A Multiparameter Polarimetric Radar Simulator

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ABSTRACT

A polarimetric multiparameter radar simulator is described here that is able to reproduce the in-phase and in-quadrature components of the radar echo associated to a population of nonspherical raindrops that is moving under the effect of fall speed, wind, and turbulence. Thanks to a detailed representation of the physical environment, the simulator provides useful information about the different impact of a number of factors (distribution of drops with size, drop shape, and motion, etc.) on radar observables. Moreover, the accuracy of rain estimates involved by the limited acquisition time of operational radars can be investigated by a statistical analysis of the synthetic sequences. The results of the first comparisons between simulated Doppler spectra and those collected by an S-band meteorological radar confirm the usefulness of this software instrument.

1. Introduction

The derivation of rain estimates starting from radar observables is a complex procedure that involves a careful evaluation of the meteorological environment under investigation: the amount and the evolution of the population of scatterers that fill the radar cell together with the microphysical properties of the hydrometeors are of main concern. Moreover, propagation effects (e.g., signal extra attenuation induced by rain present along the path toward the radar cell) could lead to wrong estimates and must be taken into account. Finally, the influence of the sensor (operating frequency, pulse repetition frequency, antenna pattern, etc.) must be considered.

Only in a number of simple cases, after assumptions and approximations have been introduced, is it possible to extract close analytic expressions that relate rain parameters to radar measurables. In this context, an accurate, physically based radar simulator proves extremely useful for the assessment of the effects of all the above-mentioned contributions to the received echoes.

In a previous work (Capsoni and D’Amico 1998) a software radar simulator, developed at Politecnico di Milano, was presented. The synthetic radar signal was generated by adding up (in amplitude and phase) all the contributions coming from each single scatterer, present in a virtual meteorological environment. The characteristics of the radar system were taken into account, as well as the effects of propagation; however, the hydrometeors were assumed to be spherical in shape and falling in stagnant air.

The simulator has been recently extended to overcome these limitations; in particular, the anisotropic effects induced by the deformation and canting of the hydrometeors have been included, as well as the perturbations of their fall motion due to turbulence and wind, and the differential propagation impairments.

In this work, we give a description of the most important modifications that have been introduced in the original simulator and present a sample of the results that can now be obtained. Last but not least, measurements collected by an S-band Doppler radar were processed and the data were compared with a database of rain echoes synthesized by the simulator.

Due to the high number of symbols introduced in the following, Table 1 gives, for the sake of clarity, a list of the most frequently used variables both with a synthetic description.

2. The radar simulator: Philosophy

The new version of the simulator retains much of the basic structure of the original one, described in Capsoni and D’Amico (1998); for this reason, only the main points will be reported here for convenience. In general, the signal received from a pulse Doppler radar (Fig. 1) is due to the superposition of signals backscattered by the hydrometeors present in the radar resolution volume (Doviak and Zrnić 1994):

\[ I(\tau_\alpha, T_\gamma) = H \sum_i \left[ \frac{\sqrt{\sigma_i} |W_i|}{r_i^2} f(\theta_i, \phi_i) \cos(\gamma_i) \right] \]

\[ Q(\tau_\alpha, T_\gamma) = H \sum_i \left[ \frac{\sqrt{\sigma_i} |W_i|}{r_i^2} f(\theta_i, \phi_i) \sin(\gamma_i) \right], \]

where

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Antenna directivity function $|F(\theta, \phi)|$

FIG. 1. Geometry of the radar environment.

The meteorological environment

A fundamental aspect of the simulator’s operation regards the creation of an “artificial rain,” with a number of particles as small as possible (to keep computing time within reasonable limits), but nevertheless capable of reproducing the scattering behavior of the real raindrop population. Stated in a different way, we have to sample properly the drop size distribution $N(D)$. To accomplish the task, in the original version of the simulator a diameter interval ($D_{\text{rain}}$, $D_{\text{max}}$), selected by the user, was divided into $N_d$ equal intervals and a compressed DSD was derived from the real DSD (discretized over the above $N_d$ points) by a logarithmic law (Capsoni and
D’Amico 1998). A good compromise between physical accuracy and computing time was assured provided that a minimum of $N_y = 300$ diameter classes and $M_y = 2 \times 10^4$ virtual particles were set respectively.

