Preliminary Steps in Assimilating SSM/I Brightness Temperatures in a Hurricane Prediction Scheme

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ABSTRACT

Different aspects of assimilating satellite-observed microwave radiances (brightness temperatures) into the initial vortex of a hurricane prediction model are discussed. The tangent linear and adjoint observation operators were developed from a computationally inexpensive and reasonably accurate radiative transfer model. These models have the advantage of being able to perform in all types of weather, including rain. The adjoint radiative transfer model was used to conduct a sensitivity analysis of brightness temperatures to different atmospheric and surface variables. The sensitivities computed by the model compare favorably with physical understandings of how brightness temperatures are affected by the atmosphere and the surface. The errors associated with some of the approximations in the radiative transfer model were estimated from comparisons with a more accurate model. These errors were found to be smaller than estimates from previous studies. The random errors associated with brightness temperature observations were also estimated from statistical structure function calculations and were found to be in line with estimates previously used. The models developed and the errors calculated for this study will be used in future work to assimilate brightness temperatures in hurricane initializations and to evaluate the performance of different microphysical schemes in hurricane prediction.

1. Introduction

Passive microwave remote sensing provides invaluable information on meteorological quantities such as precipitation, specific humidity, and surface ocean wind speeds. However, microwave-sensing instruments such as the Special Sensor Microwave Imager (SSM/I) only measure radiances, or brightness temperatures ($T_b$), which are indirectly related to the quantities used in numerical weather prediction (NWP) models. Unlike conventional observations, such as temperature and pressure, $T_b$s cannot be directly assimilated into NWP models. Early use of SSM/I observations focused on the retrieval of conventional variables from radiances through empirical regression algorithms (Hollinger 1991). Unfortunately, these methods often cannot account for the nonlinear interactions between $T_b$s and the atmospheric state, especially in the presence of hydrometeors. Instead of using retrieved products, Eyre et al. (1993) proposed a method whereby satellite radiances could be utilized in a more direct manner. The basis of this method is a variational assimilation scheme that maps conventional meteorological variables into satellite radiance space using a radiative transfer model (RTM). Phalippou (1996) extended this method to SSM/I observations in order to retrieve humidity profiles, cloud liquid water paths, and surface wind speeds over the ocean. In both studies, the retrieval could only be applied in rain-free areas because the RTM was unable to incorporate precipitation information. Aonashi and Liu (1999) studied the impact of using variational retrieval in mesoscale disturbances by using an RTM developed by Liu (1998) that included hydrometeors (both liquid and ice). All of these studies showed a positive impact on NWP forecasts using the quantities retrieved from the variational scheme. However, they all required a great amount of computational time in order to calculate the gradient of the $T_b$s with respect to conventional meteorological variables (a requirement of the variational scheme) because a perturbation method was used for this calculation. Zou et al. (2001) developed an adjoint RTM that allowed for much faster calculations of this gradient. Zou et al. (2001) developed an adjoint RTM that allowed for much faster calculations of this gradient. However, the model used in Zou et al. (2001) was similar to the model used by Eyre et al. (1993) and Phalippou (1996), so the method could only be used in precipitation-free environments. Still, improvements in the forecast of a tropical system were realized by assimilating $T_b$s surrounding the storm.

In the present study, the adjoint of Liu’s (1998) RTM is developed in an effort to eventually assimilate $T_b$s in areas where there are a large number of hydrometeors, that is, in the middle of a storm. The adjoint RTM is used for a sensitivity analysis to determine which model

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fields are most influential in radiative transfer calculations.

In order to assimilate radiances in a variational framework, information on the errors associated with the observations are necessary. There are two types of errors that need to be determined when using \( T_s \)s. The first is the error associated with the actual \( T_s \) observations. These measurements are assumed to be unbiased, and previous studies using SSM/I \( T_s \)s (Phalippou 1996; Aonashi and Liu 1999) have used the radiometric sensitivities of the SSM/I as estimates of the observational error variances. This study will further examine the observational error variances through the use of structure functions.

The second type of error is that which results from the radiative transfer calculations. There are many sources of error associated with the RTM. Here, we will quantify the error due to an approximation in the radiative transfer calculations that improves the speed of the RTM. However, there are other, potentially larger, sources of error that exist, especially when the model is used in rainy conditions. The source of these errors, as well as their possible estimates, will also be discussed.

Section 2 reviews the variational method used for assimilating \( T_s \)s proposed by Eyre et al. (1993) as well as the SSM/I microwave radiance observations used in this study. Section 3 will briefly touch on the bogus data assimilation (BDA) scheme (Zou and Xiao 2000) that was used to create the NWP model fields used in this study. Section 4 highlights results from adjoint sensitivity analyses, and section 5 investigates the errors associated with both the observations and the RTM. A summary, as well as a discussion of work planned for the future, is provided in section 6.

