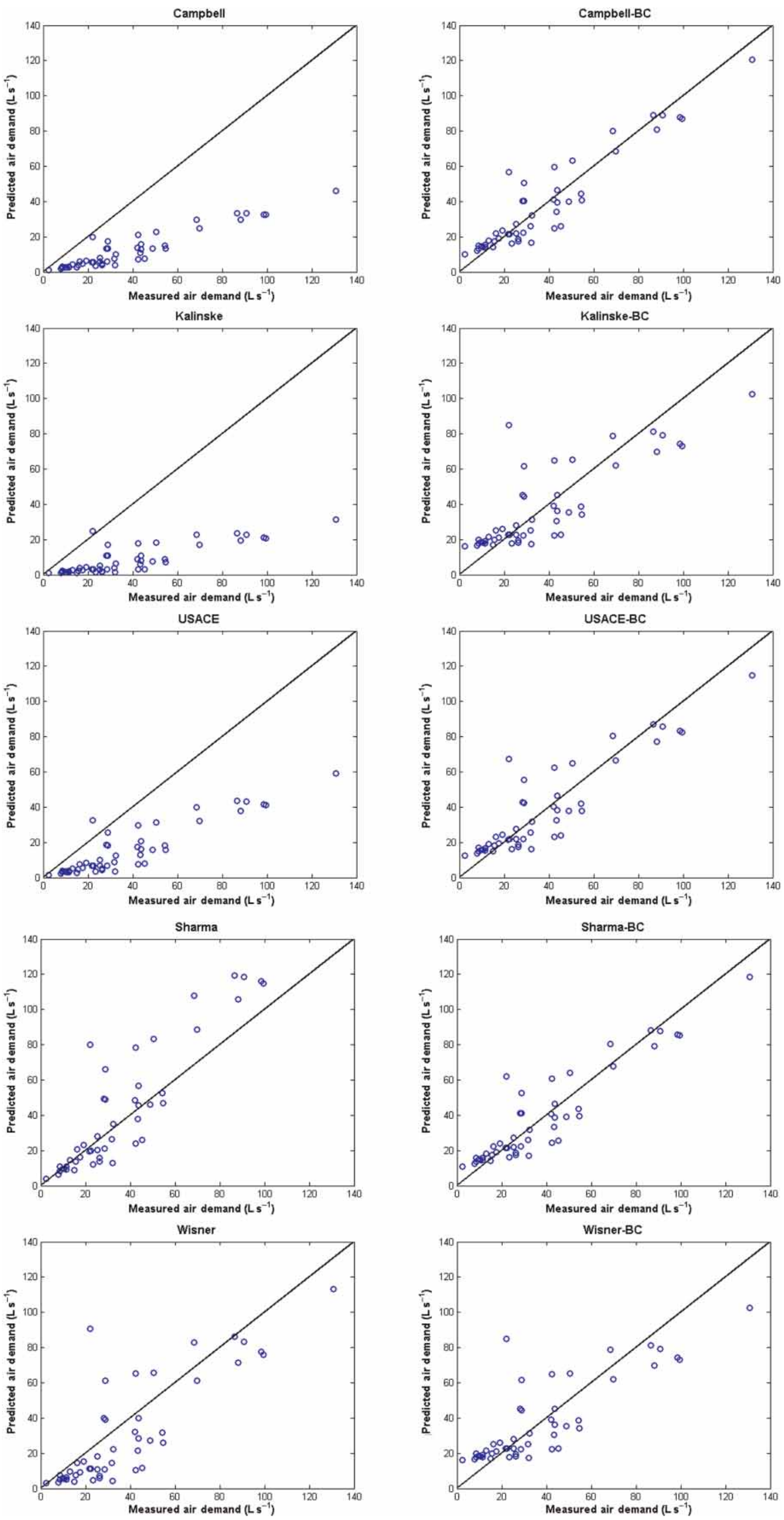

Figure 7 | Comparison of the measured air flow rate and the predictions obtained from the experimental equations for the test dataset. Right columns are the bias-corrected (BC) simulations.