From the computational point of view, the new version of the simulator is much more demanding than the original, due to the increased complexity of the meteorological model implemented. In particular, when we introduce wind and turbulence effects, we have to handle particles moving with arbitrary motion, which, in principle, may be a function of both their position and time; moreover, the particles not being spherical, their backscattering cross section is a function of orientation that, again, may be space and time dependent. Since the operations required to update the state of the “environment” have to be repeated $M_y$ times for each pulse, the execution time significantly increases. These considerations led us to search for a new compromise between computing time and physical accuracy, through the investigation of a more efficient DSD compression algorithm: our goal was reducing $M_y$ as much as possible, without losing too much in terms of physical consistency of our synthetic environment. As we will show in section 3a, the procedure adopted by the new version of the simulator permits a substantial reduction of $M_y$, and, at the same time, ensures that the Doppler spectrum associated with the synthetic signal approximates well the theoretical one. The Doppler spectrum is tightly related to the microphysical properties of the population of particles that fill the radar beam (information about their nature, shape, size, and motion can, in principle, be retrieved by the analysis of the associated Doppler spectrum). Moreover, under certain conditions, it is related to the DSD by a one-to-one relation. In fact, if we consider Eq. (1) and (i) we force both the antenna directivity function $f(\theta, \phi)$ and the range weighting function $W_r$ to be constant inside the resolution volume $V$ and zero elsewhere; (ii) the radial extent of the radar cell is negligible with respect to its range from the radar so that $r_c$ can be considered constant for all the particles inside; and (iii) the contribution $I_i$ due to attenuation along the path is constant in $V$; then it follows that the echo mean power $dP$ associated with the VN($D$)d$D$ scatterers with diameters ranging from $D$ to $D + dD$ is given by

$$dP = A\sigma_r(D)N(D)dD,$$  \hspace{2cm} (4)

where $A$ is a constant. The power density per unit of the diameter $D$ is

$$S(D) = \frac{dP}{dD} = A\sigma_r(D)N(D).$$  \hspace{2cm} (5)

Moreover, (iv) we envisage pointing the radar vertically and (v) drops to be falling at their terminal velocity $v$ (m s$^{-1}$) in stagnant air, so that $v$ reduces to its radial component. We assume with Atlas et al. (1973):

$$u(D) = 9.65 - 10.3e^{-6D},$$  \hspace{2cm} (6)

where the units for $D$ are in centimeters. Since $v$ is a strictly monotonous (increasing) function of $D$, the inverse $D = D(v)$ can be determined with no ambiguity. Finally, being the Doppler spectrum $S(v)$, the mean power density per unit of the radial velocity, we have

$$S(v) = \frac{dP}{dv} = \frac{dP}{dD}\frac{dD}{dv} = A\sigma_r[D(v)]N[D(v)]D'(v).$$  \hspace{2cm} (7)

Therefore, if we run our simulator having set the conditions as described above, and if we verify that the simulated Doppler spectrum agrees well with the theoretical expression given by Eq. (7), then we may conclude that the compressed DSD represents an accurate sample of the real DSD. Note that $S(v)$ is a random variable, depending on the relative phase between the signals [see Eq. (1)] contributed by the raindrops, which is unpredictable because they are randomly placed inside the resolution volume. Hence, the equality between theoretical and simulated spectra has to be satisfied in a statistical sense.

**DSD compression algorithm**

The compression algorithm adopted by the new simulator envisages the following steps.