2. Variational assimilation and SSM/I observations

a. Variational assimilation of brightness temperatures

The variational assimilation method proposed by Le Dimet and Talagrand (1986), and utilized by Eyre et al. (1993) to include satellite observed radiances, minimizes the value of a scalar cost function:

\[
J(x) = \frac{1}{2} [H(x) - y^{obs}]^T [O + F]^{-1} [H(x) - y^{obs}] + \frac{1}{2} (x - x^b)^T B^{-1} (x - x^b),
\]

where \( x \) is a state vector composed of atmospheric and surface variables and \( x^b \) is a background vector usually composed of values taken from a previous forecast. The observed \( T_s \)s are contained in \( y^{obs} \), \( H \) is the RTM that computes \( T_s \)s from the input values of the atmospheric state in \( x \), \( O \) is the estimated error covariance of the \( T_s \) observations, \( F \) is the estimated error covariance of the RTM, and \( B \) is the estimated error covariance of the background field.

The variational scheme is an attractive method because it includes both a priori information, \( x^b \), and error characteristics, \( O, F, \) and \( B \), the inverses of which serve as weightings. In order to assimilate a new type of observation such as SSM/I microwave radiances, \( H, O, \) and \( F \) need to be defined. The error information that is contained in \( O \) and \( F \) is discussed in section 5. Liu’s (1998) RTM, which utilizes the discrete ordinate method with 4 streams for radiative transfer calculations (Liu 1974), is used as the observation operator \( H \). The 4-stream RTM computes a \( T_s \) for any microwave frequency based on the atmospheric temperature \( T_p \), pressure \( p \), relative humidity, cloud water \( q_c \), rainwater \( q_r \), cloud ice \( q_{ic} \), and snow ice \( q_s \), as well as the surface temperature \( T_s \), and the surface wind speed (only used over oceans).

In order to find the best estimate of \( x \) that minimizes the value of \( J(x) \), the gradient of \( J(x) \) is needed. The gradient of the cost function is obtained by differentiating (1) with respect to \( x \):

\[
\nabla J(x) = \frac{\partial H(x)}{\partial x} - \frac{1}{2} (x - x^b)^T B^{-1} (x - x^b),
\]

This is the tangent linear operator of the RTM.

To obtain \( \nabla J(x) \), the gradient of the observation operator with respect to the control variables, the tangent linear operator \( H \), and the adjoint operator \( H^ T \) are also needed. Since the calculation of \( T_s \)s is a one-dimensional problem (the \( T_s \) value at a specified point depends only on the vertical atmospheric profile and the surface conditions at that point) previous studies (Eyre et al. 1993; Phalippou 1996; Aonashi and Liu 1999) have used a simple perturbation method to approximate this gradient. This method entails slightly perturbing each input variable to the RTM separately and then running the RTM in order to calculate the gradient. The computational expense of this method is directly related to the number of levels in the RTM and the number of \( T_s \)s that need to be calculated, so the expense can become relatively large as these factors grow. An adjoint RTM is an alternative and efficient tool for gradient calculations. Using an adjoint model requires only two model integrations (one forward model run and one adjoint model run) to calculate the gradient with respect to all input parameters. The tangent linear and adjoint models of Liu’s (1998) RTM have been developed and tested in order to efficiently calculate \( H \) and \( H^ T \).

b. SSM/I microwave radiance observations for Hurricane Bonnie (1998)

The SSM/I is a seven-channel passive radiometer that is part of the payload on polar-orbiting Defense Mete-
orological Satellite Program (DMSP) satellites (Hollinger et al. 1987). Both vertical and horizontal polarizations are observed at 19.350, 37.000, and 85.500 GHz (channels 19V, 19H, 37V, 37H, 85V, and 85H), while only vertically polarized measurements are recorded at 22.235 GHz (channel 22V). The resolution of the 85-GHz observations is roughly 15 km, while the observations at all other channels have a coarser resolution of 60 km.

Hurricane Bonnie (1998) was chosen as the tropical cyclone of interest for this study. Bonnie reached hurricane strength on 22 August 1998 at 0600 UTC over the tropical Atlantic Ocean. Bonnie achieved its maximum strength on 23 August at 1200 UTC as a category 3 hurricane with a minimum central sea level pressure (SLP) of 954 mb and maximum surface winds of 100 kt. It then weakened slightly before making landfall in Wilmington, North Carolina, on 27 August.

SSM/I observations from three DMSP satellites (F11, F13, and F14) were made available over an 11-day period (19–29 August 1998) for this study. Two swaths over the tropical Atlantic Ocean for each satellite were provided daily (one at approximately 0000 UTC and the other at approximately 1200 UTC).

3. The bogus data assimilation scheme

a. Overview of bogus data assimilation

The goal of this study is to investigate the properties of $T_b$'s in hurricane environments. However, it is often difficult to reproduce such an environment in NWP fields from large-scale analyses. There are not enough quality observations in hurricane regions to fully resolve the structures of the initial vortex with the right position, intensity, and size of an actual hurricane.