Table 4 | Evaluation of the ML methods in predicting air discharge for the test dataset

Models	RMSE	BIAS (%)	NSE
ANFIS (GP)	5.73	11.45	0.96
ANFIS (SC)	7.58	13.61	0.93
ANFIS (FCMC)	9.97	18.45	0.88
GFS	10.40	20.44	0.87
Fuzzy WM	15.49	28.37	0.70

box, though their structures are more meaningful than a neural network.

Bias in a model is due to deficiencies in its structure which makes it under- or overestimate a predictand in most conditions. Bias correction is a simple tool to improve the model performance without changing its structure. Bias correction is currently widely used in hydrology and climatology to enhance forecasts based on observed data (e.g. Hashino *et al.* 2006). This technique effectively enhances the model simulations based on the measurements and makes its predictions more reliable. The NSE values of all models and equations are compared before and after bias correction (Table 5).

Figures 7 and 8 show the model simulations versus the measurements before and after the bias correction. The equation results have improved significantly after bias correction, which is due to the large bias. As discussed above, these equations are based on a limited number of experiments and do not generalize to different geometric and hydraulic flow conditions. The significant improvement of the Kalinke & Robertson equation, as an example, comes from its geometrical limitation to the flow through pipes. The ML results show limited change after bias correction.

As mentioned in the previous sections, in this study the BC models were used to perform the BMA. After combining the model/equation results, the weights assigned to each simulation were also determined. In this study ANFIS gained the highest weights compared to other methods. ANFIS initialized by FCMC received the overall highest weight (Figure 9). FIS and FGS weights were close to the equation ones (after bias correction). The BMA prediction performed very closely to the best model with a NSE of 0.94. The proposed merging technique does not get biased towards several low performance equations instead it tends to be close to the best prediction. This is in contrast to the simple average of several simulations from different models, which assigns similar weights to all simulations. On the other hand BMA gets the advantage of all models and does not discard any information obtained from them. One can apply this merging method on different model simulations of a hydraulic design, and expect to gain a skillful prediction.

CONCLUSION

More than 240 data series containing the air–water flow measurements of eight physical scaled models were utilized, to predict air discharge in gated tunnels. The current experimental equations were evaluated. Considering their limitations to specific model geometries and flow conditions it was shown that they provide biased results.

Three models based on ML techniques were developed. The ML models include a simple fuzzy system without any automatic training, a fuzzy model which was trained using GAs and ANFIS. The simulations from GFS and ANFIS were more acceptable than the other models and equations. The fuzzy model with no training performed better than the experimental equations in predicting air discharge for a test data set, however, it produced the weakest ML result. This highlights the importance of training the ML algorithms to improve the predictions. Merging the fuzzy model and genetic algorithms was implemented similar to the procedure for calibrating the conceptual hydrologic models, however, the number of unknowns were much larger (170 unknowns) which significantly reduced the efficiency of the algorithm. Since the simple genetic algorithm was not effective in optimizing the fuzzy system with large number of parameters, we

Table 5 | NSE performance measure of the models before and after the bias correction

Models	FIS	GFS	GP	SC	FCMC	USACE	Sharma	Campbell & Guyton	Wisner	Kalinske & Robertson
Before bias correction	0.7	0.87	0.96	0.93	0.88	0.05	0.59	−0.24	0.56	−0.68
After bias correction	0.77	0.89	0.97	0.95	0.91	0.81	0.85	0.87	0.68	0.68