1) **Automatic selection of $D_{\text{min}}$ and $D_{\text{max}}$**

We tried to minimize the diameter interval ($D_{\text{min}}$, $D_{\text{max}}$) over which the DSD is discretized: the smaller ($D_{\text{min}}$, $D_{\text{max}}$), the lower the number of diameter classes $N_y$, hence the amount of virtual drops $M_y$ required. The procedure selects the significant portion of the theoretical mean power density (5), whose behavior depends on the expressions of the backscattering cross section and of the DSD, which are both user-selected. The options currently available for $N(D)$ are the Marshall–Palmer distribution (Marshall and Palmer 1948) and the gamma distribution (Ulbrich 1983); for $\sigma_r(D)$ the simulator allows the user to select between the analytical expression found by Rayleigh (Oguchi 1983) and a numerical solution (Oguchi 1977). The Rayleigh model provides an approximation of the backscattering cross section of distorted raindrops (oblate spheroids), in a close analytical form, that is accurate at S band and generally at C band. However, recent studies (Carey et al. 2000; Zrnić et al. 2000) show that Rayleigh scattering is not necessarily a good approximation at C band in tropical rainfall. In the above case and also at higher frequencies, the numerical algorithm based on the point-matching technique (Oguchi 1977) should be used. At the operational frequencies of meteorological radars (S, C, and X bands), the power density $S(D)$ is a single maximum function. The simulator computes the points $D_{\text{min}}$ and $D_{\text{max}}$, where $S(D)$ is 20 dB below its maximum. A physical bound on $D_{\text{max}}$ has been established taking into account the limit given by the mechanical instability of big raindrops: following Oguchi (1983), we set $D_{\text{max}}$...
≤ 8 mm. The function S(D) is therefore truncated. All the classes that give contributions below our threshold level are considered as empty. Obviously, the above procedure is correct only if the errors produced by the thresholding technique are negligible. Gordon (1995) calculated the error upper bounds when we estimate the first two spectral moments from a truncated Doppler spectrum as a function of the threshold level: application of the above-mentioned expressions shows that a level equal to 20 dB below the maximum ensures negligible errors.

2) Determination of the Compressed DSD

The segment (D_{min}, D_{max}) is divided into a user-defined number of diameter classes N_1. After that, M_d virtual drops are distributed into the N_1 classes. Named \(N_{d,m} = V N(D_w) \Delta D\) the number of particles of the mth diameter class falling inside a radar cell with volume V and \(M_{d,m}\) the corresponding amount of virtual particles, the new simulator adopts the following compression algorithm:

\[
M_{d,m} = \begin{cases} 
1, & \text{if } N_{d,m} < 1 \\
N_{d,m}, & \text{if } 1 \leq N_{d,m} \leq N^* \\
N^*, & \text{if } N_{d,m} > N^*. 
\end{cases}
\]  

(8)

Since each virtual particle represents \(Q_{d,m} = N_{d,w}/M_{d,m}\) real drops, we need to multiply its backscattering cross section by the factor \(Q_{d,m}\). As we will demonstrate, there exists a minimum number \(N^*_1\) of fictitious particles with diameter \(D_w\) that permits us to reproduce the Doppler spectrum of a corresponding population of \(N_{d,m} > N^*_1\) real drops. Therefore, if \(N_{d,m} > N^*_1\), we can force \(M_{d,m} = N^*_1\) without loss of information; if \(N(D_w) \leq N^*_1\), the (discretized) real and fictitious DSDs coincide. We also set a minimum of \(M_{d,m} = 1\) even if \(N_{d,m} < 1\) in order to avoid the presence of classes without particles, that would give rise to “holes” in the Doppler spectrum. Finally, note that the DSD compression algorithm needs two input parameters, \(N_c\) and \(N^*_1\), while \(M_d\) is determined once \(N(D)\), the expression of the backscattering cross section, and the resolution volume size have been specified by the user. An upper bound on the overall number of virtual particles \(M_d\) is given by the product between the number of diameter classes \(N_1\) and the maximum number of virtual drops per class \(N^*_1\):

\[M_d \leq N_1 N^*_1.\]  

(9)

A number of simulations have been carried out in order to determine proper values for \(N_c\) and \(N^*_1\); system parameters used for these simulations are listed in Table 2. Moreover, we forced all the conditions that lead to Eq. (7). For \(N_c\) ranging from 10 to 300 and for \(N^*_1\) from 1 to 50, we generated very long sequences of 218 samples. Figure 2 shows the relevant portion of the 256-point Doppler spectrum obtained averaging 1024 spectra, in the case of \(N_c = 300, 200, 30, 10, N^*_1 = 50\). Since the pulse repetition frequency (PRF) is 1 kHz, the spectral resolution allowed is 3.9 Hz, that is, 0.2 m s\(^{-1}\) (at 2.8 GHz). Note that the reduction of \(N_c\) seriously affects the results: when \(N_c = 30\) the sawtooth profile due to insufficient DSD sampling is clearly visible. On the other hand, we verified that the curves are rather insensitive to \(N^*_1\); the spectrum corresponding to \(N^*_1 = 1\) is very similar to that obtained with \(N^*_1 = 50\). The above result is not surprising since the Doppler spectrum represents a mean value, which is properly estimated if we generate sequences long enough to smooth the fluctuations due to the random nature of the sum of drop contributions in (1), no matter how many they actually are. As an index of the agreement between simulated and theoretical spectra we computed the relative mean-square error \(\epsilon\) at the \(N_p\) points internal to the 20-dB interval around the spectral peak previously selected:

\[\epsilon = \sqrt{\frac{1}{N_p} \sum_{p=1}^{N_p} \left(1 - \frac{S_{p,\text{sim}}}{S_{p,\text{th}}^0}\right)^2}.\]  

(10)
Whatever $N^*$, we found that $\epsilon$ remains below 10% if $N_i \geq 200$, while it grows up to about 20% if $N_i = 100$ and exceeds 50% when $N_i = 30$. If you consider that the discrepancy between simulations and theory can be partially ascribed to the finiteness of our sample, it can be stated that $N_i = 200$ warrants an acceptable DSD sampling error below 10%.

Once $N_i$ has been determined, we have to set $N^*$. For this purpose, we consider the signal contributed by the $N_{d,m}$ particles that populate the $m$th diameter class; $N_{d,m}$ determines the amplitude of the fluctuations of the signal power associated to the $m$th class (i.e., the fluctuations of the corresponding point in the Doppler spectrum): in the case $N_{d,m} > N^*$, the $m$th class is simulated by only $N^*$ drops, therefore we have to investigate the agreement between the stochastic signals produced by $N_{d,m}$ and $N^*$ particles, respectively. According to the central limit theorem, if $N_{d,m}$ is large enough (theoretically infinite), the associated in-phase and in-quadrature components at the $m$th pulse are mutually independent Gaussian variables with zero mean and equal variance and signal power shows an exponential probability density function (PDF). In order to select $N^*$, we generated sequences assuming a monodisperse DSD and compared the (estimated) power PDF to the exponential: in this special case, we have only one diameter class ($N_i = 1$) while the number of fictitious drops $N_d$ is variable. Without lack of generality, we took under examination only one diameter class, $D = 1$ mm, because the only difference between particles of different sizes is their speed $v_{\text{det}}$, which is deterministic.

Figure 3 depicts the theoretical and the simulated PDFs for $M_d = 50$, 5, and 2. As expected, we see that the larger $M_d$, the closer the simulated and the theoretical PDFs are. If we compute the mean-square error $\epsilon$ and assume 10% to be an acceptable value (as it was for $N_i$), we conclude that at least 10 virtual drops for each class are necessary. Therefore, in Eq. (8), we put $N^* = 10$. Finally, since we have chosen $N_i = 200$, we need $M_d = 200 \times 10 = 2 \times 10^3$ particles to simulate the DSD when the inequality $N_{d,m} > N^*$ holds for all the diameter classes [see Eq. (8)], or less otherwise. This implies a reduction of a factor of 10 (at least) in computing time with respect to the original DSD algorithm.

It should be noted that the above results hold under the hypothesis of a given spectral resolution (0.2 m s$^{-1}$ in our case). If we increase the number of spectrum points and/or decrease the PRF, it will be necessary to correspondingly increase $N_i$. As a general rule, the quality index $\epsilon$ previously introduced does not change when we choose the number of diameter classes $N_i$, according to the following equation:

$$N_i = \frac{K \text{PRF} \lambda}{N_{ds}},$$

where $N_{ds}$ is the number of Doppler spectrum points; PRF and $\lambda$ are expressed in hertz and meters, respectively; and $K = 478$ m$^{-1}$ Hz$^{-1}$.