The variational BDA scheme has been found to be a promising method for producing a dynamically consistent and conceptually correct initial vortex for hurricane prediction (Zou and Xiao 2000; Zou et al. 2001). The BDA scheme assimilates a bogus SLP field, which is generated from a few observed parameters, in a four-dimensional variational data assimilation (4DVAR) framework. Actual observations such as SSM/I radiances can be incorporated into the BDA procedure.

b. Producing the initial fields

Seven bogus SLP fields were created at 12-h intervals from 0000 UTC 22 August to 0000 UTC 25 August 1998. 4DVAR was performed for each of the seven initial times using the fifth-generation Pennsylvania State University–National Center for Atmospheric Research (NCAR) Mesoscale Model (MM5; Dudhia 1993) Adjoint Modeling System (Zou et al. 1997). The bogus SLP field was fitted to the original analysis in 3-min intervals during a 30-min assimilation window and was assumed to remain constant during the assimilation. All model state variables (i.e., pressure, temperature, wind, mixing ratio, etc.) adjusted to produce a surface low that was similar to the bogus low, while satisfying the dynamical and physical constraint provided by the MM5.

Each BDA field was created on 49 × 49 horizontal grid (30-km resolution) with 27 vertical levels. The BDA fields were then placed on larger grids (69 × 69 × 27 grid points) of the same resolution. Version 2 of the MM5 was run over the larger domain to create 12-h forecast fields. Grell's cumulus parameterization, along with the Blackadar planetary boundary layer scheme and the Goddard explicit moisture scheme, was used in the model integrations. The 12-h forecast fields created by the combination of BDA and MM5 model integrations were used as input for the RTM.

c. BDA results

The initial average track error for the seven forecasts was 23 km (within one model grid space of the actual hurricane center). By 6 h, the track errors had increased to 67 km, and at the 12-h mark the average track error had only risen to 68 km. The average magnitudes of the difference in the observed and forecasted minimum central SLP were 3, 7, and 11 mb for the initial and the 6- and 12-h forecast times, respectively. The differences were generally less than 10 mb in magnitude except for the first two forecast periods (beginning 0000 and 1200 UTC 22 August) when Bonnie was rapidly developing. The average magnitudes of the difference in the forecasted and observed maximum winds were 47, 11, and 13 kt for the initial and the 6- and 12-h forecast times, respectively. The greatest wind errors (as large as 60 kt) occurred at the initial time of each of the seven 12-h forecasts because the BDA scheme was unable to fully correct the winds from the large-scale analysis. By the sixth forecast hour the wind differences were generally less than 20 kt, because the model had adjusted the wind field to the pressure field.

The observed $T_b$'s and those calculated from the 12-h forecast ending 0000 UTC August 25 for the 19V and 85V channels are shown in Figs. 1 and 2. The observed and calculated 19V $T_b$'s compare quite well in both the general pattern and the difference in values. The warmest $T_b$'s are on the eastern side of the storm in both the observations and the forecast, indicating heavier precipitation (section 4) to the east of the eye (the eye is located at 26.9°N, 73.2°W). The difference between the forecasted and observed $T_b$'s is generally less than 40 K.

The differences between the observed and forecasted $T_b$'s in the 85V channel are much worse than in the 19V channel (Fig. 2). The difference between the forecast and observations is greater than 100 K just to the northeast of the eye, which is in the same area of the greatest model-produced snow-ice concentrations (not shown). As is seen in section 4, the calculated $T_b$ for the 85-GHz channels is extremely sensitive to the snow-ice...
concentration, which means that more accurate hydrometeor concentrations, especially snow ice, could reduce this large difference between the forecasted and observed $T_b$ s. Also, the RTM uses a Marshall–Palmer distribution for ice particles and assumes that each ice particle is spherical. This is not necessarily how frozen hydrometeors are distributed or shaped in the real atmosphere, so the assumptions in the RTM account for some of this difference as well.

Furthermore, the resolution of the model also plays an important role in accounting for the difference between the observed and forecasted $T_b$ s at all channels. Even if the model is correctly predicting the hydrometeor fields at 30-km resolution, there is variability in the observed hydrometeor fields on much smaller scales (1–10 km) that will have a considerable effect on the $T_b$ observations that will not be seen in the forecasted $T_b$ s if the hydrometeor fields are assumed homogeneous.
over the model grid space, as is done in the RTM. This can lead to differences on the order of tens of kelvins (Kummerow et al. 1996). More discussion on these errors is provided in section 5b(3). In future work, \( T_b \) observations will be used to evaluate the performance of different explicit moisture schemes on smaller grid spacings.

4. Adjoint sensitivity of the radiative transfer model

a. Formulation

There are a number of ways to determine the sensitivity of a model. Traditional sensitivity analyses involve perturbing a few input parameters of a model and measuring the change in the response of the model. Adjoint models allow for the efficient calculation of the sensitivity of a single response of a model to all the input parameters of the model. More recently, adjoint models have been used in data assimilation problems to determine the sensitivity of the optimal solution of the problem to the input parameters of the model (Zupanski 1995). For this study, an adjoint sensitivity analysis is performed, whereby the sensitivity of the \( T_b \) calculated by the RTM to the input of the RTM will be determined. In the case of the RTM, the response function is defined as

\[
J_a(x) = T_b(\alpha),
\]

where \( T_b(\alpha) \) is the \( T_b \) in the \( n \)th channel and \( x \) is a vector that contains values of atmospheric and surface variables used as input for the RTM: air temperature, pressure, mixing ratio, cloud water, rainwater, cloud ice, snow ice, ground temperature, and sea surface wind speed. The sensitivity of \( J_a \) with respect to \( x \) is defined as

\[
J_{as} = (\nabla J_a)^T \delta x = (\nabla x)^T \delta x,
\]

where \( \delta x \) is a perturbation to the original input vector and \( \nabla x \) is the result of the adjoint model integration with a unit input for the adjoint variable for the \( n \)th channel and zero input for all other channels. The sensitivity of the response function to the \( l \)th element of \( x \) is expressed as

\[
J_{als} = \delta x_l = \delta x_l.
\]

