Effects of Incorporating a Brightband Model in a Downward-Looking Radar Rainfall Retrieval Algorithm

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ABSTRACT

A range-profiling method proposed for nadir-looking rain radar has been investigated using aircraft measurements of a typhoon event at 10 and 35 GHz. In order to take into account the effects of change in the hydrometeor phase along the radar beam, it was necessary to modify the original method. Instead of using an analytic expression for the retrieval of radar reflectivity, a numerical solution is described that enables one to include the melting layer contribution. Results show a marked improvement in the estimated rainfall rates even when a fairly simple brightband model is used, indicating the usefulness and necessity of incorporating a brightband model in a retrieval of rain rate with a downward-looking radar. The improvement occurs mainly in the retrieved Ka-band results and highlights the importance of the melting layer in retrieval algorithms, particularly for high frequencies.

1. Introduction

There have been several retrieval methods proposed for spaceborne rain radars such as the Tropical Rainfall Measurement Mission (TRMM) precipitation radar, currently flying in a low earth orbit at 350-km height (Kawanishi et al. 1999). In order to achieve an acceptable footprint size and intensity resolution, these systems tend to operate at high frequencies; hence, to derive rain-rate profiles from their measurements, the algorithms must consider propagation effects such as rain attenuation and non-Rayleigh scattering.

One of the promising methods that deal with both these effects is the $N_0$ adjustment algorithm (Kozu et al. 1991), which determines the rainfall rate by first estimating the drop size distribution (DSD). This estimation is based on the total attenuation constraint, which enables the adjustment of the parameter $N_0$ in the DSD until the path integrated attenuation obtained from the measured reflectivity profile matches that derived from the surface echo, a technique that comes under the category of surface reference methods. In the current “standard” TRMM precipitation radar (PR) algorithm to derive 3D rain-rate distribution (code number 2A25), a similar and a simpler concept of DSD estimation is implemented to improve the retrieval accuracy (Iguchi et al. 1998).

In the original version of the $N_0$ adjustment method, the Hitschfeld–Bordan (H–B) algorithm (Hitschfeld and Bordan 1954) was used to derive the effective radar reflectivity $Z_e$ from the radar measurements. For simplicity, it was assumed that the nature of hydrometeors along a given radar beam were all the same. In this paper, we consider potential improvements to this assumption, namely, the inclusion of a melting layer. Because of certain inherent assumptions in the H–B solution, the method becomes inapplicable for cases where the hydrometeor type is variable with range, and hence
we suggest an alternative solution for the retrieval of \( Z_e \), based on a numerical iteration procedure.

The improvements obtained from these modifications are tested using airborne radar measurements of a typhoon event made at 10 and 34.5 GHz. Results are evaluated in terms of the consistency between the two sets of independently derived rain-rate profiles. The main purpose is to illustrate the effectiveness of including a melting layer model for better retrieval of rainfall profiles including the bright band. The results of this paper would also be useful to illustrate the validity of the TRMM radar algorithm that uses a similar melting layer model.

2. The retrieval method

a. DSD assumptions

One of the fundamental assumptions of the \( N_0 \) adjustment method concerns the drop size distribution in rain. The method uses the two-scale DSD model, given by the negative exponential function

\[
N(D) \, dD = N_0 \exp(-\lambda D) \, dD.\tag{1}
\]

For a given radar beam, \( N_0 \) is kept fixed while \( \lambda \) is allowed to vary with range. Under these conditions, the specific attenuation \( k \) (dB km\(^{-1}\)) and the effective radar reflectivity \( Z_e \) are related to a good approximation via the power law:

\[
k = \alpha(N_0)Z_r^{\beta(N_0)}.\tag{2}
\]

For a given \( N_0 \), coefficient \( \alpha \) and \( \beta \) are derived from linear regressions between \( \log(Z_e) \) and \( \log(k) \) obtained for several different DSDs using Mie theory. Their values are stored as a function of \( N_0 \) in a lookup table and used for the retrieval of \( Z_e \) from the modified \( Z_m \) measured reflectivity) profile.

b. Melting layer implementation

Here, \( Z_e \) and \( Z_m \) at a given range gate "r" are related via the radar equation

\[
Z_e(r) = Z_m(r) \exp \left[ 0.2 \ln 10 \int_0^r \alpha(s)Z_e^{\beta(N_0)}(s) \, ds \right].\tag{3}
\]