4. Effect of wind and turbulence

Raindrop motion affects the time autocorrelation function of rain echo. Information about echo decorrelation time is of paramount importance in the assessment of meteorological parameters through radar observables because it allows us to predict how many samples have to be collected in order to achieve accurate estimates of such parameters (in a statistical sense). If we assume that the particles move under the influence of the gravitational force only, as it was the case in the previous version of the simulator, each scatterer falls vertically at its terminal velocity, according to (6). Since the degree of echo correlation is related to the spread of radial components of raindrop speed, it is interesting to compare two opposite situations:

- the radar points vertically (maximum radial velocity spread), and
- the radar points horizontally (minimum radial velocity spread).

Simulations have been devoted to estimate the decorrelation time $\tau_d$ found at different rain rates $R$; $\tau_d$ is alternatively defined as the time delay corresponding to the values 0.7, 0.5, and 0.368 (1/$\tau_d$) of the autocovariance function $C(\tau)$ of the echo power.

At 90$^\circ$ of elevation (Fig. 4), $\tau_d$, being in the order of a few milliseconds, slightly increases with the rain rate. At $C(\tau) = 0.5$ (square dots), $\tau_d$ grows from about 5 to 9 ms as the rain rate changes from 1 to 100 mm h$^{-1}$. Figure 4 also shows a linear least squares fit that approximates quite well the results. From a theoretical point of view such a behavior can be explained if we consider the Doppler spectrum of the signal, since the decorrelation time is related to its width through an
A simple model of isotropic turbulence has been implemented: a randomly oriented speed component is added to the deterministic value for each particle. The amplitude of this vector is also random and follows a user-defined distribution. The autocorrelation properties of the process above can be taken into account by modifying both direction and amplitude at a given rate.

Simulations were run to evaluate the impact of wind and turbulence on echo decorrelation at 1° of elevation with $R = 10 \text{ mm h}^{-1}$. In the first case above, the following power law expression was assumed (Brussaard 1976):

$$U_r(h) = U_r(h_1) \left( \frac{h}{h_1} \right)^\alpha$$  

where $U_r(h)$ is the wind speed ($\text{m s}^{-1}$) at height $h$, $h_1$ is a reference height, and $\alpha$ is a coefficient that depends on surface roughness and atmospheric stability. Since Eq. (12) is effective up to a few hundred meters above the ground, the range of the simulated radar cell was chosen so that the corresponding edge heights were $h_1 = 10 \text{ m}$ and $h_2 = 300 \text{ m}$, respectively. Wind direction was assumed to be radial, that is, directed along the radial straight line that joins the radar location to the center of the radar cell we simulated. The parameter $\alpha$ was set to 0.3 while $U_r(h_1)$ was increased from 1 up to $10 \text{ m s}^{-1}$. Figure 6 reports the horizontal speed of particles as a function of diameter, with height $h$ as a parameter, when $U_r(h_1) = 3 \text{ m s}^{-1}$. Figure 7 finally shows $\tau_d$ as a function of the radial wind speed $U_r(h_1)$, when $R = 10 \text{ mm h}^{-1}$. Taking as a reference the two edge values of $\tau_d$ [at $C(\tau) = 0.5$], obtained with elevation 90° and 1°, that is, about 6 and 90 ms, respectively, in stagnant air conditions, we note that moderate winds as weak as $3 \text{ m s}^{-1}$ (15 km h$^{-1}$) at ground level produce values of decorrelation time similar to the situation of vertical radar alignment.

1) A horizontal wind profile has been reproduced, which intensity is a function of height according to a user-defined function. A vertical gradient of wind speed induces a corresponding spread of raindrop horizontal speed, which, in turn, produces faster echo decorrelation with respect to the case of free fall.

2) A simple model of isotropic turbulence has been implemented: a randomly oriented speed component

inverse relation. Apart from the effects of the antenna pattern and the receiver filter, the Doppler spectrum is given by (7): in our hypotheses (Marshall–Palmer DSD and Rayleigh scattering), the term $\sigma_2(D)N(D)$ is proportional to $e^{-2L_0D^2}$, where $A$ is a function of $R$; as $R$ increases, $e^{-2L_0D^2}$ spreads around its maximum so that the spectrum widens. Nevertheless, since such maximum shifts toward higher diameter (i.e., velocity) values, the corresponding velocity interval tightens because the curve $n(D)$ approaches its asymptotic value. The term $e^{-2L_0D^2}$ decreases signal correlation as $R$ increases, while the radial velocity profile works in the opposite direction. According to our results, the latter effect is slightly prevailing at vertical pointing.