The nondimensional relative sensitivity \( S_a \) (Zou et al. 1993) is defined as

\[
S_a = \frac{J_{als}^{-1}}{J_a}(x_l)^{-1} = \frac{(\nabla J_a)^T x_l}{J_a}.
\]

The magnitude of the relative sensitivity serves as a guide to ranking the relative importance of the different components of the input vector in the computation of the \( T_b \) at a selected channel. A vertical profile of the relative sensitivity of the \( T_b \) to the input variables indicates which variables have the biggest effect on the \( T_b \) and at what levels the effects are greatest.

b. Numerical results

Average relative sensitivities were computed for three different types of atmospheric conditions: clear, rainy, and cloudy. Each grid point from the seven 12-h MM5 forecasts was placed into one of these categories based on the following criteria. If the integrated rainwater exceeded 1.0 mm or the integrated snow ice was greater than 0.5 mm at the grid point, then it was considered rainy. These thresholds were obtained by noting that areas of large \( T_b \) depressions at 19 GHz coincided well with the respective threshold contours for integrated rainwater and snow ice. For all points that were not rainy, if the integrated cloud water was greater than 0.1 mm, then it was placed in the cloudy category. The cloudy threshold was determined by noting that areas of large \( T_b \) excesses at 19 GHz coincided well with the threshold contour of integrated cloud water. All other grid points that did not meet the requirements to be either rainy or cloudy were considered clear. From each category, 49 grid points were picked at random and the sensitivities were computed for the profile at each point, and then the average was computed for each of the three weather conditions. The average profiles of the moisture variables for each weather condition are shown in Fig. 3, the standard deviations are given by the horizontal bars. In clear conditions there are very few hydrometers of any kind. In cloudy conditions there is a maximum \( q_r \) (cloud water) concentration of 0.25 g kg\(^{-1}\) located at 850 mb, and a small amount of \( q_r \) (rainwater) is present in the lower levels. In rainy conditions, the \( q_r \) concentration is greatest (0.7 g kg\(^{-1}\)) from the surface up to 700 mb. The maximum \( q_r \) concentration is higher (at 500 mb) than the maximum in cloudy conditions, and the \( q_s \) (snow ice) concentration is greatest at 300 mb (0.75 g kg\(^{-1}\)), while the \( q_c \) (cloud ice) concentration is much smaller (0.1 g kg\(^{-1}\)) and located slightly higher at 250 mb. As would be expected, the profiles of \( q \) (mixing ratio) show that the rainy points were the moistest and the clear points were the driest. The average temperature (not shown) in the lower atmosphere is warmest in clear conditions and coolest in rainy conditions, but the difference in the mean temperature between the different weather conditions is less than 1 K at every level. Above 600 mb, the rainy atmosphere is as much as 4 K warmer than the clear atmosphere and 3 K warmer than the cloudy atmosphere.

The average relative sensitivities for three channels (19H, 19V, and 85V) are shown in Figs. 4–6. Only the largest atmospheric sensitivities (left-hand plots) are displayed here. For clear conditions the sensitivities to \( T \) and \( q \) are larger than to other variables, for rainy conditions the sensitivities to \( T \) and \( q_r \) are greater than to other variables, and in cloudy conditions the sensitivities to \( T \), \( q_r \), and \( q_s \) are greatest. The sensitivities to the other atmospheric variables were generally an order of magnitude less. In the plots on the right-hand side of the figures the sensitivities to the surface parameters...
Fig. 3. Averaged profiles of $q_r$, $q_s$, and $q_{ci}$ in (a) clear, (b) cloudy, and (c) rainy conditions, and (d) of $q$ in all conditions. Horizontal bars represent the std dev.

($T_s$, $u$, and $v$) are shown. The sensitivities to $T_s$ are much larger than the sensitivities to $u$ and $v$, and in most cases are greater than the sensitivities to any of the atmospheric variables.

Positive sensitivities indicate that an increase in the variable will lead to an increase in $T_b$, while a negative sensitivity will result in a reduced $T_b$ if the variable is increased. The sensitivities to $T$ and $T_g$ are primarily positive, meaning that under these conditions an increase in the air or ground temperature will generally lead to an increased $T_b$. However, there are small negative sensitivities to $T$ that are located near the surface in cloudy conditions, which means that an increase in the air temperature actually leads to a decrease in the $T_b$.

