“Quasi-Period” Signal Processing Technique for Doppler Sodar Measurements

IGOR V. PETENKO AND ANDREY N. BEDULIN

Institute of Atmospheric Physics, Russia Academy of Sciences, Moscow, Russia

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ABSTRACT

An approach for estimating the first and second moments of the spectrum of a narrowband signal, which will be called the “quasi-period” technique, is described. This method is based on the statistical properties of the instantaneous frequency of a quasi-sinusoidal signal. The probability density function of the instantaneous frequency is determined by the first two moments of the spectrum only and does not depend on its shape. Using statistics of the instantaneous frequency, it is possible to compensate for the bias of the mean frequency by noise and to define an objective criterion for identifying data with an inadequate signal-to-noise ratio. Using model signals with known spectra as well as a sinusoidal signal and additive noise, an evaluation of the accuracy of this method for estimating the mean frequency was obtained. The use of this technique in sodar systems for wind measurements allows better identification of interfering data and measurement of the wind variance in addition to its mean value, both during one profile and averaging over a number of profiles.

1. Introduction

The problem of estimating spectral characteristics of narrowband signals is one of the most important in the statistical analysis of radio signals. One of the earliest applications was measuring the Doppler shift of received radar signals; the same problem arises in underwater acoustics, acoustic, and radioacoustic remote sensing. The mean frequency (the first moment of the spectrum) and the frequency variance (the second moment of the spectrum) are of prime interest for Doppler velocity measurements. Existing systems for acoustic sounding of the atmosphere (sodars) use a wide variety of methods of frequency (Doppler velocity) measurements. Analysis and comparison of some of these techniques are performed by Neff and Coulter (1986). Based on previous published works as well as our own experience, we believe that the best results are given from 1) fast Fourier transforms (FFT) with peak detection (Elisei et al. 1986), 2) complex covariance estimation, and 3) the zero crossing technique with prior searching of the signal (Petenko et al. 1987). All of these methods provide acceptable results for measuring the mean horizontal wind; this has been confirmed by comparisons with in situ techniques (Kaimal et al. 1984; Petenko et al. 1987). Nevertheless, the need to improve the acoustic signal processing methods to provide more precise measurements of turbulence and mean wind with short averaging periods induced us to return to the problem of the retrieval of the Doppler shift.

In Bedulin et al. (1990) we showed that the first and second moments of the spectrum of narrowband signals could be estimated from a histogram of time intervals between zero crossings. A technique for making such measurements was also described. Here we give a more rigorous basis of this method and describe a modified algorithm that provides higher accuracy of estimating the mean frequency and bandwidth under conditions of low signal-to-noise ratio. The results of laboratory verifications are given.

2. Description of the “quasi-period” technique

The proposed method is based on the statistical properties of the derivative of phase (or instantaneous frequency) of a random narrowband process. A detailed theoretical consideration of these properties is given in Tickhonov (1966). All the results presented below are applicable to a narrowband quasi-harmonic signal that can be presented as

\[ s(t) = A(t) \cos(\omega_0 t + \phi(t)), \]

where \( A(t) \) is the envelope, changing slowly in comparison with \( \cos(\omega_0 t) \), and \( \phi(t) \) is a random phase.

The probability density function of the instantaneous frequency \( \omega(t) = \omega_0 + d\phi(t)/dt \) is described by

\[ W_s(\omega) = \rho_0^* \left[ \frac{\omega - \omega_0}{\rho_0^*} \right]^{-3/2}, \]

where \( \rho_0^* = d\rho/|d\tau| \) and \( \rho(\tau) \) is the correlation function.