If we consider the situation of (quasi) horizontal beam pointing (Fig. 5), the values of $\tau_d$ grow of a factor close to 10, since the radial component of fall speed is now very small. The linear fit highlights a modest decreasing trend of $\tau_d$ against $R$.

The very high values of $\tau_d$ at 1° elevation would in principle require a very large number of radar echo samples (about 10 times larger than if we point toward the zenith) to ensure the same statistical confidence as for 90°. More “realistic” values of $\tau_d$ can be obtained if we take into account other phenomena that increase signal decorrelation in a real environment.

The new simulator includes both the effect of wind and turbulence in the following way.
Turbulence effects were investigated adding a random zero-mean radial component to raindrop speed. Its amplitude is normally distributed with a standard deviation variable from 0.25 up to 3.0 m s⁻¹ and was refreshed once every second. From Fig. 8, we deduce that power decorrelation time decreases down to 6 ms if we assume a standard deviation of the radial speed greater than about 1 m s⁻¹.

5. The polarimetric enhancement

Several “polarimetric” techniques have been proposed so far in the literature that exploit the asymmetrical shape of raindrops in order to improve the accuracy of rain estimates obtained through radar observations (Seliga and Bringi 1976; Seliga and Bringi 1978; Sachidananda and Zrnić 1987; Scarchilli et al. 1993).

However, a correct interpretation of such measurements is made difficult by the complexity of the relations between polarimetric measurables and rain properties; moreover, power-based measurements (i.e., reflectivity $Z$ and differential reflectivity $Z_{DR}$) can be seriously affected by propagation impairments. The new version of the simulator simulates the operation of a polarimetric radar: the hydrometeors are now assumed to be oblate spheroids (see Fig. 9) whose axial ratio is related to their size through the standard equation

$$\frac{a}{b} = f - ga_0,$$

(13)

where $a$ and $b$ are the major and minor semiaxes (mm), $a_0$ is the radius (mm) of the equivolumetric sphere, and $f$ and $g$ are two constants that depend on the model adopted (Pruppacher and Beard 1970; Morrison et al.)
and extra phase shift by (Oguchi 1983); specific extra attenuation and horizontal polarizations (which coincide with the principal planes) can be related to $R$ (mm h$^{-1}$) through the simple power law relations

$$\alpha_v = a_v R^{b_v}, \quad \beta_v = c_v R^{d_v},$$

(14)

where the constants $a$, $b$, $c$, and $d$ depend on frequency. The values of the constants above have been calculated, at the frequencies of interest, by a direct integration over the DSD of the scattering parameters evaluated through a point-matching program (Oguchi 1973). The computed values are reported in Table 4 for reference; values at frequencies different than those in Table 4 can be obtained by interpolation using a logarithmic scale for frequency and coefficients ($a$ and $c$), and a linear scale for exponents ($b$ and $d$). Equation (14) must be (numerically) integrated along the path to obtain total attenuation and (extra) phase shift.

As a first application of our polarimetric simulator, we generated sequences with quasi-horizontal pointing (antenna elevation equal to 1°) in order to produce estimates of the differential reflectivity $Z_{DR}$, defined as the ratio between the horizontally and vertically polarized power signals $Z_h$ and $Z_v$. Since $Z_{DR}$ is used to predict $R$, it is interesting to determine how many uncorrelated samples have to be collected to ensure accurate estimates of $R$. To accomplish the task, after autocovariance evaluation, the sequences have been split into $N$ subsequences of $N_{inc}$ uncorrelated samples each, with $N_{inc}$ ranging from 1 to 128 assuming decorrelation when signal power autocovariance $C(\tau)$ falls below 0.5; $N$ estimates $Z_{DR}$ have been obtained and the relative standard deviation $\sigma_{ZDR}$ of the estimate has been evaluated. Following Sachidananda and Zrnić (1985), we expressed $\sigma_{ZDR}$ in decibels:

$$\sigma_{Z_{DR}}^2 = 10 \log_{10} \left( \frac{\sigma_{Z_{DR}}^2 + Z_{DR}^2}{Z_{DR}^2} \right).$$

(15)