The sensitivities to $q$ (in clear and cloudy conditions)
and to $q_c$ in cloudy conditions are both positive and greatest in the lower levels of the atmosphere where their concentrations are greatest. Increases in $q$ and $q_c$ lead to increased absorption and emission by the liquid droplets in the atmosphere, which emit more microwave radiation than the ocean surface, leading to an increased $T_b$. The sensitivity to $q_s$ is negative and has its greatest magnitude in the upper atmosphere where the $q_s$ concentration is largest. Microwave radiation is scattered by ice crystals, which results in a smaller $T_b$ when the $q_s$ concentration increases. The sensitivities to $q_r$ (not shown) are mostly positive at 19 and 37 GHz, which means that the raindrops are emitting radiation. At 85 GHz the sensitivities to $q_r$ are mostly negative, which means that the raindrops are scattering radiation at this frequency.
For the 19H channel (Fig. 4) the largest averaged sensitivity of $T_s$ to any variable is to $T_a$ in all weather conditions, although in rainy conditions the sensitivity to $T$ around 700 mb is almost as large as the sensitivity to $T_a$. The same is also true for the 19V channel (Fig. 5), although the ratio of the sensitivity to $T_a$ to the sensitivity to any other variable is much greater in clear-and cloudy-sky conditions than it is for the 19H channel.

The same patterns are seen in the 22- and 37-GHz channels (figures omitted), where the greatest sensitivity is to $T_a$, and for the 37-GHz channels the vertically polarized channel has a larger ratio of the sensitivity to $T_a$ to the sensitivity of any other variable when compared to the horizontally polarized channel.

The sensitivity results also show that 19V $T_s$s are less sensitive to the surface wind speed than are the 19H
$T_o$s (Figs. 4 and 5). This was also seen in the 37- and 85-GHz channels. This sensitivity of the model is consistent with observations (Hollinger 1971; Swift 1974) that show horizontally polarized microwave radiation being more sensitive to ocean surface wind speeds than vertically polarized microwave radiation.

For the 85-GHz channels, the sensitivity is again largest for the ground temperature in clear- and cloudy-sky conditions (Fig. 6). However, in rainy conditions, the magnitude of the sensitivities to the air temperature and to the snow-ice concentrations are both greater than the sensitivity to the ground temperature. The greatest sensitivities to the air temperature are also located higher in the atmosphere, where the ice concentrations are greatest, than they are for the other channels. Also, the ratio of the magnitude of the sensitivity to $q_s$ to the
magnitude of the sensitivity to any other variable is considerably greater for the 85-GHz channels than it is for any of the other channels.

Brightness temperatures are thought to also be largely sensitive to liquid precipitation, especially for the lower channels (19V/H and 37V/H), but the sensitivities to $q_r$ in rainy conditions are much smaller than the sensitivities to $T$. However, the variability in the $q_r$ fields is much larger than in the $T$ fields. Therefore, a realistic change in $q_r$ (of the order of 100%) could produce as much or more of a change in the $T$, than could a realistic change in the $T$ field (of the order of only a few percent). The same is also true for the other hydrometeor fields. Even though the sensitivities to these fields are smaller than the sensitivity to $T$, realistic changes in these variables can create larger changes in the $T$, than can realistic changes in $T$.

The sensitivities of the model are largely what would be expected from physical understanding of radiative transfer in the atmosphere. Therefore, this model is well representing those characteristics of radiative transfer and is an adequate observation operator to be used in a data assimilation scheme.

5. Error characteristics of SSM/I observations and the radiative transfer model

In order to assimilate $T_i$s in a variational scheme, the statistical characteristics of the errors associated with the actual radiance observations (observation errors), as well as the errors that occur when mapping conventional meteorological variables to $T_i$ space (model errors), need to be known. In this section, we will first investigate the errors associated with SSM/I radiance observations by calculating the structure functions for these values. Then we will focus on the errors that arise from 4-stream approximation in the RTM calculations for a selected sample of atmospheric profiles and surface conditions. However, the error associated with this approximation is not the only and probably not the largest error associated with RTM. Other sources of error in the RTM will also be discussed.

a. Observational errors

Previous studies (Phlipppou 1996; Aonashi and Liu 1999) have assumed that SSM/I radiance observations are unbiased and that the radiometric sensitivities of the SSM/I can be used as estimates of the rms error of the observations. Here, the observations will be assumed unbiased; however, structure functions will be utilized to determine new estimates of the rms errors of SSM/I observations for each channel.

The structure function for SSM/I $T_i$s is defined as (Gandin 1963; Hillger and Vonder Haar 1979)

$$b(\rho_i) = \frac{\sum_{i,j \neq i} [T_i(r_i) - T_j(r_j)]^2}{\sum_{i,j \neq i} 1},$$

where $\rho_i$ is the structure function of the $i$th channel, $T_i(r)$ is the observation at point $r$, and $b(\rho_i)$ is the average value of the structure function for all pairs of points $r_i$ and $r_j$ separated by a distance within the interval. The above expression assumes that the field of $T_i$ deviations from a spatial average is both homogeneous and isotropic so that $b$ can be expressed as a function of separation distance $\rho$.

Gandin (1963) showed that the uncertainty in the measurement can be estimated using the following formula:

$$b^*(\rho) = b(\rho) + 2\sigma^2,$$

where $b^*(\rho)$ is the structure function calculated from actual observations that contain random errors. The variance of the random errors in the measurements is expressed as $\sigma^2$. When $\rho$ approaches zero, $b(0) = 0$ and $b^*(0) = 2\sigma^2$. This means that the measurement uncertainty can be estimated by extrapolating the calculated structure function values to a separation distance of zero. Here, it is assumed that the structure function flattens out as $\rho$ approaches zero, so that the structure function value calculated at the smallest separation distance (25 km) will be equal to twice the maximum estimate of $\sigma^2$.