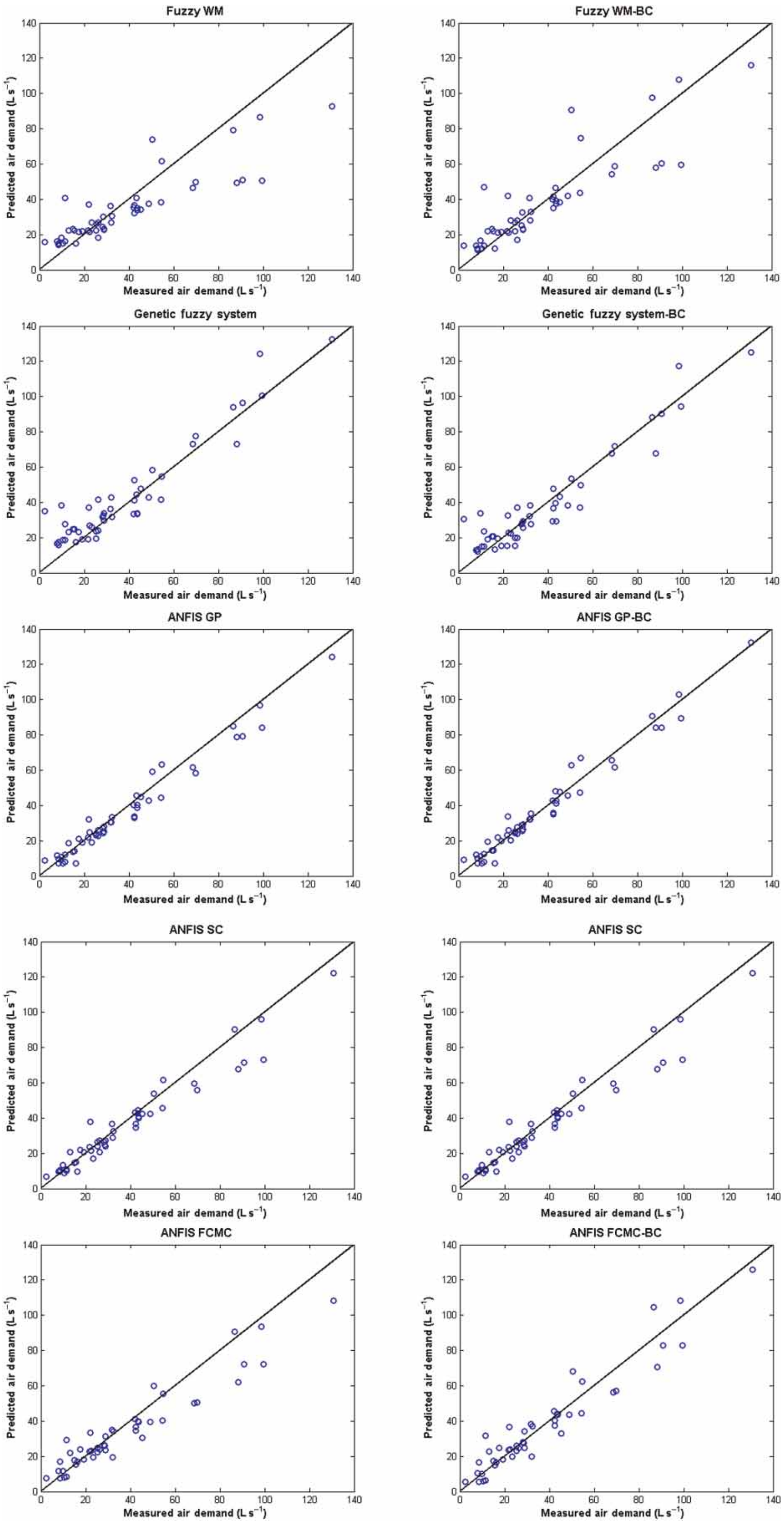

Figure 8 | Similar to Figure 7, representing the ML simulations.

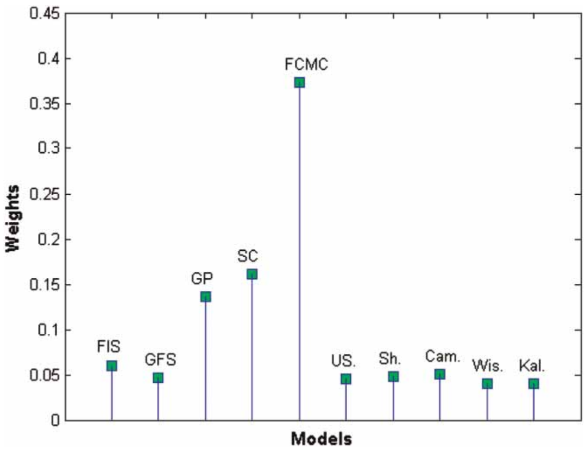

Figure 9 | BMA weights assigned to each model; the equation labels are abbreviated and show the first few letters of their names e.g., US: USACE.

made some modifications to the algorithm which enhanced its 'exploring' and 'exploiting' capabilities. Overall ANFIS showed the most effective and efficient ML technique among the others.