If \( \beta \) is independent of range, then Eq. (3) can be solved by the standard H–B method (Hitschfeld and Bordan 1954), which gives an analytic solution for any range. In the previous version of the retrieval method (Kozu et al. 1991), it was possible to use this solution because, for a given \( N_0 \), it assumed that the same \( k-Z \) relationship held throughout the radar measured \( Z_m \) profile undergoing retrieval. However, if a melting layer model is to be implemented within the overall algorithm, \( \alpha \) and \( \beta \) become range dependent, particularly within the melting layer, and hence Eq. (3) cannot be solved analytically.

An alternative solution is to convert Eq. (3) into decibel units. If \( dBZ_e \) and \( dBZ_m \) represent the two reflectivities in decibels, then Eq. (3) at the \( n \)th range gate can be rewritten as

\[
dBZ_e(n) = dBZ_m(n) + \sum_{i=0}^{n-1} \delta A_i + \alpha(N_0)Z_e^{\beta(N_0)}(n) \Delta s,\tag{4}
\]

where

\[
\delta A_i = 2\alpha(N_0)Z_e^{\beta(N_0)} \Delta s.\tag{5}
\]

For \( n = 1 \), \( \Sigma \delta A_i \) will be zero and Eq. (4) can be solved for \( dBZ_e(1) \) numerically, for example, by the Newton–Raphson iteration procedure. Here, \( \delta A_i \) is then determined using Eq. (5) and included in Eq. (4) to solve for \( dBZ_e(2) \). [Note, in solving for \( dBZ_e(n) \) in Eq. (4), it is important to set the initial estimate to \( dBZ_m(n) + \Sigma \delta A_i \). Problems with solution divergence may arise if other estimates are used.] The procedure is repeated until \( n \) reaches the surface echo range gate \( n_s \). This finally gives \( Z_e \) and \( k \) profiles as output for a given \( N_0 \).

The total path attenuation \( A(N_0) \) at a given frequency is then derived using another power-law relation between \( k \) and \( Z_e \) whose coefficients can be obtained from the appropriate linear regressions for the chosen \( N_0 \). The total path attenuation \( A(N_0) \) at a given frequency is then derived using

\[
A(N_0) = \sum_{i=1}^{n_s} 2k(r_i, N_0) \Delta s,\tag{6}
\]

and compared with the appropriate surface echo-derived attenuation \( A_{opt} \). The solution for \( N_0 \) is obtained when the quantity \([k(N_0) - A_{opt}] \) reaches a minimum value. The \( Z_e \) profile for the optimum \( N_0 \) is then used to derive the rainfall rates \( R(n) \) using another power-law relation between \( k \) and \( Z_e \) whose coefficients can be obtained from the appropriate linear regressions for the chosen \( N_0 \). This power law is the form

\[
R_e(n) = \alpha_1(N_{0, opt})Z_e(n)Z_r^{\beta_1(N_{0, opt})},\tag{7}
\]

where \( \alpha_1(N_{0, opt}) \) and \( \beta_1(N_{0, opt}) \) are calculated from the \( Z-R \) relationship derived from the DSD containing the optimum value of \( N_0 \) (\( N_{0, opt} \)). Examples of these coefficients can be found in Table 1 of Kozu et al. (1991).

The melting layer model considered here is the non-coalescence–nonbreakup model as described by Awaka et al. (1985). This model assumes spherical brightband particles composed of a homogenous mixture of water, ice, and air. A description of the model is given in the appendix. In the analysis presented here, a melting layer thickness of 900 m is chosen in which the fractional volume content of water is assumed to change from 0.011 to 0.85. The choice of the melting layer thickness was made after examining several measured copolar and cross-polar reflectivity profiles at X band. This melting region is divided into six equal segments and in each segment, the best-fit \( k-Z \) coefficients are derived by computing their Mie scattering and extinction cross sections for the melting particles. The same range of DSD’s
as in rain is used and, as before, the coefficients are derived for various values of $N_0$.