In calculating the structure functions, much of the procedure of Hillger and Vonder Haar (1979) was followed. The separation distances were grouped into 50-km intervals (1–50, 51–100 km, etc.), and the value calculated was set at the middle of the interval (i.e., the first value is set at 25 km). For this study, data were not available over the same area for the time period investigated, so different 10° latitude by 10° longitude boxes were chosen for each of 11 days of data over the tropical Atlantic basin (19–29 August 1998). Table 1 lists the areas for each of the DMSP satellites (F11, F13, and F14) that were chosen for the analysis. Each box was chosen in an effort to maximize the amount of area of the box covered by SSM/I observations. In order to limit the amount of variation in the function values at small distances a rain flag (Colton and Poe 1999) was applied to the data to remove values in areas of precipitation. At each scan position, if the following conditions were met, $T_{b19V} - T_{b37H} > 50$ K, $T_{b19V} < 185$ K, and $T_{b37H} < 210$ K, then the $T_i$s were kept in the analysis; otherwise they were removed.

Figure 7 shows the structure functions for each channel and for each of the three satellites. The structure functions show similar patterns for each satellite, but the values at larger separation distances show a great deal of variation between satellites. The values differed by as much as 50 K² for the horizontally polarized channels. In order to determine the validity of the values calculated, the same procedure was repeated using only the first 5 days of data. The patterns were similar to those seen in Fig. 7.

The basic pattern seen in the structure functions is that the vertically polarized channels (except 22V) show much less structure than the other channels (the hori-
zontally polarized channels and the 22V channel). By
removing areas of precipitation from the analysis, areas of large variability of the $T_b$ in the 19V, 37V, and 85V channels are also removed because of the large sensitivity of these channels to the atmospheric temperature and snow-ice concentrations in rainy areas (section 4). On the other hand, the 22V channel is sensitive to water vapor, and areas of varying water vapor concentration remain after the rain flag has been applied to the data, which explains why the structure functions are much greater for the 22V channel than they are for the 19V, 37V, and 85V channels. Furthermore, the horizontally polarized channels are more sensitive to surface wind speeds and atmospheric variables, while the vertically polarized channels are most sensitive to the ground temperature. This leads to more structure in the horizontally polarized channels (19H, 37H, and 85H) than in the vertically polarized channels (19V, 37V, and 85V).

Estimates of $\sigma$ are given in Table 2 as well as the radiometric sensitivity provided by the manufacturer (last column, from Colton and Poe 1999). As mentioned above, the estimates were also computed for only the first 5 days of data and were close to the values calculated (within 20%) when all 11 days of data were utilized. Overall, the estimated values are slightly larger than the radiometric sensitivities. In addition, the estimates changed very little between satellites, except for the 22V and 37V channels of the F11 satellite, which differed by more than 1 K from the F13 and F14 satellites. Those channels that have the smallest structure function values (19V, 37V, 85V) also have relatively smaller estimates of the uncertainty of the measurement. Also, these estimates have been made by assuming that the structure function becomes flat at the smallest separation distance, when in actuality it most likely decreases, which would further reduce the estimates. The differences between these estimates and the radiometric sensitivities can also be attributed to the variability caused by the surface wind speed, the water vapor gradients, and possibly by areas of precipitation that were not removed by the rain flag. In Hillger and Vonder Haar (1979) those channels that were sensitive to surface conditions and water vapor gave estimates of the error that were larger than the manufacturer-specified sensitivity. Therefore, SSM/I observation errors are well represented by the radiometric sensitivities of the instrument.

b. Errors in the RTM

The errors associated with $T_b$ observations have been discussed; however, the errors associated with the observation operator must also be considered because of large differences between the observed and calculated $T_b$s (section 3c) caused by the errors in the RTM. The ideal situation for assessing the errors associated with the RTM would be to compare observed $T_b$s with $T_b$s calculated from the RTM using conventional meteorological observations, such as radiosonde data, as input to the RTM, as was done in Eyre (1992). Unfortunately, there are very few conventional observations over oceans where hurricanes are located, and the RTM used in this study requires more than just conventional observations as input, namely, cloud/rainwater and cloud/snow ice are also needed. Therefore, NWP model fields are used as input to the RTM. However, observed $T_b$s cannot be used to assess the errors of the RTM because the errors will also include errors associated with the NWP model fields. This especially becomes a problem in rainy areas, where it is difficult to ascertain the true atmospheric state. Differences between observed $T_b$s and those calculated from the RTM can be greater than 100 K in these areas (section 3c). Instead, the results of the simplified RTM will be compared with a more complicated and accurate RTM, as was done in Zou et al. (2002) for GPS/Meteorology (MET) bending angle data. Here $T_b$s calculated from the 4-stream RTM are compared with the results from a 32-stream RTM to determine both the bias and the error variance of the 4-stream RTM. Therefore, only the error associated with the differences between the 32- and 4-stream models is being estimated. The differences in the two models are that the 4-stream RTM determines the scattering source term using a 4-stream discrete ordinate solution and a Heneyy–Greenstein scattering phase function (cross-polarization scattering is neglected), while the 32-stream

### Table 1. Areas used to calculate the structure functions.

<table>
<thead>
<tr>
<th>Date</th>
<th>Lat</th>
<th>Lon</th>
<th>Lat</th>
<th>Lon</th>
<th>Lat</th>
<th>Lon</th>
</tr>
</thead>
</table>
RTM determines the scattering source term using a 32-stream discrete ordinate solution and a Mie scattering phase function (Liu 1998).