For the normalized frequency \( y = d\phi dt (\rho_0^*)^{-1/2} \)
It should be noted that the variance of the instantaneous frequency does not exist (it tends to infinity owing to the effect of a "phase jump"). Therefore, as a quantitative parameter characterizing the random frequency does not exist (it tends to infinity owing to the effect of a "phase jump"), we use the following:

\[
\langle |\omega - \omega_0| \rangle = \int |\omega - \omega_0| W_\omega(\omega) \, d\omega = (-\rho_0^2)^{1/2}.
\]

When the process is the sum of a sinusoidal signal \(s(t) = A_\omega \cos(\omega t)\) and additive narrowband noise \(\xi(t) = A(t) \cos[\omega_0 t - \varphi(t)]\), we can write \(s(t) + \xi(t) = V(t) \cos[\omega_0 t - \theta(t)]\). The formula for the probability density function is given by Tickhonov (1966) and Zhukov (1962):

\[
W_{x+N}(y) = W_\omega(y) \exp\left[\frac{-\alpha^2(1 + \nu)}{4}\right] \times \left[1 + \alpha^2\left(\frac{y}{2}\right)I_0(\alpha^2(1 - \nu) / 4) + \frac{1}{2}\alpha^2\gamma I_1(\alpha^2(1 - \nu) / 4)\right], \tag{2}
\]

where

\[
\Delta \omega = (\omega_s - \omega_b), \quad y = \frac{d\theta}{dt} (-\rho_0^2)^{-1/2}, \quad q = \Delta \omega (-\rho_0^2)^{-1/2},
\]

\[
\gamma = \frac{1 + q^2 - qy}{1 + (q - y)^2}, \quad \nu = \frac{y^2}{1 + (q - y)^2}.
\]

Here \(\alpha = A_\omega/\alpha\), \(\sigma^2\) is the variance of noise, and \(I_0(z)\) and \(I_1(z)\) are the Bessel functions of imaginary arguments.

The above formulas give the distribution of the instantaneous frequency, but in practice we measure the time interval \(T\) between two zero level crossings of the signal (crossings with the same sign of the derivative). In the first approximation, when \(\Delta \omega \omega \ll 1\), they are related by Levin (1968) as

\[
\omega(t) = \omega_b + \frac{d\varphi}{dt} - \frac{2\pi}{T}. \tag{3}
\]

For the typical bandwidth of received acoustic signals of 10–20 Hz, the second-order correction is about \(10^{-4}\) (Levin 1968), so the approximation in Eq. (3) can be used.

The value \((-\rho_0^2)^{1/2}\), characterizing the width of the signal spectrum, determines the distribution of the instantaneous frequency Eq. (1), which does not depend on the shape and energy of the spectrum (assuming it is symmetrical). The maximum of the probability density function is achieved at \(\omega = \omega_b = 2\pi f_0\) and is equal to \((-\rho_0^2)^{1/2}/2\). For Gaussian spectra the value \((-\rho_0^2)^{1/2}\)

is equal to the second central moment (Tickhonov 1966).

In Bedulin et al. (1991) we were not quite correct in assuming a similarity of the shape of the spectrum and the probability density function. Later, a more careful experimental study confirmed the correctness of Eq. (1) and its independence on a spectral shape. Figure 1 illustrates histograms of the instantaneous frequency for different spectral shapes and widths. The theoretical curves corresponding to Eq. (1b) are also presented. Thus, measuring the intervals \(T\) between zero crossings (quasi-periods) and computing the histogram of \(f = 1/T\), we can estimate the mean frequency \(f_0\) of the spectrum (as the position of the maximum of the histogram) as well the spectral width [from the value of the maximum of the probability density function \(W_{\text{max}}(f)\)] as

\[
\Delta f_c = (-\rho_0^2)^{1/2}/\pi = (W_{\text{max}})^{-1}. \tag{4}
\]

Approximate estimates are also possible by use of relations

\[
f_0 = \frac{1}{N} \sum_i \left(\frac{1}{T_i}\right) \quad \text{and} \quad \Delta f_c = \frac{2}{N} \sum_i \text{abs}(f_0 - f_i).
\]
In practice, we deal with a mix of a narrowband (quasisinusoidal) signal and wider bandwidth noise, but that also can be considered as a quasi-sinusoidal signal. However, theoretical description of these statistics is laborious. So, we restrict the consideration to a limiting case of a sum of a sinusoidal signal and narrowband noise, whose instantaneous frequency distribution \( W_{s,N}(y) \) is described by Eq. (2). Generally, the presence of noise affects the measured frequency of signal so that it is biased toward the center frequency of the noise. This effect is common for mean frequency measurements (Neff and Coulter 1986). Here we suggest a way of improving estimation of the frequency of a sinusoidal signal to compensate for this bias by noise. We define this estimator as the weighted center of the positive part of the difference between the total distribution \( W_{s+N} \) and the noise distribution \( W_{N} \):