To complete the lookup table, a temperature variation in the rain region is also considered. However, since the attenuation coefficients are not so sensitive to temperature (as compared with the melting process), it was adequate to segment the rain region in steps of 10°C. Starting from the bottom of the melting layer, which was assigned to be rain at 0°C, region up to the sea surface was divided into equal segments, with a thickness depending on the ground temperature. In the region above the melting layer, dry snow is assumed, and hence no attenuation correction is applied.

### 3. Results

An example profile of $Z_m$, taken through a stratiform rain event is shown in Fig. 1. These data were taken as part of a campaign to continue the study of algorithm development for the TRMM radar. The experiment was conducted in the western Pacific in September 1990 (Kumagai et al. 1993) with the National Aeronautics and Space Administration DC-8 aircraft that was flying at an altitude of 12 km above sea level. The bright band is clearly visible at around 7.5-km range. In this region and in the ice above, the differences in $Z_m$ between X and Ka band are mainly due to Mie scattering while in the rain region below, the attenuation effects become increasingly dominant.

The X-band signal suffers very little attenuation except for high rainfall rates. In the case of Fig. 1, the total two-way path attenuation is around 1.7 dB, which is, in fact, comparable to the fluctuations observed normally in the sea surface echoes. For such cases, the solution for $N_0$ is best found by deriving $Z_{m_{ka}}$ from the $Z_m$ profile and applying the total attenuation constraint at Ka band (Kozu et al. 1991). The only extra parameter this requires is the attenuation coefficient that relates $k$ at 35 GHz to $Z_r$ at 10 GHz. Figure 2 compares the $Z_{m_{ka}}$ profile so obtained with the corresponding profile derived using the $Z_m$ measurements.

Close agreement is seen between the two curves, which suggests that the Ka-band attenuation is being estimated fairly accurately. The rain-rate profiles obtained using the $R$-$Z_e$ relations for the two $N_0$ solutions are shown on the left-hand side of Fig. 2. On average, the curves have rainfall rates of 4.3 (X band) and 5.4 (Ka band) mm h$^{-1}$. It also appears that the vertical structure of retrieved rain rate does not have significant brightband peak (different from $Z$-factor profile), suggesting that the application of this melting layer model is sufficient at these frequencies. This has also been found to be generally the case with the TRMM radar retrieved data for stratiform precipitation (T. Iguchi 1999, private communication).

A summary of the results from a continuous set of measurements from the typhoon experiment is presented in Fig. 3. The top graph (Fig. 3a) shows the two-way total attenuation at Ka band and the two remaining graphs show the comparison of results. The variation of $N_0$ is compared in Fig. 3b, while the path-averaged rainfall rate (averaged over the rain region only) is shown in Fig. 3c.

Qualitatively, the results appear to be quite well correlated. The two independent estimates of $N_0$ agree reasonably well with each other as do the averaged rainfall rates. Furthermore, high attenuation seems to correspond to high rainfall rates, which in turn, correspond generally to low $N_0$. The range of $N_0$ values also lies within expected values. It is noticeable, however, that the agreement gets a little poorer for rainfall rates approaching 10 mm h$^{-1}$. To examine this further, a large section of radar measurements (around 800 records) were analyzed. Some quality check was applied to the analysis to eliminate the observations taken during aircraft attitude fluctuation and banking. In addition, it was found that in some cases the Ka-band signal suffered
so high an attenuation that the $Z_{\text{meas}}$ measurements just above the sea surface were indistinguishable from the system noise. To avoid these cases, a minimum requirement of a 3-dB signal-to-noise ratio was imposed.

Figure 4a compares the retrieved rainfall rates from X- and Ka-band data independently. The rainfall rates are path averaged, that is, each point represents the mean value averaged over the vertical profile of the retrieved rain fall rates. The rms error for these points is 25%. Also drawn is the line of equality, which is included for reference. While there is good agreement below 5 mm h$^{-1}$, the deviation is once again apparent at high rain rates. The fact that this corresponds to low values of $N_0$ indicates the presence of large drops that are probably Mie scatterers at Ka band in rain but not at X band. If this is the case, the use of a constant attenuation coefficient that relates $k$ at 35 GHz to $Z_e$ at 10 GHz may not be entirely accurate.

The accuracy of the brightband model used in this study should also be examined. However, to illustrate the improvement obtained by incorporating the brightband model correction, rainfall rates were retrieved using the earlier version of the $N_0$-adjustment algorithm, that is, with no melting layer, where the H–B solution is used to retrieve $Z_e$ using coefficients derived solely for rain at 0°C. Figure 4b shows the resulting comparisons. The results show a significantly large rms error (51%).