The experimental equations, although very simple, inherit a lot of uncertainties. Therefore in practice they are rarely accepted as the true representations of reality. Usually either a scaled model is set up to measure the air discharge, or some factors are applied to increase the predicted value and reduce the potential risk due to the uncertainties. On the other hand the ML methods consist of more complex structures than the simple equations, and they are highly dependent on the available data, which may be costly and time consuming to prepare. However when a model like ANFIS is properly trained and validated based on sufficient data, its application for further predictions would be undemanding, as illustrated by Habibagahi (2002). Considering these we believe that the ML techniques are applicable for hydraulic structure design, besides they reduce the overall cost of the projects by producing more reliable results.

Kavianpour & Najafi (2008) showed the application of fuzzy GA in optimizing the parameters of aeration ratio equation through the sensitivity analysis of the effective predictors. However the application of other methods based on machine learning (such as the symbolic regression through genetic programming (Koza 1992) and evolutionary polynomial regression (EPR) (Giustolisi & Savic 2006) which combines numerical and symbolic regression) need to be investigated more in hydraulic structure design. These algorithms provide the components of the equations besides optimizing the parameters.

Bias correcting the simulations improved their performances significantly, especially for the experimental equations. Bias correction does not enhance the equation or model, instead it applies a linear/non-linear expression to the predictions to improve the biases due to structural deficiencies. This technique can be used for the design of hydraulic structures, when measured data are available. Each equation and model, can be bias corrected based on the previous measurements, and then applied for future predictions. Besides, since in the current study the equations have been applied for various physical models, the resulting bias correction factors shown in Table 2 can be used for future predictions.

No single model is complete enough to produce perfect simulations for any design purposes. The application of several models and relationships allows for incorporating the strengths of each model. BMA was applied to merge several air discharge predictions as an example of multi-modeling in hydraulics. This statistical method assigned weights to each single simulation and then calculated the weighted average for the final prediction. In this study, the BMA performance was close to the best model. It should be noted that, although the fuzzy and Bayesian approaches deal with the uncertainties in two different ways (i.e. representing vagueness and randomness), the BMA model does not consider the structure of the fuzzy models. It takes into account the predictions (either they come from fuzzy/numerical models or simple equations), and assigns weights to them in a probabilistic approach, and the structure of the model/equation is not considered. Therefore BMA can be applied for merging predictions from any models.

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APPENDIX

According to Figure 5, the training process of ANFIS is illustrated for each layer.

The nodes in each layer perform similar functions:

Layer 1: The predictors are input to this layer. Each node represents one fuzzy set of a predictor, which contains a linguistic label (e.g. low). The output of the node is the degree to which the given predictor belongs to the fuzzy set. The membership degrees range from 0 to 1:

$$O_i^1 = \mu_{Low}(H) \tag{A1}$$

where O_i^1 is the value of the membership degree (i is the number of the node). The parameters of the membership functions are called the 'premise' parameters. The Gaussian and Bell-shape functions were analysed in this study.

Layer 2: The node function in this layer is a t-norm fuzzy operator, such as a minimum or a multiplication, which performs the generalized 'AND' in each rule:

$$w_i = \mu_{Low}(H) \times \mu_{Low}(G) \tag{A2}$$

The nodes' outputs represent the firing strengths of the rules.

Layer 3: The normalized firing strengths of the rules are calculated in this layer:

$$\overline{w_i} = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2 \tag{A3}$$

Layer 4: The normalized firing strength is multiplied by the linear function of the predictors:

$$O_i^4 = \overline{w_i} \cdot Q_{ai} = \overline{w_i}(p_i H + q_i G + r_i) \tag{A4}$$

p_i, q_i and r_i are the consequent parameters.

Layer 5: All the results from layer 4 are summed up in one single node in this layer which results in the final air flow rate:

$$O_1^5 = Q_a = \sum_{i=1}^{N} \overline{w_i} Q_{ai} \tag{A5}$$

In the hybrid learning algorithm which is utilized in ANFIS (Jang 1993, 1996; Jang *et al.* 1997), the consequent parameters are determined by the least square estimator, while the premise parameters are obtained by the back propagation gradient descent learning rule. The hybrid learning rule has the advantage of decreasing the dimension of the search space in the gradient method; besides it shortens the convergence time.