Since $N = 100$, $\sigma_{Z_{DR}}^2 \approx \sigma_{Z_{DR}}$. Figure 10 shows $\sigma_{Z_{DR}}$ as a function of $N_{inc}$ for an exponential DSD with maximum equivolumetric diameter of 8 mm, at $R = 1, 10,$ and $50$ mm h$^{-1}$, respectively. In Sachidananda and Zrnić (1985) it is also stated that, for the chosen DSD, a standard error in the rain rate less than 25% is guaranteed if $\sigma_{Z_{DR}}$ does not exceed 0.1 dB: according to the results of our simulations, the above requirement is satisfied if at least 20 uncorrelated samples are collected at 1 mm h$^{-1}$, 64 at 10 mm h$^{-1}$, and 128 at 50 mm h$^{-1}$, respectively. A correspondingly very high number of pulses must be transmitted: as an example, with a PRF of 1 kHz and assuming a decorrelation time $\tau_d$ in the order

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<th>Frequency (GHz)</th>
<th>$a_h$</th>
<th>$b_h$</th>
<th>$a_v$</th>
<th>$b_v$</th>
<th>$c_v$</th>
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</table>

Figure 10. Relative standard deviation of $Z_{DR}$ estimates as a function of the number of uncorrelated samples $N_{inc}$, for a Marshall–Palmer DSD at three different rain rates.
of 10 ms, two successive uncorrelated samples occur once every 10 pulses, therefore we need to collect up to 200 samples at 1 mm h$^{-1}$, 640 at 10 mm h$^{-1}$, and 1280 at 50 mm h$^{-1}$.

In the derivation of the above results, we have implemented a polarimetric radar equipped with two separate channels, one for each of the two orthogonally polarized signals, in order to perform simultaneous estimates of $Z_H$ and $Z_V$. Single-channel radars are able to provide polarimetric measurements based on a different technique, which consists of alternate sampling of horizontally and vertically polarized components (Seliga and Brinigi 1976). In Fig. 11 we draw a comparison of the $\sigma_{ZDR}$ values obtained via the two techniques in the case of rain rate $R$ equal to 10 mm h$^{-1}$: subsequences of $N_{inc}$ = 64 uncorrelated samples each have been considered to satisfy the bound $\sigma_{ZDR} < 0.1$ dB for simultaneous sampling (method 1). A single $Z_{DR}$ estimate achieved via the alternate sampling technique (method 2) corresponds to two estimates with method 1. Bound $\sigma'_{ZDR}$ is plotted against the ratio $\tau_j / T_s$, $\tau_j$ being the signal power decorrelation time and $T_s$ the pulse repetition period, respectively. Notice how, as expected, the accuracy of method 2 dramatically decreases as two successive samples become uncorrelated ($\tau_j / T_s = 1$). Curve 2 of Fig. 11 remains close to the asymptotic value of 0.1 dB if $\tau_j / T_s \geq 2$, that is, given $\tau_j$, the alternate sampling technique requires a sufficiently high PRF (PRF being the inverse of $T_s$) in order to warrant the same statistical accuracy as the simultaneous sampling technique.

6. Comparison with measurements

The synthetic signals have been compared with data collected by the S-band meteorological radar located at Spino D’Adda, about 20 km east of Milan, Italy, during several rain events from November 1999 to May 2000. The basic characteristics of the radar are listed in the upper section of Table 2. Radar antenna was sequentially pointed at 10°, 20°, and 90° of elevation, while the azimuth was fixed. For each of the three pointings, sequences of 4096 pulses were recorded at a PRF of 1 kHz. The cycle above was repeated at regular intervals of 15 min in order to detect the evolution of the rain events investigated. The receiver has both a logarithmic and a linear IF chain; LOG, I, and Q outputs were simultaneously available and stored for later offline processing. Radar cells were classified on the basis of the corresponding Doppler spectra after suppression of clutter components (Passarelli et al. 1981). Only cells with measured rain rates $R \geq 0.5$ mm h$^{-1}$ were retained. A large database of simulated signals was created at the three elevations mentioned above, reproducing a large variety of rain-rate conditions, DSDs, and rainfall velocity fields owing to wind and turbulence. Measured and synthetic Doppler spectra were compared through a best-fit procedure.