The improvement obtained with the present modifications occurs mainly because the estimates from the 35-GHz data gets closer to the 10-GHz estimates, while the latter exhibits significantly smaller differences between the old and the new methods. This suggests that the melting layer modeling is particularly important at high frequencies. If the brightband model is not incorporated within the retrieval method, a tendency to underestimate the total attenuation would occur, which when constrained by the surface-echo derived total path integrated attenuation would result in increased estimates of $N_0$ and a corresponding increase in the retrieved rainrates. This overestimation is evident in Fig. 4b and also in Fig. 4a but to a much lesser extent.
Further improvements may be possible if one uses a more realistic, but more complicated, model for the melting layer (e.g., those considered in Goddard and D’Amico 1993). One problem with such methods is that they require the initial mass density as model input, which is usually unknown. However, as shown in that article, this could be estimated from the dBZ enhancement observed within the melting layer, at least for stratiform rain.

For convective storms, the problem is more acute since the convection process causes the melting particles to mix with ice and rain, which results in an extended melting region. In the absence of cross-polar and Doppler velocity information (as in TRMM), it may perhaps be adequate to assume some form of monotonic variation in the k–Z coefficients from the top of the ice region down to the sea surface.

Finally, it should be mentioned that although the new method uses an iteration procedure to retrieve Z_e, many test simulations have shown that the method gives stable solutions if the initial conditions are set to the correct values (as noted earlier in section 2b) and if the radar calibration is set to the correct value. The present method also gives almost identical results to the H–B solution and data analyses show that it achieves stable solutions, provided the initial values are set to appropriate estimates.

4. Conclusions

A rain retrieval method proposed for nadir-looking radar has been modified to include the effects of melting layer in the rain region. Analysis using typhoon data at 10 and 35 GHz shows that these modifications provide significant improvement in the estimated rainfall rates. The improvement occurs mainly in the retrieval of the 35-GHz measurements.

Unlike the analytic solution (the Hitschfeld–Bordan equation) used in the original method, the modified version uses an iteration procedure to obtain the effective radar reflectivity profile. Nevertheless, various simulations and data analyses show that it achieves stable solutions, provided the initial values are set to appropriate estimates.

The improved results highlight the importance of the melting layer in the rain retrieval. Since a similar melting layer model is used in the standard TRMM PR algorithm (2A25) to retrieve the 3D rain-rate distribution, the results provide some form of justification for the 2A25 algorithm. Further work is needed to investigate the effects of using more complicated models, in terms of the accuracy achieved in the retrieved rainrate profiles.

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APPENDIX

The Brightband Model

**a. Composite dielectrics model**

There are several brightband models that have been developed in the recent past [e.g., Hardaker et al. (1995) and references therein]. The brightband model used in this paper is outlined below referring to Awaka et al. (1985). As a first step, a composite dielectrics model is used to calculate the dielectric constant of the brightband particle. We assume that bright band particle (snowflake) is composed of a uniform mixture of water, ice, and air. Dielectric constants of water, ice, and air \( (\varepsilon_w, \varepsilon_i, \text{and} \varepsilon_a) \) respectively) are related to the dielectric constant of their mixture, that is, snow, \( (\varepsilon_s) \) with

\[
\frac{\varepsilon_s - 1}{\varepsilon_s + 1} = P_w \frac{\varepsilon_w - 1}{\varepsilon_w + 1} + P_i \frac{\varepsilon_i - 1}{\varepsilon_i + 1} + P_a \frac{\varepsilon_a - 1}{\varepsilon_a + 1}.
\]

where \( U \) is a form factor; and \( P_w, P_i, \text{and} P_a \) are fractional volume contents of water, ice, and air, respectively. By definition, \( P_w + P_i + P_a = 1 \). Also, letting \( \rho_w, \rho_i, \text{and} \rho_a \) be the density of water, ice, and air, respectively, the density of snow \( \rho_s \) can be written as

\[
\rho_s = \rho_w P_w + \rho_i P_i + \rho_a P_a.
\]