\[
f_s = \frac{\int [W_{s+N}(f) - W_{N}(f)]^+ df}{\int [W_{s+N}(f) - W_{N}(f)]^+ df},
\]

where

\[
[W_{s+N}(f) - W_{N}(f)]^+ = \begin{cases} 
  W_{s+N}(f) - W_{N}(f), & W_{s+N}(f) - W_{N}(f) > 0 \\
  0, & W_{s+N}(f) - W_{N}(f) \leq 0.
\end{cases}
\]
for a particular device is determined by its frequency response only and can be measured when calibrating.

Thus, the proposed method gives an unbiased estimate of the frequency of a sinusoidal signal mixed with narrowband noise under any signal-to-noise ratio.

Now we consider some other characteristics of this method. In Fig. 4 time variations of the instantaneous frequency of narrowband signals are shown. Sharp outliers are seen in these plots. They are not the result of instrumental errors, but due to the effect of a “phase jump” (Tickhonov 1966). In the conventional zero crossing technique, when the number of crossings during a given time interval or the time interval corresponding to a given number of crossings are measured, this effect can cause deviation in the measured frequency, especially when short sampling intervals are used. Our method gives an opportunity to check and reject these outliers. To demonstrate an advantage of the new approach we consider the variance of the mean frequency estimates, which usually characterizes the accuracy of an estimator. We use simulated data obtained from measuring the mean frequency of a narrowband signal with the center frequency of 2000 Hz and the bandwidth of 46 Hz in two ways: the conventional zero crossing technique and quasi-period one. In Fig. 5 both values estimated for different sampling intervals used in sodar measurements are presented. It can be seen that the quasi-period standard deviation (the square root of the variance) of the mean frequency estimates is 25%—30% less than that for the zero crossing one.

Profiles of vertical wind calculated by the conventional zero crossing procedure and the quasi-period technique are presented in Fig. 6. It is seen the new method gives a more consistent profile estimate.

In our opinion, the main advantage of this method is that the phase characteristic (frequency) of a signal is measured almost directly, while any spectral or correlation approach calculates phase parameters from amplitude measurements using some model assumptions. Thus, for precise measurement of the frequency of a sinusoidal signal by this technique, only one period is needed; this is not sufficient for any spectral method.

On the other hand, in comparison with a traditional spectral approach, this method is free of spectral broadening due to finite sampling time. For typical sodar

Fig. 4. (a) Time variations of the instantaneous frequency of narrowband signals and (b) a received sodar signal.

Fig. 5. Comparison of the standard deviation of the mean frequency estimates for a narrowband signal, using the quasi-period and zero crossing techniques, vs different sample intervals.

Fig. 6. Vertical wind profile measured by the quasi-period technique (△) and the conventional zero crossing technique (+).
parameters, such broadening is comparable to the actual bandwidth of the received signal.

3. Conclusions

The quasi-period technique described above consists in estimating the probability density function of the instantaneous frequency by measuring the statistics of time intervals between zero crossings.

1) The form of the function Eq. (1) that results from the quasi-period histogram is universal for any spectral shape and is determined by the mean frequency of the signal and its bandwidth only.

2) The quasi-period estimate is free of the effect of spectral broadening caused by a finite sampling interval. This broadening is present when using conventional spectral approaches.

3) The quasi-period estimator allows us to check and reject outliers caused by the phase-jump effect.

4) In the presence of additive narrowband noise, it is possible to obtain an unbiased estimate of the mean frequency of the signal by subtracting the noise distribution [Eq. (1)] from the total distribution [Eq. (2)] and calculating the mean frequency with Eq. (5). Such an estimate is unbiased at any signal-to-noise ratio.

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REFERENCES