1) Bias

As mentioned in section 2, the relatively few number of streams in the 4-stream RTM does not fully account for the amount of radiation that is being scattered, meaning that 4-stream $T_s$, $T_s^{4}$, will generally be less than $T_s$ computed from an RTM with a higher stream number, $T_s^{32}$. Figure 8 shows the difference between the 32-stream and 4-stream calculated $T_s$ [$T_s^{32} - T_s^{4}$]. The 12-h MM5 forecast ending at 0000 UTC August 24 was used as input for both RTMs. There are two areas where the differences are significant [$|T_s^{32} - T_s^{4}| > 0.5$ K]. Both of these areas coincide with the areas of maximum integrated rainwater in Fig. 9. The situation is
TABLE 2. Estimated rms errors (in K) for SSM/I $T_b$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$F_{11}$</th>
<th>$F_{13}$</th>
<th>$F_{14}$</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>19V</td>
<td>0.79</td>
<td>0.82</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>19H</td>
<td>1.19</td>
<td>1.16</td>
<td>1.29</td>
<td>0.80</td>
</tr>
<tr>
<td>22V</td>
<td>1.22</td>
<td>1.05</td>
<td>2.95</td>
<td>0.80</td>
</tr>
<tr>
<td>37V</td>
<td>1.85</td>
<td>0.89</td>
<td>0.91</td>
<td>0.60</td>
</tr>
<tr>
<td>37H</td>
<td>1.57</td>
<td>1.54</td>
<td>1.63</td>
<td>0.60</td>
</tr>
<tr>
<td>85V</td>
<td>1.82</td>
<td>1.17</td>
<td>1.28</td>
<td>1.10</td>
</tr>
<tr>
<td>85V</td>
<td>3.03</td>
<td>2.27</td>
<td>2.63</td>
<td>1.10</td>
</tr>
</tbody>
</table>

similar for the other channels; in areas of high integrated rainwater and snow ice, the 4-stream model consistently underpredicts $T_b$.

Eyre (1992) proposed a method for removing biases in calculated radiances that was partly airmass dependent (i.e., for different atmospheric conditions, different bias values were removed from the data). The same approach is undertaken here. The differences between the 32-stream and 4-stream RTMs were calculated at each grid point over a $69 \times 69$ grid for four different time periods (1200 UTC on 22, 23, 24, and 25 August). The bias in rainy areas was then determined by taking the average of all differences at grid points where the integrated rainwater was greater than 1.0 mm and/or the integrated snow ice exceeded 0.5 mm. The values of these thresholds were obtained by noting that areas of large $T_b$ differences were outlined by contours of the integrated rainwater and snow-ice thresholds. At all other grid points the bias was considered negligible (the average difference between the RTMs was less than 0.05 K for every channel). The bias corrections to be added to 4-stream $T_b$s in rainy areas are given in Table 3.

The bias corrections in Table 3 were then added to the 4-stream $T_b$s calculated from an independent 12-h MM5 forecast ending at 0000 UTC 24 August (these $T_b$s were not used in the calculation of the bias correction). The difference between the 32-stream and the corrected 4-stream $T_b$s are shown in Fig. 10 (this figure can be compared with Fig. 8 to see the effects of the bias correction). Overall, the negative bias of the 4-stream model has been reduced. The absolute difference has generally been reduced to less than 1 K. However, in a small area near 24°N, 71°W, the integrated rainwater and snow-ice thresholds were too large, so the bias correction was not applied in an area of relatively large bias. A number of bias estimates were calculated using

![Fig. 8. Difference between the 32-stream and 4-stream T_bs for the 85V channel. Contoured at -0.1, 0.1, 0.4, 0.8, 1.2, 1.6, and 2.0 K.](http://journals.ametsoc.org/jtech/article-pdf/20/8/1154/3319287/1520-0426(2003)020_1154_psiaib_2_0_co_2.pdf)
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Fig. 9. The 12-h forecast of integrated rainwater (solid contour every 1.5 mm) and SLP (dashed contour every 4 mb) valid at 0000 UTC 24 Aug 1998.

Table 3. Bias corrections (in K) for the 4-stream RTM.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>19V</td>
<td>0.66</td>
</tr>
<tr>
<td>19H</td>
<td>0.75</td>
</tr>
<tr>
<td>22V</td>
<td>0.45</td>
</tr>
<tr>
<td>37V</td>
<td>0.78</td>
</tr>
<tr>
<td>37H</td>
<td>0.85</td>
</tr>
<tr>
<td>85V</td>
<td>1.36</td>
</tr>
<tr>
<td>85H</td>
<td>1.33</td>
</tr>
</tbody>
</table>

different combinations of the time periods available. In each case the estimate for each channel varied by less than 0.1 K from the values listed in Table 3.