Noting that \( \rho_w = 1 \text{ g cm}^{-3}, \rho_i = 0.92 \text{ g cm}^{-3}, \text{and} \rho_a \ll 1 \text{ g cm}^{-3} \), we have \( P_s \approx (\rho_w - \rho_a) / 0.92 \). Since there is an empirical relation \( \rho_s = \sqrt{P_w} \) (Matsumoto and Nishitsuji 1971; Nishitsuji and Hiyajama 1971), we have the relation \( P_s = (\sqrt{P_w} - P_a) / 0.92 \). Noting that the last term of Eq. (A1) can be omitted because \( \varepsilon_s \approx 1 \), and that \( \varepsilon_w \text{ and} \varepsilon_i \) are uniquely determined by specifying the temperature of the particle \( T \), the quantities necessary to determine the dielectric constant of snow \( \varepsilon_s \) are \( P_w, T, \text{and} U \). Table A1 lists those parameters in the bright band as well as the resultant refractive indices of snow at several frequencies.

**b. Noncoalescence–nonbreakup (N/N) model**

The second step of the modeling is to determine the drop size distribution. For simplicity, we assume that a particle changes its phase without any coalescence or breakup during its fall within the bright band, so that its melted diameter, \( D_s \), is unchanged. It follows that the flux, that is, rain rate, is constant over the bright band. Therefore, letting \( D_0, N(D_0), V_i(D) \) being the diameter, DSD, and falling velocity of a snowflake, and
Letting $D_m$, $N_s(D_m)$, $V_s(D_m)$ being the corresponding quantities of a melted particle (i.e., raindrop), we have

$$N_s(D_m)V_s(D_m)\,dD_m = N_h(D_m)V_h(D_m)\,dD_m.$$  \hspace{1cm} (A3)

Since $D_s = D_m\rho_s^{-1/3}$,

$$N_s(D_s) = \rho_s^{1/3}V_h(D_s)V_s(D_s)^{-1}N_h(D_m).$$  \hspace{1cm} (A4)

Equation (A4) states that the DSD in the bright band is determined automatically from the dropsize and fall velocity distributions of raindrops, if we know the fall velocity distribution in the bright band.

The only remaining unknown quantity to determine the DSD in the bright band with this N/N model is the fall velocity distribution in the bright band. This is estimated from the fall velocity of dry to wet snowflakes obtained by Magono and Nakamura (1965), given by the following (where the units of $V_r$ and $D_s$ are m s$^{-1}$ and mm, respectively):

$$V_r(D_s) = 8.8 \times \left[0.1D_s(\rho_r - \rho_s)^{1/2} \right]$$

for $(0.05\rho_r \leq 0.3 \text{ g cm}^{-3})$. \hspace{1cm} (A5a)

$$V_r(D_s) = 3.3 \times [\rho_r - \rho_s]^{1/2}$$

for $(0.05 \rho_r \geq 1 \text{ g cm}^{-3})$. \hspace{1cm} (A5b)

and the falling velocity of raindrops $V_r$, using the following interpolation scheme. Since usually $\rho_r \gg \rho_s$, Eq. (A5a) can be approximated as $V_r = 2.8\rho_r^{1/3}\sqrt{D_m}$. [Note that $D_s = D_m(\rho_r/\rho_s)^{-1/3}$ where $\rho_s = 1$ in cgs units.] This means that the fall velocity of a snowflake having a given $D_m$ is proportional to $\rho_r^{-1/3}$ in the region, $0.05 \leq \rho_r \leq 0.3 \text{ g cm}^{-3}$. If we assume that this $\rho_r$ dependence of the fall velocity holds also in the region, $0.3 < \rho_r < 1 \text{ g cm}^{-3}$, the following relation is obtained for a given $D_m$:

$$V_r = \frac{(V_s - V_0)(\rho_r^{1/3} - 0.3^{1/3})(1 - 0.3^{1/3}) + V_0}{(0.3 < \rho_r < 1 \text{ g cm}^{-3})},$$

where $V_0$ is the fall velocity of a snowflake at $\rho_r = 0.3 \text{ g cm}^{-3}$ given by Eq. (A5a). Since there is the relation $\rho_r = \sqrt{P_w}$, the fall velocity distribution at a specified altitude can be calculated from the $P_w$ data given in Table A1.

**REFERENCES**