Figure 12 shows some examples of spectral fit. Doppler velocity is positive for particles moving toward the radar. At vertical elevation, we mainly found asymmetrical curves as that depicted in Fig. 12a, with a decreasing edge that is sharper than the increasing part. They were well simulated assuming raindrops moving only under the action of fall speed, according to Eq. (6). Best matches were always found imposing a gamma DSD whose shape parameter $\mu$ was between −1 and 1. We also detected symmetrical shapes (Fig. 12b): measured spectra were well approximated by adding a normally distributed vertical component of velocity with standard deviation up to 0.8 m s$^{-1}$.

Figures 12c–12f report a sample of spectra at 10° and 20° of elevation: a much higher variability of shapes is noticeable. In Figs. 12c and 12d the shape resembles a Gaussian: in the former the curve is narrow around its maximum; the best fit was obtained considering the fall velocity and a constant horizontal component of wind, directed from the radar to the projection on the ground of the resolution volume midpoint and equal to −3.8 m s$^{-1}$. Note that, if the wind speed is constant, the particles assume the same horizontal velocity as the surrounding air mass, that is, all the raindrops in our simulation had a radial velocity equal to the algebraic sum of wind speed and fall speed radial components. In Fig. 12d we had to include a radial component of drop speed with normal distribution and standard deviation equal to 1.0 m s$^{-1}$, plus an horizontal component of wind equal to −10.7 m s$^{-1}$. Figure 12e depicts a situation where a rectangular distribution (with standard deviation of 2.3 m s$^{-1}$) better fits the data; the constant wind component was here of −4.4 m s$^{-1}$. Finally, Fig. 12f reports the occurrence of an asymmetrical spectrum at 10° of elevation: the asymmetry can be explained by the effect of an horizontal wind profile $U_0(h)$ variable with height according to a nonlinear law. In fact, if we con-
sider that the raindrops are uniformly distributed inside the resolution volume and that the horizontal component due to wind mainly contributes to their Doppler velocity, the Doppler spectrum will exhibit a maximum at the velocity corresponding to the minimum of wind speed gradient with height since a large height interval will be filled with particles with similar velocity (assuming that the particles can follow the variations of wind speed); on the other hand, as the above gradient increases, so does the velocity interval of particles on equal height intervals, and the spectral power density decreases. The Doppler spectrum in Fig. 12f was approximated simulating a wind profile increasing with height with an expression of the form $U_H(h) = C_1(h - h_1)^\beta + U_0(h_1)$, where $C_1 = 1.69 \text{ m}^{-\beta} \text{ s}^{-1}$, $\beta = 0.25$, $h_1 = 1370 \text{ m}$, and $U_0(h_1) = 1.7 \text{ m s}^{-1}$, respectively.
7. Conclusions

A software tool able to generate a synthetic meteorological radar signal has been presented. The simulator has been developed on a physical basis: a radar cell filled with raindrops was reproduced and the associated radar echo was generated adding up the contributions of all the scatterers in the resolution volume. The effects of both antenna directivity function and receiver filter were included.

We have shown that the agreement between simulated and theoretical Doppler spectra is acceptable (mean square error within 10%) when we generate a discretized DSD with at most 10 particles per diameter interval and a number of diameter intervals that is a function both of the PRF and of the wavelength, according to Eq. (11).

Since particle motion influences the behavior of radar signals, we modeled a number of atmospheric effects, that is, fall speed, wind, and turbulence. The software also simulates polarimetric radar operation considering spheroidal raindrops and including differential propagation impairments due to an arbitrary rain profile along the path. All the parameters associated to the models we implemented are tunable by the user.

Two possible simulator applications were also shown. (i) We evaluated the impact of each contribution of particle velocity (fall speed, wind, and turbulence) on signal decorrelation process; and (ii) we investigated the accuracy of differential reflectivity estimates performed by polarimetric radars during normal operational conditions.

Finally, we presented the first results of a comparison between a database of synthetic signals and the data collected by an S-band meteorological radar. By overlapping simulated and measured Doppler spectra we found, in most cases, a good agreement. Since the precipitation parameters of the simulated signals are known, a characterization of rain may be, in principle, derived. Future work will be addressed in the above course.

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