2) Variance

Aonashi and Liu (1999) estimated the rms errors of the 4-stream RTM to be 5 K for the 85-GHz channels and 3 K for the other channels, based on the results of Liu (1998). However, the results from the bias correction suggest that these estimates might be too high, especially once the bias has been removed. The rms errors

Fig. 10. Difference between the 32-stream and 4-stream $T_b$s for the 85V channel after the bias corrections have been added to the 4-stream $T_b$s. Contoured at $-0.8$, $-0.4$, $-0.1$, $0.1$, $0.4$, $0.8$, and $1.2$ K.
of the bias-corrected 4-stream $T_b$s were calculated for the three time periods not used in the bias-correction calculations (0000 UTC on 23, 24, and 25 August). The results are given in Table 4.

The rms errors were calculated for all grid points and then separately in rainy and nonrainy areas. The rainy and nonrainy areas were defined using the threshold-integrated rainwater and snow-ice values from above. The rms errors were also calculated in the rainy areas without removing the bias. Those values are listed in the last column of Table 4.

The rms errors in rainy areas of the uncorrected $T_b$s are slightly larger than the bias estimates, showing that much of the error in the 4-stream model can be attributed to the bias. However, even with the biases removed from the 4-stream $T_b$s, the rms errors are greater in rainy areas than they are in nonrainy areas. The estimated rms errors for all grid points are closer to the values of the nonrainy rms errors than they are to the rainy rms errors, mainly because there are many more nonrainy points than rainy points used in the calculation (13,587 nonrainy points versus 285 rainy points). These results indicate that separate estimates of the error variances should be considered for both rainy and nonrainy areas, or the estimate for the rainy rms error could be used as a maximum estimate of the rms error for all areas.

3) OTHER ERROR SOURCES

There are two other potentially large sources of error that arise in rainy conditions that were eluded to in section 3c. One source of error results from the plane-parallel assumption in the RTM. The RTM assumes that the NWP fields are horizontally homogeneous over the model grid. In actuality, fields such as rainfall can be highly variable over model grids, which can lead to large differences between observed and calculated $T_b$s. These differences have been estimated to be on the order of tens of kelvins (Kummerow et al. 1996).

Another possibly large source of error is due to assumptions made on the particle size distributions for both rain and snow. The RTM assumes that both precipitating liquid and ice are spherical and follow Marshall–Palmer distributions. McKague et al. (1998) estimated the rms error in the 19-GHz $T_b$ to be 4.9 K when the rainfall was assumed to follow a Marshall–Palmer distribution. We are unaware of any studies that have estimated the error associated with assuming a Marshall–Palmer distribution for ice particles, but it is expected that these errors would be larger than errors associated with rainfall and would be greatest for the 85-GHz channels.

The two errors discussed above are likely to be correlated between channels and in space, meaning that the forward model (RTM) error covariance matrix would be nondiagonal. Assimilation schemes usually make use of simplifying assumptions, such as that the spatial errors are homogeneous and isotropic, in order to reduce the burden of computing full error covariance matrices. The same approach would likely be needed to assimilate microwave $T_b$s in rainy environments because some information from the off-diagonal elements is necessary for the assimilation scheme to distinguish between differences due to errors in the model fields and those due to the uncertainty in the RTM, but it would be too cumbersome to compute the entire error covariance matrix and its inverse.

6. Summary

Some of the many considerations that must be dealt with when assimilating indirect observations into an NWP model have been discussed in this work. The tangent linear and adjoint RTMs were developed so that the gradient of $T_b$s with respect to conventional meteorological variables can be calculated efficiently. These models were shown to give correct results for a selected sample of atmospheric and surface conditions. In formulating these models, a 4-stream RTM was chosen as the observation operator because of its ability to estimate $T_b$s in rainy environments because some information from the off-diagonal elements is necessary for the assimilation scheme to distinguish between differences due to errors in the model fields and those due to the uncertainty in the RTM, but it would be too cumbersome to compute the entire error covariance matrix and its inverse.

Estimates of the errors associated with $T_b$s were also calculated. The observational error was found to be in line with values that had been used in previous studies. The bias of the 4-stream RTM was estimated based on the type of weather (rainy versus not rainy). With the bias removed, the estimated error variance associated with the 4-stream approximation was found to be much smaller than that found in previous studies.

The results of this work will be used in future studies to directly assimilate SSM/I radiances into NWP forecasts. However, there are still some considerations that must be addressed. As was shown in this study, $T_b$s are sensitive to hydrometeor concentrations, and the differences between observed $T_b$s and those calculated from model fields are still quite large in rainy areas. Therefore, it is important to have an accurate depiction of the hydrometeor concentrations in order to assimilate $T_b$s in the middle of the hurricane. In work planned for the future, $T_b$ observations will be used to evaluate the performance of different microphysical schemes used...
in NWP. Also, sensitivity analyses will be conducted for some of these schemes to determine which parameters most influence the calculation of hydrometeor concentrations. Hopefully, once these tasks have been completed the actual impact of the assimilation of $T_b$ s in hurricane prediction can be investigated.

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